

# Spike timing-dependent plasticity induces non-trivial topology in the brain



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## ABSTRACT

We study the capacity of Hodgkin–Huxley neuron in a network to change temporarily or permanently their connections and behavior, the so called spike timing-dependent plasticity (STDP), as a function of their synchronous behavior. We consider STDP of excitatory and inhibitory synapses driven by Hebbian rules. We show that the final state of networks evolved by a STDP depend on the initial network configuration. Specifically, an initial all-to-all topology evolves to a complex topology. Moreover, external perturbations can induce co-existence of clusters, those whose neurons are synchronous and those whose neurons are desynchronous. This work reveals that STDP based on Hebbian rules leads to a change in the direction of the synapses between high and low frequency neurons, and therefore, Hebbian learning can be explained in terms of preferential attachment between these two diverse communities of neurons, those with low-frequency spiking neurons, and those with higher-frequency spiking neurons.

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## 1. Introduction

Neuroplasticity, also known as brain plasticity or brain malleability (Brenner, Bialek, & Van Steveninck, 2000; Strong, Koberle, Van Steveninck, & Bialek, 1998), refers to the ability of the brain to reorganize neural pathways in response to new information, environment, development, sensory stimulation, or damage (Draganski et al., 2004; James, 1890; Lashley, 1923). The term neuroplasticity was firstly introduced in 1948 by neuroscientist J. Konorski in a work (Konorski, 1948) that showed the associative learning as a result of the adaptation of the brain to external stimuli. In 1949, D. O. Hebb, in his book entitled “The Organization of Behavior” (Hebb, 1949), proposed a plasticity rule, today known as Hebb’s rule.

Scientific advances in neuroimaging and in noninvasive brain stimulation have provided insights to understand better neuroplasticity. Learning-induced structural alterations in gray and

white matter have been documented in human brain (Dayan & Cohen, 2011). Draganski et al. (2004) used whole-brain magnetic-resonance imaging to observe learning-induced neuroplasticity. They verified structural changes in areas of the brain associated with the processing and storage of complex visual motion. Lu et al. (2003) demonstrated that neuroplasticity is affected by environmental stimuli. In addition, neuroimaging studies have showed alterations of neuroplasticity in depression, namely depressive disorder may be associated with impairment of neuroplasticity (Fuchs, Czéh, Kole, Michaelis, & Lucassen, 2004).

Aiming at understanding the fundamental mechanisms behind plasticity, Popovych and collaborators studied the effect of noise on synchronous behavior in globally-coupled spiking Hodgkin–Huxley neurons with spike timing-dependent plasticity (STDP) and excitatory synapses (Borges et al., 2016; Popovych, Yanchuk, & Tass, 2013). STDP networks have nodes that adapt their synaptic strength according to some rule based on their spike timings (Gilson, Burkitt, & Van Hemmen, 2010; Markram, Gerstner, & Sjostrom, 2011, 2012). Abarbanel and Talathi (2006) studied a neural circuit responsible for recognizing interspike interval sequences by means of STDP of inhibitory synapses. Similar results, though

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using different kinds of neural models, have been reported earlier by Kalitzin, Van Dijk, and Spekreijse (2000), where it was shown that coherent input can enhance synapses inducing high connectivity, whereas mutually anti-correlated inputs to individual neurons weaken connectivity. On the contrary, the work in Popovych et al. (2013) however shows that a fully uncorrelated input can enhance connectivity. Sadeh, Clopath, and Rotter (2015) studied the emergence of functionally specific connectivity in the visual cortex with Hebbian plasticity based on visual experience. They showed that plasticity can lead to functionally specific and stable connections in random networks composed of leaky integrate-and-fire neurons. In our work, we focus the attention on the network dependence on plasticity. To do that, we consider an initial all-to-all topology and focus on the changes in synchronous and non synchronous states caused in a Hodgkin–Huxley neural network with excitatory (eSTDP) and inhibitory synapses (iSTDP).

Neural spike synchronization is responsible for information transfer (Antonopoulos, Srivastava, Pinto, & Baptista, 2015; Baptista, Szmoski, Pereira, & Pinto, 2016), and can be associated with forms of dysfunction. For instance, abnormally synchronized oscillatory activity has been reported in Parkinson's disease (Hammond, Bergman, & Brown, 2007), epilepsy (Uhlhaas & Singer, 2006) and some other neurological disorders. Synchronous behavior was analyzed in systems of synfire chains to solve binding problems. It was verified that dynamics of binding may be modeled by competitive synchronization among synfire chains (Abeles, Hayon, & Lehmann, 2004; Hayon & Lehmann, 2005). Moreover, synfire chains have been considered to describe information transfer phenomena and coherent spiking (Wang, Sornborger, & Tao, 2016).

