Modeling non-stationary, non-axisymmetric heat patterns in DIII-D

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Motivation

Homoclinic orbits (and surfaces) are structurally unstable in three dimensional Hamiltonian systems.

The plasma edge of single and double-null discharges is composed of homoclinic or heteroclinic magnetic field lines.

The geometry of the plasma edge will change under any symmetry break.
Saddles and Manifolds

Saddle orbits are structurally stable in three dimensions, but the homoclinic surface splits into separated invariant manifolds.

When axial symmetry breaks the magnetic perturbations destroy the homoclinic field lines.

The new invariant surfaces define the plasma edge.

The lobes define multiple strike regions with helical patterns.
The non-axisymmetric lobes have been observed experimentally and can be modeled using vacuum field perturbations.

Any asymmetric magnetic perturbation creates the lobes.

\[
\tilde{B} = \nabla \psi \times \nabla \phi + F(\psi) \nabla \psi + \tilde{B}_v
\]
Coherent structures

If manifolds determine the plasma edge geometry for stationary magnetic fields, it may do so for time-dependent fields.

MHD oscillations and instabilities break the axisymmetry and evolve in a regular fashion.

At each time-slice the magnetic field pattern creates new invariant manifolds.

Transition from one manifold geometry to the next must be smooth.
Experimental Signatures

In DIII-D discharge #158826 the magnetic signals and divertor heat pattern evolution are synchronized.

The coupling continues until the plasma disrupts.
Modeling the plasma response

The perturbation field has internal and external sources but is dominated by the plasma response.

The magnetic probes reveal a dominant \( m=1, n=1 \) mode. It can be due to an internal kink (ideal), tearing mode (resistive), or a helical core.

The magnetic pattern appears to rotate toroidally with frequency around 7Hz.

The amplitude of the fluctuations grows monotonically at a small rate.
We consider that the response currents are concentrated in the \( q=1 \) surface and that rotate toroidally at a constant angular frequency.

\[
\delta \vec{j}(\vec{r}, t) = \sum_{i=1}^{N_h} I_i \vec{b}(\vec{r}, t) \delta (\vec{r} - \vec{r}_h(\phi_i - \omega t))
\]

\( \delta \vec{j}(\vec{r}, t) \) : Plasma response current.

\( \vec{r}_h(\phi_0) \) : Helical filament starting on the LFS mid-plane of the \( q=1 \) surface at toroidal angle \( \phi_0 \).

\( \delta (\vec{r} - \vec{r}') \) : The usual Dirac delta.

\( \vec{b}(\vec{r}, t) \) : Direction of the magnetic field in \((\vec{r}, t)\).

\( I_i \) : Current on the \( i^{th} \) filament.

\( N_h \) : Number of filaments.

Taking the rotational we recover the Biot-Savart law for a set of rotating filaments.
A minimum model requires two filaments. Their currents and relative phase were adjusted numerically.

The modeled wave-forms adjust well the magnetic probes measurements.

- probes signal.
- modeled signal.
Building the manifolds

Identify the Unstable Periodic Orbit (UPO), or saddle orbit for the perturbed system.

Identify the eigenvectors of the Poincaré map and build an elementary segment near the saddle point.

Map forward and backwards the elementary segments.
Some advertisements...

The UPO is located within a precision of $10^{-10}$ m, using a modified Levenberg-Marquardt procedure.

The field line integration method is adaptive five order and package independent.

Each manifold section segments is calculated from the previous instead of the first (CPU time reduces >700%).

Lagrange polynomials are used to start new orbits on the fly.

The manifold can be calculated along the confining chamber to determine the weted area.
The instants of the heat measurements are converted to phases of the rotating frame and compared with the rotating manifold at the strike plane.

The wetted area from the manifold calculation matches well with the peak locations and the extension of the heat deposition profile.
Conclusions

The invariant manifolds created by time-dependent magnetic fields are coherent structures evolving smoothly in time.

The plasma edge evolution can be described in terms of the changing invariant manifolds created by the internal and external non-axisymmetric currents.

Invariant manifold calculations describe well the deposition patterns with few assumptions and minimalist models.

Perspectives

Transport calculations can be used along simple models of the magnetic field to calculate the heat deposition profiles.