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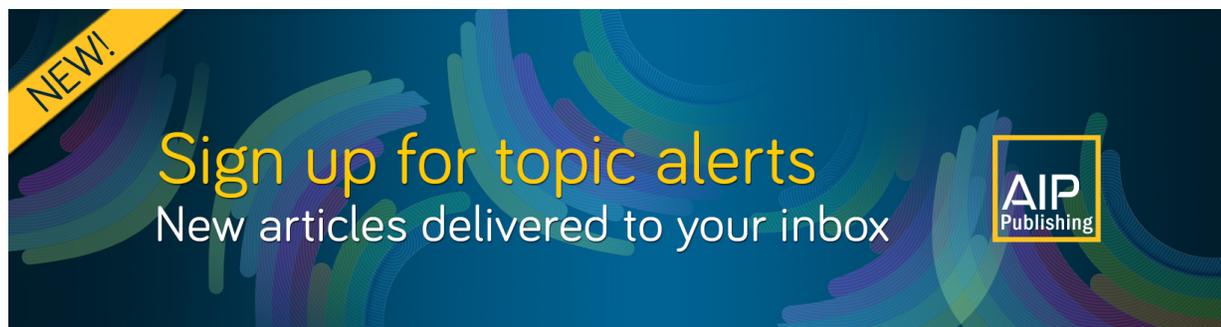
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ABSTRACT

Chimera states are spatiotemporal patterns in which coherent and incoherent dynamics coexist simultaneously. These patterns were observed in both locally and nonlocally coupled oscillators. We study the existence of chimera states in networks of coupled Rössler oscillators. The Rössler oscillator can exhibit periodic or chaotic behavior depending on the control parameters. In this work, we show that the existence of coherent, incoherent, and chimera states depends not only on the coupling strength, but also on the initial state of the network. The initial states can belong to complex basins of attraction that are not homogeneously distributed. Due to this fact, we characterize the basins by means of the uncertainty exponent and basin stability. In our simulations, we find basin boundaries with smooth, fractal, and riddled structures.

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Chimera states are spatiotemporal patterns in which the dynamics are simultaneously composed of coherent and incoherent domains. They have been observed in mathematical models of networks, e.g., coupled Kuramoto phase oscillators and networks composed of Landau–Stuart oscillators. Experimental evidences of chimera states were reported in mechanical oscillator networks, coupled optoelectronic oscillators, and chemical setups. We study chimera states in a network of coupled Rössler oscillators. The Rössler model is a set of ordinary differential equations that can exhibit chaotic dynamics. Circuits based on the Rössler system have been used in secure communication. In this work, we find basins of attraction for chimera states; i.e., we compute the sets of initial conditions that go to chimera states. We calculate the relative sizes of these basins of attraction for chimera using the basin stability. Furthermore, we characterize

the basin boundaries between the coexisting spatial states in the phase space. By calculating the uncertainty exponent, we show the existence of smooth, fractal, and riddled basin boundaries.

I. INTRODUCTION

Mathematical models of oscillators have been employed to describe the dynamical behavior of various types of complex systems, such as electronic circuits,¹ circadian clocks,² and oscillatory chemical systems.³ Zou *et al.*⁴ reported complex chaotic chemical reactions in electrochemical oscillators. In experimental studies, chaotic oscillators were implemented in secure communication systems.⁵

In the 1970s, Rössler⁶ introduced a three-dimensional dynamical system composed of one nonlinear and two linear equations. The Rössler system is a classical chaotic oscillator in the field of nonlinear dynamics and has been used to investigate, among other things, phase synchronization,⁷ chaotic synchronization,⁸ communication scheme based on coupled chaotic circuits,⁹ chaotic signals in electronic circuits,¹⁰ and spatial coherence in networks.¹¹

Networks of coupled oscillators have been considered to study synchronization in power grids,^{12,13} neuronal oscillations,¹⁴ and control of networks.¹⁵ Synchronization transitions in coupled Rössler oscillators were studied by Rosenblum *et al.*¹⁶ They reported transition from a nonsynchronous state to phase synchronization and transition to lag synchronization.

In 1989, Umberger *et al.*¹⁷ found coexistence of periodic and chaotic regimes in a dispersively coupled chain of oscillators. The investigation of this type of spatiotemporal dynamics was not further noticed until 2002 when Kuramoto and Battogtokh¹⁸ reported

patterns with coexisting coherence and incoherence domains in nonlocally coupled Ginzburg–Landau oscillators. In 2004, these patterns were named chimera states by Abrams and Strogatz.¹⁹ Since then, many researchers have experimentally observed the existence of chimera states in coupled mechanical systems,²⁰ opto-electronic oscillators,²¹ and chemical oscillators.²² Santos *et al.* found chimeras in neuronal models based on the cat cerebral cortex²³ and in coupled adaptive exponential integrate-and-fire neurons.²⁴ They identified two different chimera patterns according to the desynchronized spikes and bursts.