Our main goal is to show that spike timing-dependent plasticity of excitatory and inhibitory synapses induces non-trivial topologies in the plastic brain. Initial networks of neurons fully connected, evolve to a non trivial complex network. Consequently, this non-trivial topology alters the synchronous behavior. In our results, we have observed for some parameter conditions not only the improvement of neural spiking synchronization, but also for other parameter conditions that promote desynchronization. We have also observed concurrent synchronous and non synchronous behavior in the neurons of a network constructed for a particular set of parameters. Therefore, the onset of synchronicity comes along side with desynchronicity in the plastic brain. This balance between different synchronous behaviors is vital to maintain a fundamental property of a brain network. Clusters need to be sufficiently synchronous for information to be efficiently exchanged, but at the same time sufficiently desynchronous to behave independently. Finally, we show that when there is an external perturbation, the plastic neural network has an abrupt change in behavior characterized by a first-order transition.

This paper is organized as follows: In Section 2 we introduce the neural network described by coupled Hodgkin–Huxley neural model. In Section 3, we discuss our results about neural synchronization considering eSTDP and iSTDP. In Section 4 we draw our conclusions.

## 2. Neural network

In this work, we focus on eSTDP and iSTDP based on Hebbian theory proposed in Ref. Hebb (1949). These plastic mechanisms consist of synapses that become stronger or weaker depending on the pre and postsynaptic neurons' activity. We have considered an initial network with a global coupling, with chemical synapses where the connections are unidirectional, and the local dynamics is described by the Hodgkin–Huxley model (Hodgkin & Huxley, 1952;

Izhikevich, 2004). The system is given by

$$\begin{aligned} C\dot{V}_i = & I_i - g_K n_i^4 (V_i - E_K) - g_{Na} m_i^3 h_i (V_i - E_{Na}) \\ & - g_L (V_i - E_L) + \frac{(V_r^{Exc} - V_i)}{\omega_{Exc}} \sum_{j=1}^{N_{Exc}} \varepsilon_{ij} S_j \\ & + \frac{(V_r^{Inhib} - V_i)}{\omega_{Inhib}} \sum_{j=1}^{N_{Inhib}} \sigma_{ij} S_j + \Gamma_i, \end{aligned} \quad (1)$$

$$\dot{n}_i = \alpha_{n_i}(V_i)(1 - n_i) - \beta_{n_i}(V_i)n_i, \quad (2)$$

$$\dot{m}_i = \alpha_{m_i}(V_i)(1 - m_i) - \beta_{m_i}(V_i)m_i, \quad (3)$$

$$\dot{h}_i = \alpha_{h_i}(V_i)(1 - h_i) - \beta_{h_i}(V_i)h_i, \quad (4)$$

where  $C$  is the membrane capacitance ( $\mu\text{F}/\text{cm}^2$ ),  $V_i$  is the membrane potential (mV) of neuron  $i$  ( $i = 1, \dots, N$ ),  $I_i$  is a constant current density randomly distributed in the interval  $[9.0, 10.0]$ ,  $\omega_{Exc}$  (excitatory) and  $\omega_{Inhib}$  (inhibitory) are the average degree connectivities,  $\varepsilon_{ij}$  and  $\sigma_{ij}$  are the excitatory and inhibitory coupling strengths from the presynaptic neuron  $j$  to the postsynaptic neuron  $i$  (Gray, 1959). The  $\varepsilon_{ij}$  values are in the interval  $[0, 0.5]$  and the  $\sigma_{ij}$  values are in the interval  $[0, 2\sigma_M]$ . In our simulations, the maximum value for  $\varepsilon_{ij}$  is equal to 0.5 according to Ref. Popovych et al. (2013), and we consider the maximum value for  $\sigma_M$  equal to 0.75 due to the fact that for  $\sigma_{ij} < 1.5$  we observe a transition from synchronized to desynchronized states. In addition, we have discarded a transient of  $1.95 \times 10^6$  ms. We consider that 80% of the neurons are excitatorily coupled ( $N_{Exc}$ ) and 20% of them are inhibitorily coupled ( $N_{Inhib}$ ) according to anatomical estimates for the neocortex (Noback, Strominger, Demarest, & Ruggiero, 2005). Both populations receive input from all other neurons in own population and from the other population. We also consider an external perturbation  $\Gamma_i$ , so that each neuron randomly chosen receives an input with a constant intensity  $\gamma = 10 \mu\text{A}/\text{cm}^2$  during 1 ms. In each time step  $t_{step} = 0.01$  ms a random input with amplitude  $\gamma$  is applied to each neuron with a probability equal to  $t_{step}/14$ , where 14 ms approximately corresponds to the inter-spike interval of the Hodgkin–Huxley neuron. Functions  $m(V_i)$  and  $n(V_i)$  represent the activation for sodium and potassium, respectively, and  $h(V_i)$  is the function for the inactivation of sodium. Functions  $\alpha_n$ ,  $\beta_n$ ,  $\alpha_m$ ,  $\beta_m$ ,  $\alpha_h$ ,  $\beta_h$  are given by

