Fractal structures in the chaotic advection of passive scalars in leaky planar hydrodynamical flows

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ABSTRACT

The advection of passive scalars in time-independent two-dimensional incompressible fluid flows is an integrable Hamiltonian system. It becomes non-integrable if the corresponding stream function depends explicitly on time, allowing the possibility of chaotic advection of particles. We consider for a specific model (double gyre flow), a given number of exits through which advected particles can leak, without disturbing the flow itself. We investigate fractal escape basins in this problem and characterize fractality by computing the uncertainty exponent and basin entropy. Furthermore, we observe the presence of basin boundaries with points exhibiting the Wada property, i.e., boundary points that separate three or more escape basins.

Chaotic advection of particles in two-dimensional hydrodynamical flows has various applications in many fields, from meteorology and oceanography to chemical engineering. One of the outstanding properties of chaotic trajectories of advected particles is large-scale transport and mixing. In some situations, time-dependent hydrodynamical flows can be translated to non-integrable open Hamiltonian systems. The underlying dynamics of chaotic advections in leaky two-dimensional flows leads to a number of fractal structures that can be characterized using suitable diagnostics. In the case of two escape basins, we used an entropy-like quantity to quantify the final-state uncertainty and observed that the latter increases with the perturbation strength. For three or more escape basins, the corresponding basin structure is considerably more involved, presenting interior points, boundary points, and Wada points. These are points belonging to boundaries common to all basins and indicate an extreme form of fractality.

I. INTRODUCTION

Advection is the transport of a substance or quantity by the flow of a fluid, i.e., the properties of the substance are transported by the flow.[1] There are several engineering applications of advection, like the transport of pollutants, sedimentation, mixtures, and emulsions.[2,3] In the biological context, the advection of blood particles can cause the formation of atherosclerotic plaques.[4] Other applications of advection can be found in meteorology and oceanography, from the transport of quantities like heat, moisture, salinity, etc., due to atmospheric and oceanic flows, respectively.[5–8] Active processes in chemical and biological systems are also strongly affected by chaotic advection.[9,10]

A simplification in the modeling of these phenomena considers the advection of passive scalars in such a way that they are transported by the flow but without interacting with the flow itself.[1] In this approximation, the speed of the advected particles is equal to the fluid velocity at each point. If, moreover, we suppose an incompressible two-dimensional flow, the equations of motion are formally identical with Hamilton's equations.[9,11] This fact enables us to use the powerful machinery of Hamiltonian dynamics to investigate advected particle motion. In particular, if the fluid flow presents an explicit time-dependence, the resulting Hamiltonian system is non-integrable and its phase space turns out to be neither entirely regular nor entirely chaotic.[12] The regular part consists of periodic and quasiperiodic orbits, while chaotic orbits fill densely a finite region of the phase space.[13]