The coexistence of coherence and incoherence domains in spatiotemporal patterns was also observed in networks of coupled Rössler oscillators.²⁵ In 2011, Omelchenko *et al.*²⁶ reported that chaotic chimera states arise in nonlocally coupled Rössler systems. They identified a mechanism for the coherence–incoherence transition²⁷ that occurs in networks with nonlocal connections. In 2014, Chandrasekar *et al.*²⁸ demonstrated a mechanism for intensity-induced chimera states in globally coupled Rössler oscillators, in which the coexistence depends on the initial state. Recently, chimera states were found in a network of chaotically oscillating Rössler systems with small-world topology and randomly switching nonlocal connections.²⁹ An experimental verification of chimeras states was shown by Meena *et al.*³⁰ in a circuit implementation of a chaotic Rössler-type oscillator. Considering star networks, they also computed the basin of attraction and found that it has greater prevalence of chimeras. Ghosh and Jalan³¹ demonstrated a method to engineer a chimera state by means of a distribution of heterogeneous time delays on the edges. They suggested the spatial inverse participation ratio as a measure to identify chimera states.

The initial conditions play a crucial role in the emergence of chimera states.³² With this in mind, many researchers analyzed

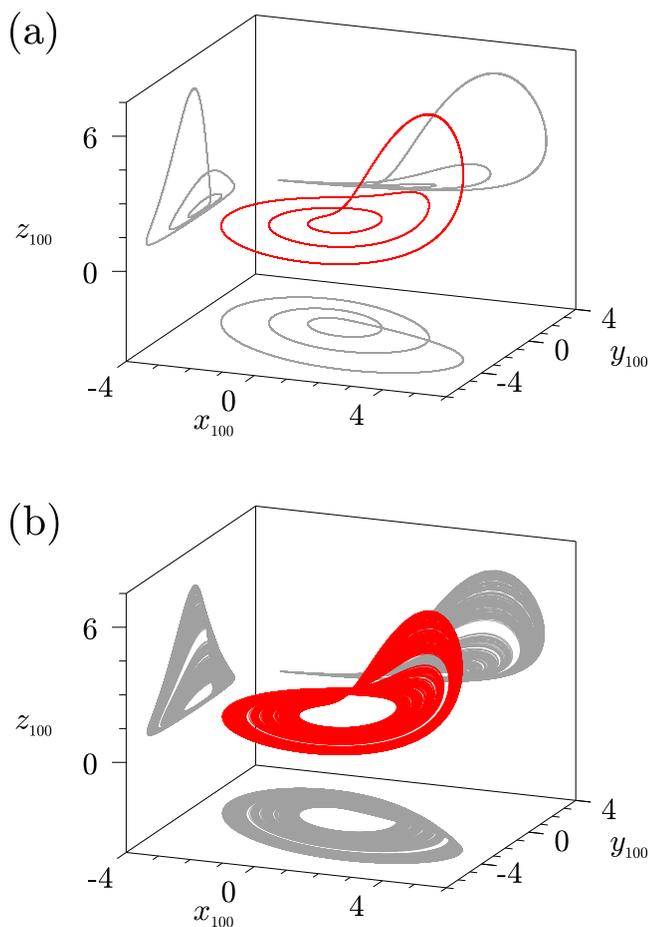


FIG. 1. Trajectories of the $i = 100$ Rössler system for $a = 0.42$, $b = 2$, $c = 3.9$, $N = 200$, and $\sigma = 0.05$. (a) and (b) display periodic and chaotic behavior, respectively.

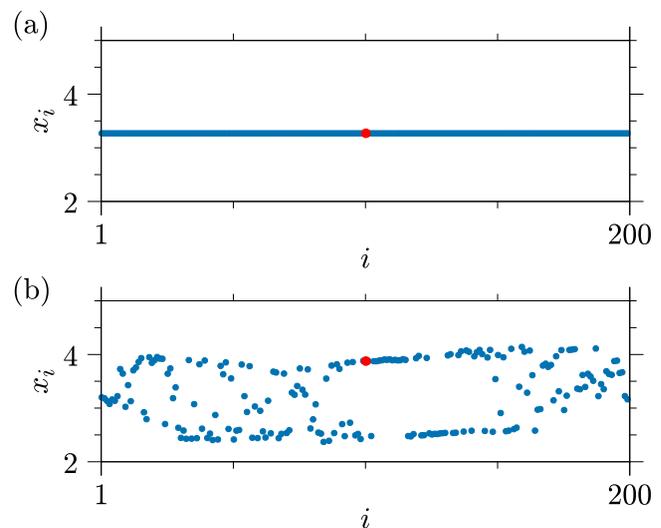


FIG. 2. Snapshot of (a) synchronized and (b) desynchronized states for $\sigma = 0.05$ and $r = 0.28$. We consider $a = 0.42$, $b = 2$, $c = 3.9$, $N = 200$: (a) oscillators are initialized with the same initial conditions and (b) different initial conditions. The red circle corresponds to the oscillator shown in Fig. 1.