$$\alpha_n(v) = \frac{0.01v + 0.55}{1 - \exp(-0.1v - 5.5)}, \quad (5)$$

$$\beta_n(v) = 0.125 \exp\left(\frac{-v - 65}{80}\right), \quad (6)$$

$$\alpha_m(v) = \frac{0.1v + 4}{1 - \exp(-0.1v - 4)}, \quad (7)$$

$$\beta_m(v) = 4 \exp\left(\frac{-v - 65}{18}\right), \quad (8)$$

$$\alpha_h(v) = 0.07 \exp\left(\frac{-v - 65}{20}\right), \quad (9)$$

$$\beta_h(v) = \frac{1}{1 + \exp(-0.1v - 3.5)}, \quad (10)$$

where  $v = V/[mV]$ . Parameter  $g$  is the conductance and  $E$  the reversal potentials for each ion. Depending on the value of the external current density  $I_i$  ( $\mu\text{A}/\text{cm}^2$ ) the neuron can present single spike activity or periodic spikings. In the case of periodic spikes, if the constant  $I_i$  increases, the spiking frequency also increases. In this work, we consider  $C = 1 \mu\text{F}/\text{cm}^2$ ,  $E_{Na} = 50$  mV,  $E_K = -77$  mV,  $E_L = -54.4$  mV,  $g_{Na} = 120$  mS/cm<sup>2</sup>,  $g_K = 36$  mS/cm<sup>2</sup>,

$g_L = 0.3 \text{ mS/cm}^2$ . The neurons are excitatorily coupled with a reversal potential  $V_r^{\text{Exc}} = 20 \text{ mV}$ , and inhibitorily coupled with a reversal potential  $V_r^{\text{Inhib}} = -75 \text{ mV}$ . The presynaptic potential  $s_i$  is given by Destexhe, Mainen, and Sejnowski (1994) and Golomb and Rinzel (1993)

$$\frac{ds_i}{dt} = \frac{5(1 - s_i)}{1 + \exp\left(-\frac{v_i + 3}{8}\right)} - s_i, \quad (11)$$

where  $v_i = V_i/[mV]$ .

One of the key principles of behavioral neuroscience is that experience can modify the brain structure, what is known as neuroplasticity (Ramon & Cajal, 1928). Although the idea that experience may modify the brain structure can probably be traced back to the 1890s (Bliss & Collingridge, 1993; Bliss & Gardner-Medwin, 1973), it was Hebb who made this a central feature in his neuropsychological theory (Hebb, 1961). With this in mind, we consider excitatory and inhibitory spike timing-dependent plasticity according to the Hebbian rule. The coupling strengths  $\varepsilon_{ij}$  and  $\sigma_{ij}$  are adjusted based on the relative timing between the spikes of presynaptic and postsynaptic neurons (Bi & Poo, 1998; Haas, Nowotny, & Abarbanel, 2006).

The plasticity dynamics can be mathematically defined as

$$\frac{d\Delta\varepsilon(t)}{dt} = f(\Delta\varepsilon, V, t), \quad (12)$$

where  $\Delta\varepsilon$  is the update value of the synaptic weight. Kalitzin et al. (2000) considered a function  $f$  that depends on the activation of the synapse, the transmembrane potential of the postsynaptic neuron, and the thresholds for switching on long-term potentiation and the long-term depression (Artola, Bröcher, & Singer, 1990). In this work, we consider a linear function  $f$  of the form  $f(\Delta\varepsilon, t) = (a + c/t)\Delta\varepsilon$ . The solution to the differential equation (12), is given by  $\Delta\varepsilon = bt^c \exp(at)$ , where  $a$ ,  $b$  and  $c$  are constants. For  $c = 0$  and  $c \neq 0$ , eSTDP and iSTDP are obtained, respectively. The plasticity dynamics introduced by means of this linear function is not fundamentally related to physiological processes (Artola et al., 1990), but, by means of this function it is possible to find a fit that describes experimental results of eSTDP and iSTDP obtained in Refs. Bi and Poo (1998) and Haas et al. (2006).