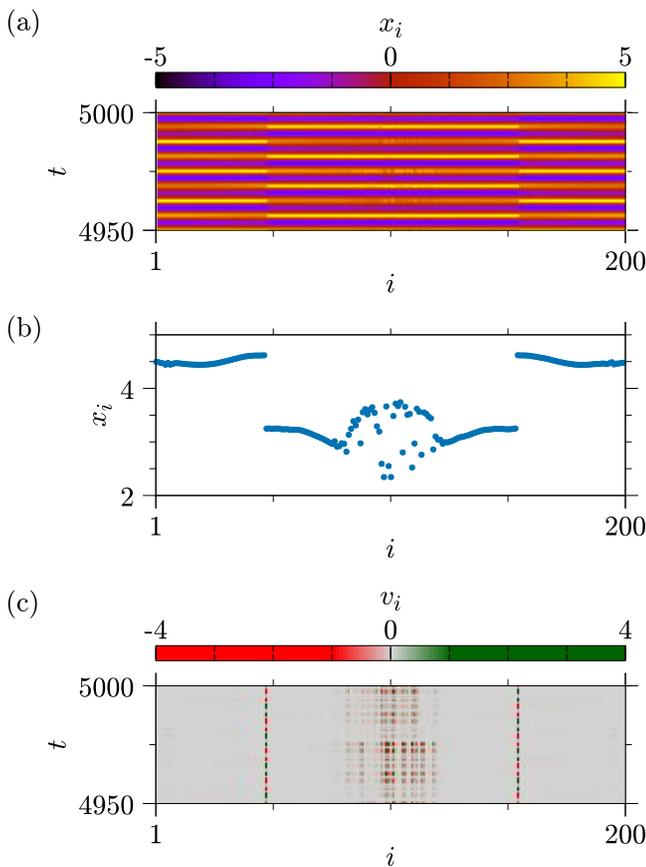


FIG. 3. (a) Space-time plot of a chimera state for $N = 200$, $a = 0.42$, $b = 2$, $c = 3.9$, $\sigma = 0.05$, and $r = 0.28$, where the color bar corresponds to the value of x_i . (b) Snapshot of the chimera state. (c) Space-time plot of v_i .

the basin of attraction for chimera states, namely, the set of initial condition that goes to chimera states. Martens *et al.*³³ considered two populations of Kuramoto–Sakaguchi oscillators and investigated the basins of attraction for states in which one population was synchronized, while the other was desynchronized. A network of nonlocally coupled Mackey–Glass systems was used to explore the basin stability related to the coexistence of domains.³⁴ In 2018, Santos *et al.*³⁵ determined the basins of attraction of spatial states observed in a network of coupled Hénon maps. The basins were separated into sets of initial conditions that led to coherent, incoherent, and chimera states. Through the uncertainty exponent, they found fractal and riddled basin boundaries. In dissipative systems with more than one attractor, the basin is said to be riddled when it is punctured with holes that belong to the basins of other attractors. Some mathematical conditions are required to identify riddled basin.³⁶ On the other hand, it has been often the case that, when the uncertainty exponent is about zero, the basins are also called riddled.

In this work, we study the basin of attraction for chimera states in a network of coupled Rössler oscillators. In our simulations, we find coherent and incoherent domains composed of synchronized and desynchronized systems, respectively. We show that the boundary between the basin of spatially coherent and chimera states can be smooth or fractal. We also observe basins with riddled structures, where there are points that belong to chimera and incoherent states. The basins of attraction for chimera states in a network of coupled Rössler oscillators have characteristics similar to the ones observed in a network of coupled Hénon maps.³⁵ Meena *et al.*³⁰ considered star networks and computed the basin of attraction. They found a greater prevalence of chimeras states in the basin of attraction. Depending on the parameters of our network, we also identify large regions in the basin of attraction in which the initial conditions go to chimera states.

This paper is organized as follows. In Sec. II, we introduce the network of coupled Rössler oscillators. Section III shows the