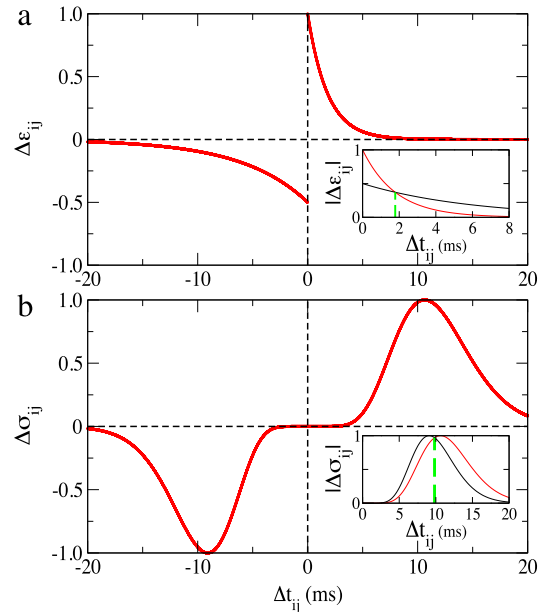
The excitatory eSTDP is given by

$$\Delta\varepsilon_{ij} = \begin{cases} A_1 \exp(-\Delta t_{ij}/\tau_1), & \Delta t_{ij} \geq 0 \\ -A_2 \exp(\Delta t_{ij}/\tau_2), & \Delta t_{ij} < 0, \end{cases} \quad (13)$$

where  $\Delta t_{ij} = t_i - t_j = t_{\text{pos}} - t_{\text{pre}}$ ,  $t_{\text{pre}}$  is the spike time of the presynaptic and  $t_{\text{pos}}$  the spike time of the postsynaptic neuron. Fig. 1(a) exhibits the result obtained from Eq. (13) for  $A_1 = 1.0$ ,  $A_2 = 0.5$ ,  $\tau_1 = 1.8 \text{ ms}$ , and  $\tau_2 = 6.0 \text{ ms}$ . The initial synaptic weights  $\varepsilon_{ij}$  are normally distributed with mean and standard deviation equal to  $\varepsilon_M = 0.25$  and  $0.02$ , respectively ( $0 \leq \varepsilon_{ij} \leq 2\varepsilon_M$ ). Then, they are updated according to Eq. (13), where  $\varepsilon_{ij} \rightarrow \varepsilon_{ij} + 10^{-3}\Delta\varepsilon_{ij}$ . The insets in Fig. 1 show the absolute value of the plasticity function, where the red and black lines are the potentiation and depression values, respectively, as a function of  $\Delta t_{ij}$ . The green dashed line in the inset figures denotes the  $\Delta t_{ij}$  value at which the curves of potentiation and depression intersect. The inset in Fig. 1(a) shows that for  $|\Delta t_{ij}| < 1.8 \text{ ms}$  the potentiation of  $\varepsilon_{ij}$  is bigger than the depression. Whereas in the case of iSTDP (inset in Fig. 1(b)) the potentiation of  $\sigma_{ij}$  is bigger than the depression for  $|\Delta t_{ij}| > 9.8 \text{ ms}$ .

For the inhibitory iSTDP synapses, the coupling strength  $\sigma_{ij}$  is adjusted based on the equation

$$\Delta\sigma_{ij} = \frac{g_0}{g_{\text{norm}}} \alpha^\beta |\Delta t_{ij}| \Delta t_{ij}^{\beta-1} \exp(-\alpha|\Delta t_{ij}|), \quad (14)$$



**Fig. 1.** (Color online) Plasticity as a function of the difference of spike timing of post and presynaptic (a) excitatory (eSTDP) and (b) inhibitory (iSTDP) synapse. The insets show the absolute value of the plasticity function.

where  $g_0$  is the scaling factor accounting for the amount of change in inhibitory conductance induced by the synaptic plasticity rule, and  $g_{\text{norm}} = \beta^\beta \exp(-\beta)$  is the normalizing constant. Fig. 1(b) exhibits the result obtained from Eq. (14) for  $g_0 = 0.02$ ,  $\beta = 10.0$ ,  $\alpha = 0.94$  if  $\Delta t_{ij} > 0$ , and for  $\alpha = 1.1$  if  $\Delta t_{ij} < 0$  (Talathi, Hwang, & Ditto, 2008). As a consequence,  $\Delta\sigma_{ij} > 0$  for  $\Delta t_{ij} > 0$ , and  $\Delta\sigma_{ij} < 0$  for  $\Delta t_{ij} < 0$ . The initial inhibitory synaptic weights  $\sigma_{ij}$  are normally distributed with mean and standard deviation equal to  $\sigma_M$  and  $0.02$ , respectively ( $0 \leq \sigma_{ij} \leq 2\sigma_M$ ). Then, the coupling strengths are updated according to Eq. (14), where  $\sigma_{ij} \rightarrow \sigma_{ij} + 10^{-3}\Delta\sigma_{ij}$ . The updates for  $\varepsilon_{ij}$  and  $\sigma_{ij}$  are applied for the last postsynaptic spike.

### 3. Spiking neuron synchronization

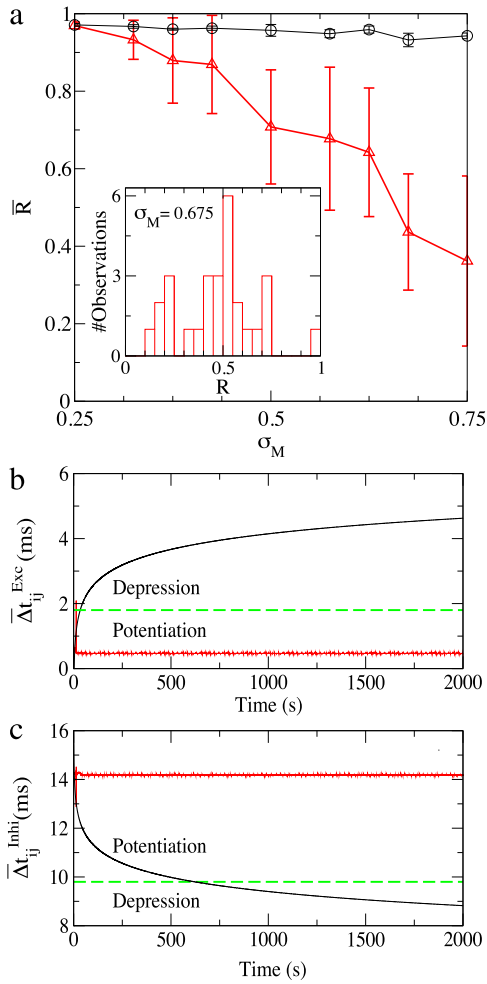
To study the effect of plasticity on the neural network, we have calculated the coupling strengths, and used the time-average order-parameter as a probe of spike synchronization, a quantity expressed by

$$R = \frac{1}{t_{\text{final}} - t_{\text{initial}}} \sum_{t_{\text{initial}}}^{t_{\text{final}}} \left| \frac{1}{N} \sum_{j=1}^N \exp(i\psi_j) \right|, \quad (15)$$

where  $t_{\text{final}} - t_{\text{initial}}$  is the time window for our estimation and the phases are calculated by

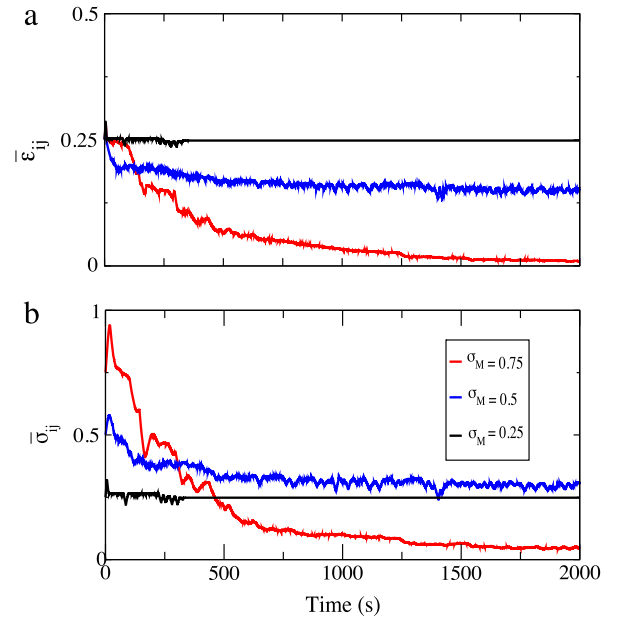
$$\psi_j(t) = m + \frac{t - t_{j,m}}{t_{j,m+1} - t_{j,m}}, \quad (16)$$

where  $t_{j,m}$  represents the time when a spike  $m$  ( $m = 0, 1, 2, \dots$ ) in neuron  $j$  occurs ( $t_{j,m} < t < t_{j,m+1}$ ), with the beginning of each spike being when  $V_j > 0$ . In synchronous behavior, the order-parameter magnitude approaches unity. In addition, if the spike times are uncorrelated, the order-parameter magnitude is typically small and vanishes for  $N \rightarrow \infty$ . When identical neurons are coupled, the neural network may exhibit complete synchronization among spiking neurons, in other words, all other neurons may present identical time evolution of their action potentials. In this work, we are not considering identical neurons, and as result it is not possible to observe complete synchronization. However, an almost-complete synchronization may be observed.