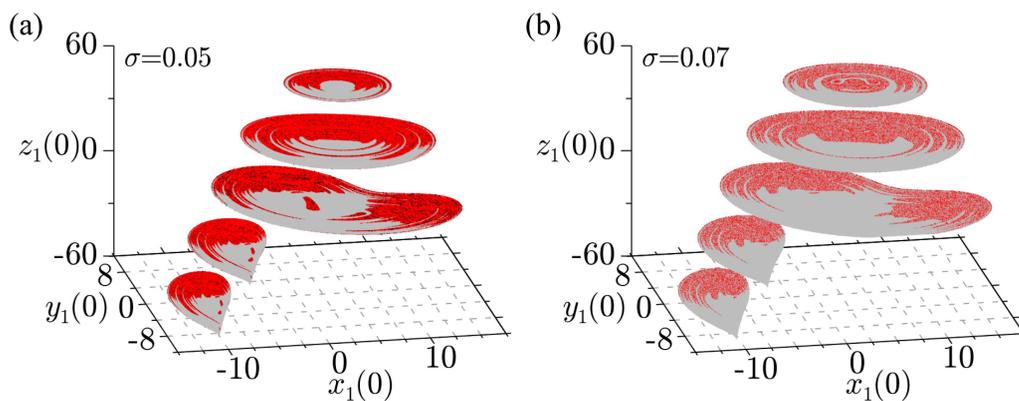


FIG. 4. Basin of attraction of Eq. (1) for five slices $[z_1(0) = -60, -30, 0, 30, 60]$, where all oscillators have initial conditions equal to 0 except for the oscillator $i = 1$. The coherent, chimera, and incoherent states are denoted by gray, red, and black colors. We consider (a) $\sigma = 0.05$ and (b) $\sigma = 0.07$.

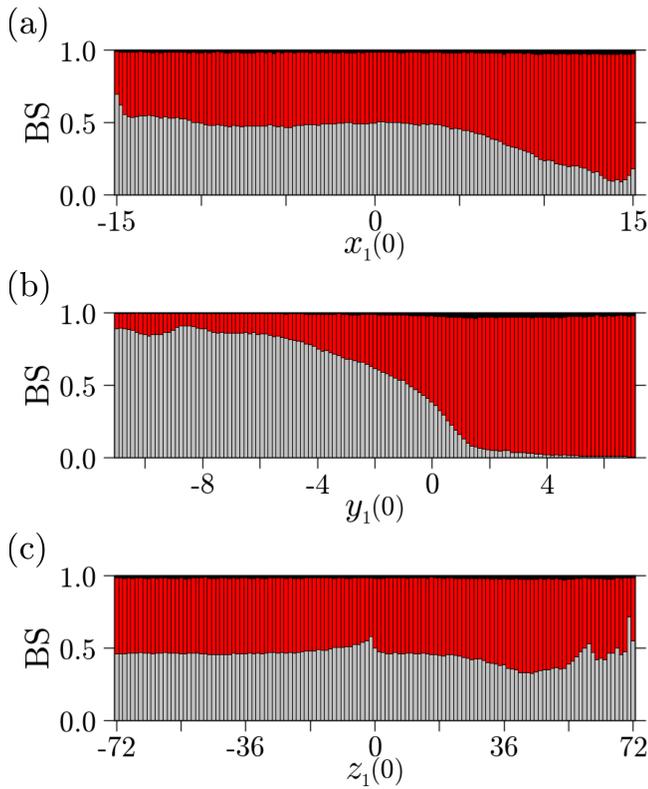


FIG. 5. Fraction of initial conditions that converges to each state as a function of (a) $x_1(0)$, (b) $y_1(0)$, and (c) $z_1(0)$ for $\sigma = 0.05$. Coherent states in gray color, chimera states in red color, and incoherent states in black color.

existence of chimera states in the coupled Rössler systems and how the spatiotemporal patterns are identified. In Sec. IV, we calculate the basins of attraction and characterize the boundaries between the spatiotemporal patterns. We draw our conclusions in Sec. V.

II. NETWORK OF RÖSSLER OSCILLATORS

Aiming to investigate coherence and incoherence domains, as well as chimera states, we consider a network composed of N Rössler oscillators coupled according to a ring topology, which is given by

$$\begin{aligned} \dot{x}_i &= -y_i - z_i + \frac{\sigma}{2rN} \sum_{j=i-rN}^{i+rN} (x_j - x_i), \\ \dot{y}_i &= x_i + ay_i + \frac{\sigma}{2rN} \sum_{j=i-rN}^{i+rN} (y_j - y_i), \\ \dot{z}_i &= b + z_i(x_i - c) + \frac{\sigma}{2rN} \sum_{j=i-rN}^{i+rN} (z_j - z_i), \end{aligned} \quad (1)$$

where $i = 1, \dots, N$ is the index of each oscillator. a , b , and c are the parameters of the Rössler oscillator, and r and σ are the coupling radius and the coupling strength, respectively. In this work, we consider $N = 200$ and $c = 3.9$. Omelchenko *et al.*²⁷ verified the existence of chimera states in a network of coupled Rössler oscillators for $c = 4$ and different values of r and σ .

In Fig. 1, we plot the numerical trajectory of the oscillator $i = 100$ [Eq. (1)] for $r = 0.28$ and $\sigma = 0.05$. Figures 1(a) and 1(b) display periodic and chaotic attractors, respectively. The periodic attractor emerges from a synchronized behavior of the network when we consider the same initial conditions for all oscillators, as shown in Fig. 2(a). For different initial conditions, all oscillators exhibit chaotic behavior [Fig. 2(b)]. The red circle corresponds to the trajectories of the oscillator $i = 100$, as shown in Fig. 1.

III. CHIMERA STATES

In order to identify the chimera states, we use the strength of incoherence that was proposed by Gopal *et al.*³⁷ To calculate the strength of incoherence, we separate the network into M boxes with $n = N/M$ oscillators and compute

$$s_m = \Theta[\delta - \chi(m)], \quad (2)$$

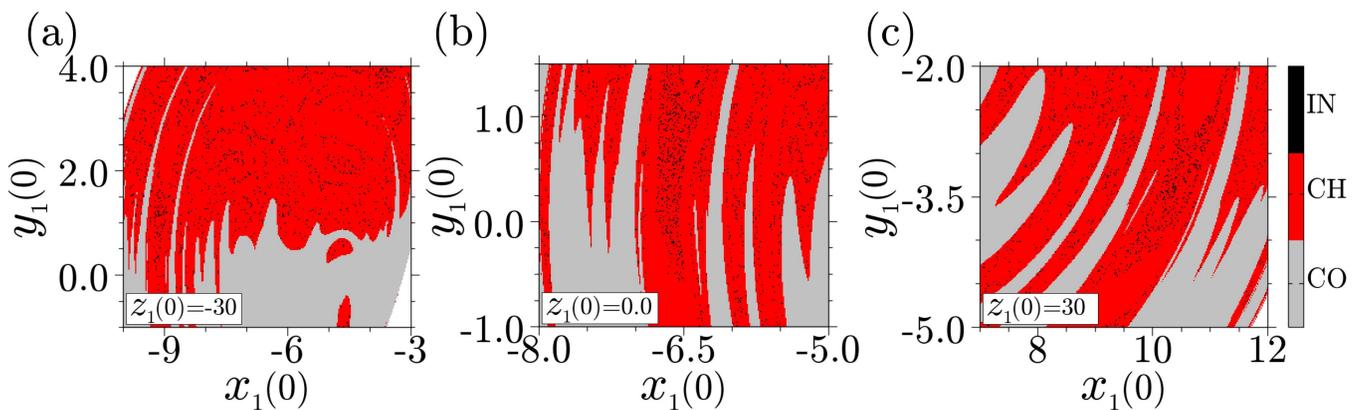


FIG. 6. Magnifications of the basin boundaries for (a) $z_1(0) = -30$, (b) $z_1(0) = 0$, and (c) $z_1(0) = 30$, where we consider $\sigma = 0.05$, $r = 0.28$, $a = 0.42$, $b = 2$, $c = 3.9$, and $N = 200$.

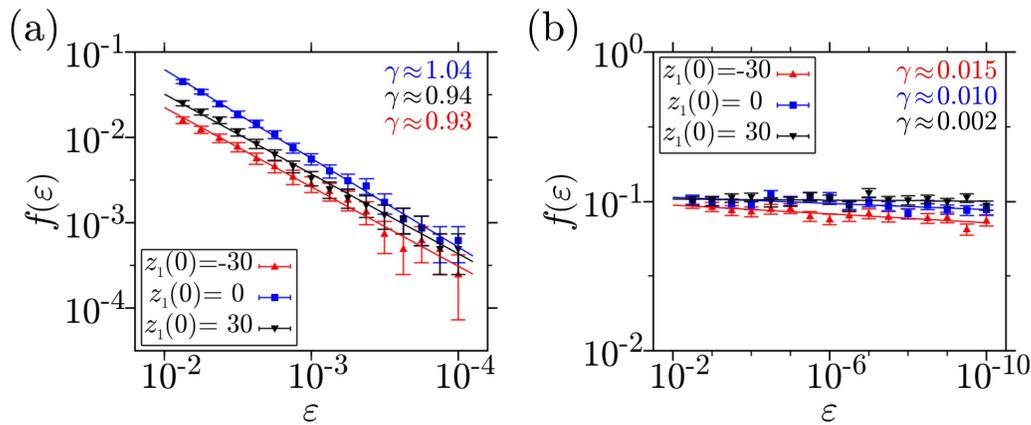


FIG. 7. Fraction of trajectories $f(\epsilon)$ generated by the uncertain points for the basin boundary between (a) coherent and chimera states and (b) incoherent and chimera states. We calculate $f(\epsilon)$ for $z_1(0) = -30$ (red points), $z_1(0) = 0$ (blue points), and $z_1(0) = 30$ (black points), where we consider $\sigma = 0.05$. The γ values are the exponents obtained by the fitting of each colored line.

for $m = 1, \dots, M$, where Θ is the Heaviside step function and δ is a predetermined threshold. $\chi(m)$ is the local standard deviation given by

$$\chi(m) = \left\langle \sqrt{\frac{1}{n} \sum_{j=n(m-1)+1}^{nm} [v_j - \bar{v}]^2} \right\rangle, \quad (3)$$

where $v_i = x_i - x_{i+1}$. If a box k has a $\chi(k)$ value lower than δ , Eq. (2) results in $s_k = 1$, which means that the oscillators in the box are coherent. The value of strength of incoherence is given by

$$SI = 1 - \frac{1}{M} \sum_{m=1}^M s_m \quad (4)$$

and is equal to 0 or 1 when the network exhibits coherent or incoherent states, respectively. When $0 < SI < 1$, the state is characterized as chimera.³⁷