**Fig. 2.** (a) Mean order-parameter  $\bar{R}$  versus  $\sigma_M$  for  $\gamma = 0.0$  and  $\varepsilon_M = 0.25$ , a result without STDP (black circles) and the other one with STDP (red triangles). The bar is the standard deviation for 30 different initial conditions. In the inset we consider  $\sigma_M = 0.675$ . Panels (b) and (c) exhibit the time evolution of the average time-difference for excitatory and inhibitory connections, respectively. The black and red lines, for  $\sigma_M = 0.675$ , correspond to  $\bar{R} \approx 0.1$  and  $\bar{R} \approx 1$ , respectively. The green line represents the separation between potentiation and depression. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 2(a) shows the mean order-parameter ( $\bar{R}$ ) that is calculated for different initial conditions, as a function of the inhibitory coupling strength  $\sigma_M$  for a neural network with excitatory and inhibitory synapses, where we consider one case without STDP (black circles) and another with STDP (red triangles). For  $\varepsilon_M$  equal to 0.25 and varying  $\sigma_M$ , we do not observe a significant alteration of the  $\bar{R}$  value without STDP, due to the fact that initially the network has an all-to-all topology. Nevertheless, considering STDP we verify that the  $\bar{R}$  values decrease with the increase of  $\sigma_M$  and present a large standard deviation. This standard deviation occurs due to the existence of different synchronization states. Then, both the upper border of the inhibitory coupling  $2\sigma$  and the different initial conditions are important to change the dynamics of the network with STDP and without external perturbation. This is verified by means of the decay of the  $R$  values and the large standard deviation bar. In the inset (Fig. 2(a)), we consider  $\sigma_M = 0.675$  and calculate the order-parameter for different initial conditions. As a result, we can see a distribution presenting different synchronization states, including desynchronization and synchronization. In Fig. 2(b) and (c) we consider  $\sigma_M = 0.675$  according to the inset, and calculate the time evolution of the



**Fig. 3.** (Color online) Time courses of the mean (a) excitatory and (b) inhibitory coupling strengths from some regimes shown in Fig. 2(a).

average time-difference for excitatory and inhibitory connections,

$$\bar{\Delta t}_{ij}^{\text{Exc}} = \frac{1}{\tau} \sum_{i \neq j} |t_{\text{pre}}^{\text{Exc}} - t_{\text{pos}}|, \quad (17)$$

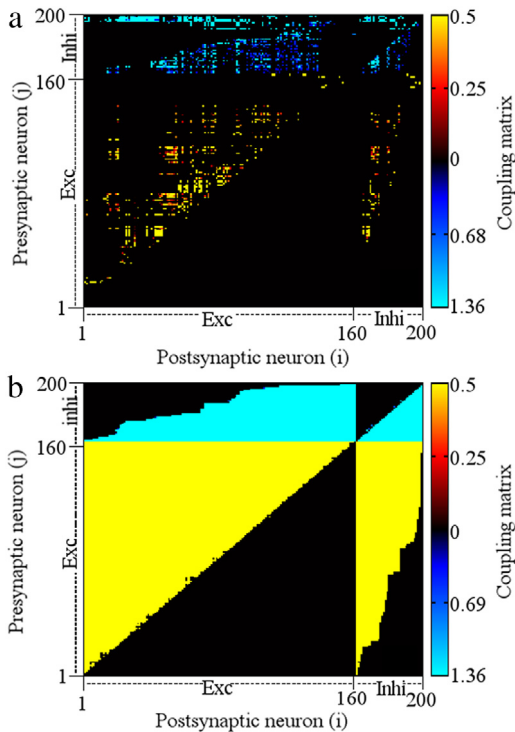
$$\bar{\Delta t}_{ij}^{\text{Inhib}} = \frac{1}{\tau} \sum_{i \neq j} |t_{\text{pre}}^{\text{Inhib}} - t_{\text{pos}}|, \quad (18)$$

respectively, for different configurations of the initial networks and  $\tau = 100$  ms. The black line shows the case in which the network goes to a desynchronized state ( $\bar{R} \approx 0.1$ ), whereas the red line exhibits the case of a network that presents synchronous behavior ( $\bar{R} \approx 1$ ). In both cases, we consider the same parameters, except the seed to generate the random distribution of the constant current density  $I_i$ . Through Fig. 2(b) and (c) it is possible to verify why and when the coupling matrix suffer substantial changes. The transition occurs when the black or red curves cross the green line. At this time, depreciation induces weak strength in the coupling matrix, and potentiation induces strong strength.

Fig. 3 exhibits the time courses of the mean excitatory (Fig. 3(a)) and inhibitory (Fig. 3(b)) coupling strengths from the multiple coexisting regimes that are shown in Fig. 2(a). We see that for  $\sigma_M = 0.25$  both  $\bar{\varepsilon}_{ij}$  and  $\bar{\sigma}_{ij}$  have constant values for the time approximately greater than 700 s, and the learning produces a triangular-type connecting matrix (as shown in Fig. 4), meaning that the connections among all neurons become preferentially directed. For  $\sigma_M = 0.5$  the  $\bar{\varepsilon}_{ij}$  values decrease to approximately 0.15, while  $\bar{\sigma}_{ij}$  values oscillate about 0.25, and the coupling matrix becomes partitioned, indicating the existence of larger clusters. Increasing the upper border  $\sigma_M$  to 0.75 both  $\bar{\varepsilon}_{ij}$  and  $\bar{\sigma}_{ij}$  tend to 0, and the coupling matrix becomes sparse.

In Fig. 4, the synaptic weights  $\varepsilon_{ij}$  and  $\sigma_{ij}$  are encoded in color for  $\gamma = 0.0$ ,  $\varepsilon_M = 0.25$ , and  $\sigma_M = 0.675$ , where we choose values of the parameters that provide the cases for (a)  $\bar{R} \approx 0.1$  and (b)  $\bar{R} \approx 1$  according to the inset in Fig. 2(a). The synaptic weights are suppressed in the desynchronized regime (Fig. 4(a)), and consequently the coupling matrix presents a small number of connections. This behavior can be verified by means of the black lines in Fig. 2(b) and (c). In addition, the synaptic weights are potentiated (red lines in Fig. 2(b) and (c)) in the synchronized regime (Fig. 4(b)), and the coupling matrix exhibits a triangular





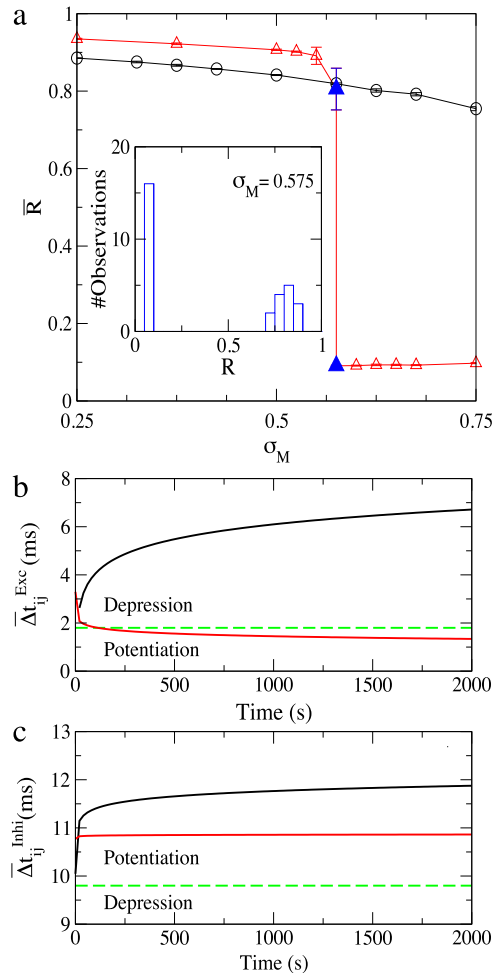
**Fig. 4.** Coupling matrix for  $\gamma = 0.0$ ,  $\varepsilon_M = 0.25$ , and  $\sigma_M = 0.675$ , where we choose values for parameters to provide the cases for (a)  $\bar{R} \approx 0.1$  showing many uncoupled neurons, and (b)  $\bar{R} \approx 1$  exhibiting many directed couplings, according to the inset in Fig. 2(a). The synaptic weights are encoded in color, where the maximum value for  $\varepsilon_{ij}$  (yellow) is 0.5 and the maximum value for  $\sigma_{ij}$  (blue) is 1.36. The neurons are ordered according to  $I_i \leq I_j$  for  $i < j$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

shape. We have verified that, in this case, the synchronous behavior has a dependence on the direction of synapses. In other words, when the presynaptic neurons are excitatory the synapses from the high frequency to the low frequency neurons become stronger. When the presynaptic neurons are inhibitory, the synapses from the low frequency to the high frequency neurons become stronger.

Fig. 4 shows the final topologies for two networks initially set with a global coupling topologies after being evolved by a STDP process. We see that the STDP induces a non-trivial topology in the network resulting in networks sparsely connected, moderately connected (Fig. 4(a)), or densely connected with strong preferential attachment (Fig. 4(b)).

Considering an external perturbation ( $I_i > 0$ ), we also study the cases without and with plasticity. In the case without STDP, we verify that the mean order-parameter has a small decay when  $\sigma_M$  increases, as shown in Fig. 5(a) with black circles. The red triangles represent the case with STDP, and unlike the case without perturbation (Fig. 2(a)), there is an abrupt transition (blue triangles), due to a first-order transition in the average order parameter. First-order transition is a term that comes from Thermodynamics and here represents a discontinuity of the mean order-parameter function with respect to the inhibitory coupling strength. In this case, the upper border of the inhibitory coupling is relevant to produce alteration in the dynamics, while the different initial conditions are important only at the transition. Based on the results in the inset (Fig. 5(a)), we verify that the network in the transition can be either in one of the states: (i) high  $\bar{R}$  with potentiation of the average-time difference for excitatory and inhibitory connections (red lines in Fig. 5(b) and (c)), or (ii) low  $\bar{R}$  with excitatory average time-difference in the depression region and inhibitory in the potentiation region (black lines).