Figure 3 shows a chimera state for 200 coupled Rössler oscillators; $a = 0.42$, $b = 2$, $c = 3.9$, $\sigma = 0.05$, and $r = 0.28$. In Fig. 3(a), the color scale corresponds to the x values of each oscillator i as a function of t . Figure 3(b) displays the snapshot of x_i in a chimera state. We see a group of oscillators around $i = 100$ with incoherent dynamics. Through the space–time plot of v_i , it is possible to see the domains with coherent and incoherent patterns, as shown in Fig. 3(c). In addition, the strength of the incoherence value is equal to 0.25.

IV. BASIN OF ATTRACTION FOR CHIMERA STATES

In our simulations, we observe different dynamical patterns for the same parameter values in the network given by Eq. (1) for different configurations of initial conditions. Due to this fact, we compute the basins of attraction to characterize the patterns. To do that, we set to zero the initial conditions of all the oscillators except for one. After changing the initial condition of one oscillator, we track the trajectories to know if the network goes out of the synchronized

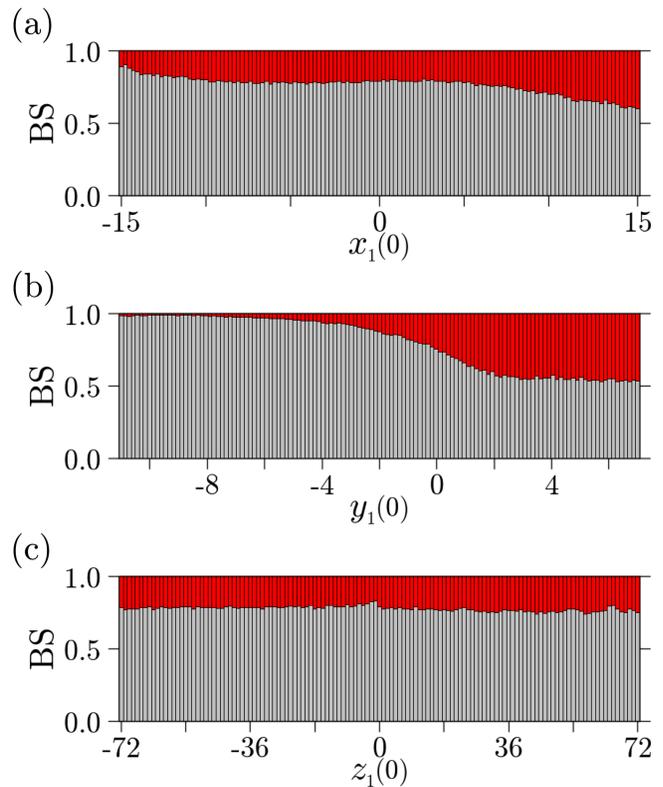


FIG. 8. Basin stability (BS) of each state as a function of (a) $x_1(0)$, (b) $y_1(0)$, and (c) $z_1(0)$, where the gray and red colors denote the coherent and chimera states, respectively, for $\sigma = 0.07$.

state. The choice of changing one initial condition is a strategy to study chimera states based on a single control parameter; otherwise, the chimera could be conditional to several other initial conditions, making it even more frequent, but much more difficult to study. The initial conditions and control parameters play a crucial role in the existence of chimera states.

Figure 4 exhibits five slices $[z_1(0) = -60, -30, 0, 30, 60]$ of initial conditions of the oscillator $i = 1$ that do not lead to divergent trajectories of the network. The gray dots correspond to initial conditions, leading the network to coherent states, the red to chimera, and black to incoherent states. In Fig. 4(a), considering $\sigma = 0.05$, we see that the slices show coherent, chimera, incoherent states. For $\sigma = 0.07$, we do not find black dots, as shown in Fig. 4(b); namely, there are no incoherent states. Therefore, the spatiotemporal pattern can depend on the initial conditions.

Changes in the basin size can be estimated by means of the basin stability (BS).³⁸ Figures 5(a), 5(b), and 5(c) display the basin stability for $x_1(0)$, $y_1(0)$, and $z_1(0)$, respectively, which is also called single node basin stability.^{39,40} To calculate the basin stability, we consider 4096 random initial conditions and verify if the system converges to incoherent (black), chimera (red), or coherent (gray) states. Comparing the results of basin stability for $x_1(0)$, $y_1(0)$, and $z_1(0)$, we see that the BS has a larger variation for $y_1(0)$ than

$x_1(0)$ and $z_1(0)$. The fraction of initial conditions that converges to chimera is small and large for negative and positive $y_1(0)$ values, respectively.

Another important feature in this system is the boundary structure of the attraction basin. In Fig. 6, we plot the magnifications of regions of the basin for $z_1(0)$ equal to (a) -30 , (b) 0 , and (c) 30 . We verify that there is a border between the basins of the coherent and chimera states, while inside the chimera basin, there are points belonging to the basin of incoherent states.

We compute the uncertainty exponent^{41,42} to investigate the existence of fractal basin boundaries. The uncertainty exponent was proposed by Grebogi *et al.*⁴³ to characterize basin boundaries. They analyzed the dependence of the predictability on the fractal and smooth structure of the basin boundaries. To calculate the uncertainty exponent, first, we consider a number of random initial conditions in a region of the basin and compute the final state. An initial condition is ε -uncertain when at least one of its neighbors within a circle of radius ε goes to a different final state. It is expected that the fraction of trajectories $f(\varepsilon)$ generated by the uncertain points scales as

$$f(\varepsilon) \sim \varepsilon^\gamma, \tag{5}$$

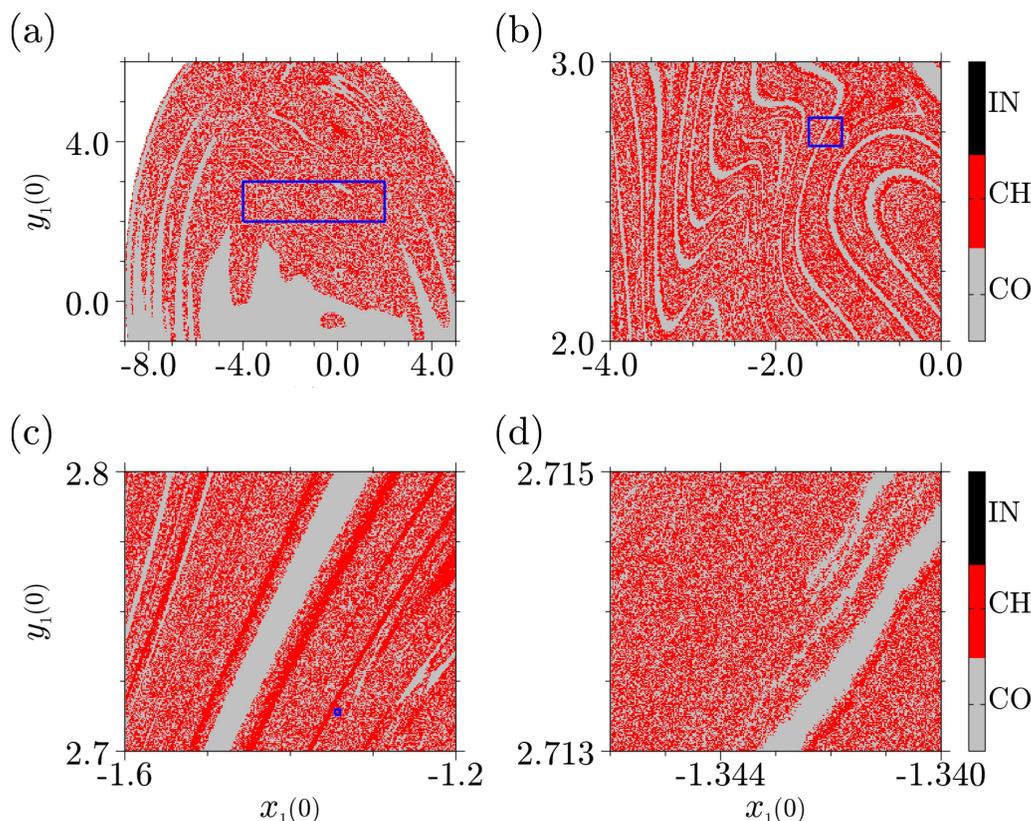


FIG. 9. (a) Basin of attraction for $\sigma = 0.07$, $r = 0.28$, $a = 0.42$, $b = 2$, $c = 3.9$, $N = 200$, and $z_1(0) = 0$. (b), (c), and (d) are magnifications.

where γ is the uncertainty exponent. The γ value is related to the box-counting dimension d of the basin boundary by

$$\gamma = D - d, \tag{6}$$

where D is the phase space dimension.

The fractal boundaries are more sensitive to initial uncertainty and exhibit γ less than 1.³⁵ In our network of coupled Rössler oscillators, the γ value allows characterizing the boundaries between the basins related to the coherent, chimera, and incoherent states. Figure 7 exhibits $f(\epsilon)$ as a function of ϵ for the three magnifications plotted in Fig. 6. To calculate $f(\epsilon)$, we sort $N_0 = 8192$ random initial conditions in the range according to Fig. 6, record the final state of each initial condition, and compare it with the final state of two distinct initial conditions inside the ϵ -neighborhood. We compute the uncertainty fraction by means of $f(\epsilon) = N_\epsilon/N_0$, where N_ϵ is the total number of uncertain initial conditions for a given ϵ .