The transition from the synchronized to the desynchronized states was reported in studies on how stimulation impact on



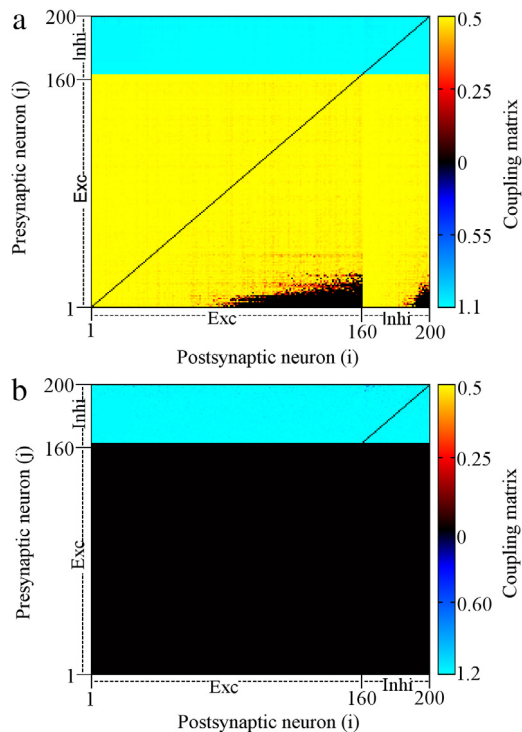
**Fig. 5.** (a) Mean order-parameter versus  $\sigma_M$  for  $\gamma = 10.0 \mu\text{A}/\text{cm}^2$ ,  $\varepsilon_M = 0.25$ , a result without STDP (black circles) and another one with STDP (red triangles). Inset plot for  $\sigma_M = 0.575$  (blue triangles) and 30 values of  $R$ , where each  $R$  is calculated from a different initial configuration. Figures (b) and (c) exhibit the time evolution of the average time-difference for excitatory and inhibitory connections, respectively, where  $\sigma_M$  is equal to 0.575. The black and red lines correspond to  $\bar{R} \approx 0.1$  and  $\bar{R} \approx 0.8$ , respectively. The green dash represents the separation between potentiation and depression. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

neurological disorders induced by an abnormal neuronal synchronization (Popovych & Tass, 2012; Tass & Majtanik, 2006). A first order transition was also observed in Popovych et al. (2013) when the stimulation intensity varies in a neural network with eSTDP. In our simulations, we observe the transition to desynchronization caused by a variation in the inhibitory coupling in neural networks with both eSTDP and iSTDP.

Fig. 6 illustrates the coupling matrix for the two states of the first-order transition. In Fig. 6(a), we can see the coupling configuration that corresponds to high  $\bar{R}$ . The network presents high connectivity, and for this reason it is possible to observe synchronous behavior. For the case of low  $\bar{R}$ , we verify that the network has only connections from neurons belonging to the inhibitory population to any other neuron, as shown in Fig. 6(b).

#### 4. Conclusion

In conclusion, we have studied the effects of spike timing-dependent plasticity on the synchronous behavior and the evolved connecting topology of neural networks constructed with Hodgkin–Huxley neurons. In our simulations, we considered parameter values for the Hodgkin–Huxley system and STDP



**Fig. 6.** Coupling matrix for  $\gamma = 10.0 \mu\text{A}/\text{cm}^2$ ,  $\varepsilon_M = 0.25$ . (a)  $\sigma_M = 0.55$  ( $\bar{R} \approx 1$ ) showing a large quantity of coupled neurons, and (b)  $\sigma_M = 0.6$  ( $\bar{R} \approx 0.1$ ) exhibiting connections from inhibitory to excitatory neurons. The synaptic weights are encoded in color, where the maximum value for  $\varepsilon_{ij}$  (yellow) is 0.5, the maximum value for  $\sigma_{ij}$  (blue) is 1.1 in (a) and 1.2 in (b). The neurons are ordered according to  $i_i \leq i_j$  for  $i < j$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

according to experimental values found in the neuroscience literature (Bi & Poo, 1998; Haas et al., 2006). Regarding the evolved topology, our main conclusion is that learning under a STDP results in evolved networks that present complex topology. Concerning the dynamic synchronous behavior of the evolved networks, we observe that the studied networks exhibit concurrent synchronous and non synchronous states with characteristics that depend on both the upper border of the inhibitory coupling and the initial conditions. Specifically, we verify that the main role of the inhibitory connections is to produce a delay in the spiking time of the postsynaptic neurons. As a consequence, the increase of the inhibitory coupling strength can suppress synchronous behavior, which contributes to a decrease in the mean order parameter. Moreover, the transition from low to a high synchronous state is smooth by alterations of the inhibitory synapses. When a random external perturbation is introduced in the network, this transition becomes discontinuous, i.e., we observe a first-order transition. Similarly to the non-perturbed network, we also find coexistence of synchronous and non-synchronous neurons in the perturbed networks.

In future works, we plan to study synchronous states in the brain considering plasticity dynamics in terms of the thresholds for switching on the long-term potentiation and the long-term depression. We also plan to investigate how the final behavior of the network depends on the initial population of excitatory neuron.

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