In Fig. 7(a), we compute $f(\epsilon)$ as a function of ϵ for the basin boundary between the coherent and chimera states. For $z_1(0) = -30$ (red points), $z_1(0) = 0$ (blue points), and $z_1(0) = 30$ (black points), we find uncertainty exponent values very close to 1. According to Ref. 41, $D = 2 - \gamma$, the dimension is approximately equal to 1, indicating that the boundary is smooth. Figure 7(b) displays $f(\epsilon)$ vs ϵ for the boundary basin between the incoherent and chimera states. The values of γ are very small, and the dimensions are approximately equal to 2. Therefore, the uncertainty of the final state remains unchangeable even when the precision of the initial conditions is increased. In this situation, the basin of chimera states is riddled with points of the basin of the incoherent states; namely, the basin boundary is riddled.

For $\sigma = 0.07$, the trajectories do not go to incoherent states [Fig. 4(b)]. As a consequence, basin stability exhibits the absence of incoherent states (black color) in $x_1(0)$, $y_1(0)$, and $z_1(0)$, as shown in Figs. 8(a), 8(b), and 8(c), respectively. Then, the basin boundary exists only between the coherent and chimera states.

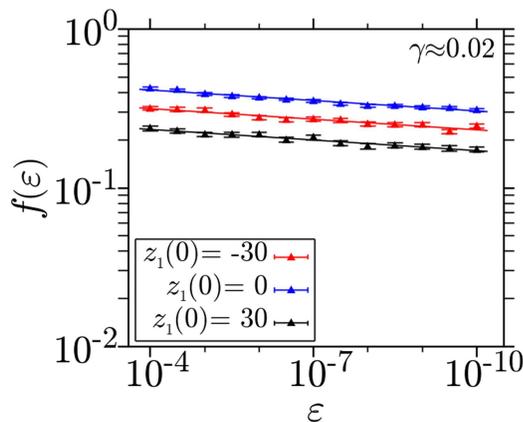


FIG. 10. Fraction of trajectories $f(\epsilon)$ generated by the uncertain points for the basin boundary between the coherent and chimera states for $\sigma = 0.07$. We calculate $f(\epsilon)$ for $z_1(0) = -30$ (red points), $z_1(0) = 0$ (blue points), and $z_1(0) = 30$ (black points). We find $\gamma \approx 0.02$ for the three cases.

Figure 9 displays magnifications of Fig. 4(b) for $z_1(0)$. We see complex structures that are caused by the crossing of the stable manifold that bounds the basin of attraction with an unstable manifold. To characterize the basin boundary, we compute the value of γ . In Fig. 10, we plot $f(\epsilon)$ as a function of ϵ for $z_1(0) = -30$ (red points), $z_1 = 0$ (blue points), and $z_1 = 30$ (black points), where the lines correspond to the respective fittings. We find $\gamma \approx 0.02$ for the three cases. This γ value implies a basin boundary with a fractal dimension $D \approx 1.98$.

V. CONCLUSIONS

In the 1970s, Rössler introduced a set of differential equations that exhibits periodic and chaotic attractors. Networks composed of Rössler systems have been used to analyze synchronization in coupled chaotic oscillators. In this work, we study basins of attraction for chimera states in a network of Rössler oscillators. Depending on the system parameters and initial conditions, we show that the spatiotemporal dynamics of nonlocally coupled Rössler oscillators can converge to coherent, incoherent, or chimera states.

We investigate the basin of attraction considering the same initial conditions for all oscillators except for one. In the chosen oscillator, we separate the initial conditions space into slices and compute the strength of incoherence. By means of the strength of incoherence, it is possible to identify coherence, incoherence, and chimera. To characterize the boundary structure of the basin of attraction, we use the uncertainty exponent γ .

For the coupling parameter equal to 0.05 and varying the initial conditions, we observe the existence of coherent, incoherent, and chimera states. We calculate the relative size of each basin through the single node basin stability. The basin stability remains approximately constant for $x_1(0)$ and $z_1(0)$, and it changes for $y_1(0)$. The fraction of initial conditions that converges to chimera is smaller for negative values of $y_1(0)$. With regard to the basin boundary, the uncertainty exponent shows that the boundary between the coherent and chimera states is smooth, while the boundary between the incoherent and chimera states is riddled. Considering the coupling parameter equal to 0.07 and varying the initial conditions, we verify the existence of coherent and chimera states and the absence of incoherent states. In our simulations using the uncertainty exponent, we find a fractal structure of the basin boundary between the coherent and chimera states.

All in all, depending on the coupling parameter of a network composed of nonlocally coupled Rösslers, we observe in our simulations the existence of the basin boundary with different structures. For three basins of attraction, the smooth boundary appears between coherent and chimera states, while the riddled boundary occurs between incoherent and chimera states. When there are two basins of attraction with the absence of incoherence, the basin boundary has a fractal structure.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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