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Dealing with final state sensitivity for synchronous communication

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Abstract

We show differences of the synchronization basins of two Matsumoto-Chua's circuit (operating in either the Rössler-type attractor regime or the Double-Scroll-type attractor regime), coupled unidirectionally or bidirectionally through a negative feed-back controller. The knowledge of the structure of the synchronization basins allows one to design a communication system robust to imperfections on the setting of the initial condition. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

In using nonlinear dynamical systems for communication systems we must deal with the following dynamical characteristics: (a) The sensitivity of the final state on the initial condition; (b) grammatical limitations imposed by the dynamics; (c) the recovery of the noisy and dumped signal. In this paper we address the first problem.

One basic problem in using chaos to communicate is the fact that the trajectory initial condition changes as we turn on-and-off the circuit [1]. In communicating with chaos [2–8], the message is encoded somehow by the chaotic trajectory. Thus, after the nonlinear wave-signal generator is turned on, the message is successfully transmitted if the final state of the trajectory is robust to imperfections on the definition of the initial condition.

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The sensitivity dependence of chaotic system to the initial condition would not be a problem if the basins of attraction (the sets of initial conditions that go toward attractors) were not so complex as it is often the case, when they have fractal basin boundaries [9] (*strong* sensitivity of the final state on the initial condition), riddled basins [10,11] (*extreme* sensitivity), or other mixed types of basins [12].

In this work we are mainly interested in dealing with situations for which we find riddled basins, with extreme dependence on the initial condition, like those observed for electronic circuits [1,13] and mechanical machines with impacts [14,15] that present some sort of gear.

Designing and implementing cheap and efficient techniques to overcome the problem of final state sensitivity is of fundamental importance to any communication systems based on chaos. Our main intention here is to present a communication system where information is transmitted by synchronizing [16–20] a circuit A with a circuit B, using unidirectional or bidirectional coupling.

The circuit considered in this work is the Matsumoto-Chua circuit [21,22]. We choose it because it represents a large class of electronic circuits that present typical behavior that can be potentially exploited in applications to communication systems. One useful characteristic is the coexistence of two symmetric chaotic attractors, which can be exploited for multi-channel, multi-user systems [23,24]. Another is the existence of one single more complex chaotic attractor, which offers capacity for sending higher rates of information.

Our main intention is to understand the operation of this circuit when used as a signal wave generator of a communication system based on synchronization.

The unidirectional coupling used in this work is inspired in the work proposed in Ref. [25], and it is a type of negative feedback. This type of coupling is used for a one-way transmission of information, that is, information transmitted from A to B. We show that the bidirectional coupling may diminish the sensitivity dependence on initial conditions. In addition, this type of coupling can be used to implement a two-way transmission of information system, for which the response of the receiver depends on the information transmitted by the transmitter.

We show that the synchronization basins may be riddled depending on the coupling. Understanding how these basins appear guide us to discriminate initial conditions that lead to final states suitable for communicating.

2. Description of the system

We represent the elements of our communication system by two coupled Matsumoto-Chua circuits, each one composed by two capacitors, C_1 and C_2, one inductor, L, one linear resistor, R, represented by $g = 1/R$, and one piecewise nonlinear resistor, represented by i_{NR}. The evolution of circuit A is described by the equations

$$C_1 \frac{\mathrm{d}V_{C_1}^A}{\mathrm{d}t} = g(V_{C_2}^A - V_{C_1}^A) - i_{NR}^A(V_{C_1}^A)$$

$$C_2 \frac{dV_{C_2}^A}{dt} = g(V_{C_1}^A - V_{C_2}^A) + i_L^A + \sigma_A K(V_{C_2}^B - V_{C_2}^A)$$

$$L \frac{di_L^A}{dt} = -V_{C_2}^A \tag{1}$$

and circuit B is described by

$$C_1 \frac{dV_{C_1}^B}{dt} = g(V_{C_2}^B - V_{C_1}^B) - i_{NR}^B(V_{C_1}^B)$$

$$C_2 \frac{dV_{C_2}^B}{dt} = g(V_{C_1}^B - V_{C_2}^B) + i_L^B + \sigma_B K(V_{C_2}^A - V_{C_2}^B)$$

$$L \frac{di_L^B}{dt} = -V_{C_2}^B , \tag{2}$$

where V_{C_1}, V_{C_2}, and i_L, represent the tension across the capacitor with $C_1 = 10.0$, the tension across the capacitor with $C_2 = 1.0$, and the current across the inductor with $L = 6.0$, respectively. The current in the piecewise linear resistor i_{NR} is

$$i_{NR} = m_0 V_{C1} + \tfrac{1}{2}(m_1 - m_0)(|V_{C1} + B_p| - |V_{C1} - B_p|) , \tag{3}$$

where $m_0 = -0,5$, $m_1 = -0,8$, and $B_p = 1,0$.

Integration is performed using the fourth-order Range–Kutta method with the integration time step of 0.04.

Circuits A and B are coupled according to the values of the constants σ_A and σ_B. Two configurations are used in this work. Unidirectional coupling, for $\sigma_A = 0$ and $\sigma_B = 1$, and bidirectional coupling, for $\sigma_A = 1$ and $\sigma_B = 1$. K is the coupling amplitude. We allow g to assume two values, $g = 0.575$ (Rössler-type attractor for uncoupled circuits), and $g = 0.6$ (Double-Scroll attractor for uncoupled circuits). The unidirectional coupling was first proposed by Pyragas [25] and it is a type of negative feedback. We place the coupling term in the equation for the derivative of the tension across the capacitor C_2 because the values of the conditional Lyapunov exponent stay negative even for higher values of the coupling amplitude.

It is appropriate to make a changing of coordinates such that the (non)synchronized state is easily recognized and characterized. Thus,

$$V_{C_1\perp} = V_{C_1^A} - V_{C_1^B}, \quad V_{C_1\|} = V_{C_1^A} + V_{C_1^B} \tag{4}$$

$$V_{C_2\perp} = V_{C_2^A} - V_{C_2^B}, \quad V_{C_2\|} = V_{C_2^A} + V_{C_2^B} \tag{5}$$

$$i_{L\perp} = i_{L^A} - i_{L^B}, \quad i_{L\|} = i_{L^A} + i_{L^B} .$$

The variables with indexes $\|$ are on the synchronization manifold while the variables with indexes $\perp$ are on the transversal manifold. If circuit A and circuit B are synchronized, the transversal variables are all null, and the trajectory of the coupled 6-D system (Eqs. (1) and (2)) is on a reduced 3-D dimension subspace on the synchronization manifold. Otherwise, the transversal variables are different than zero. Stability analysis of Eqs. (1) and (2) are performed calculating the conditional Lyapunov exponent [18] of the new variables (5). If there is one transversal conditional exponent higher

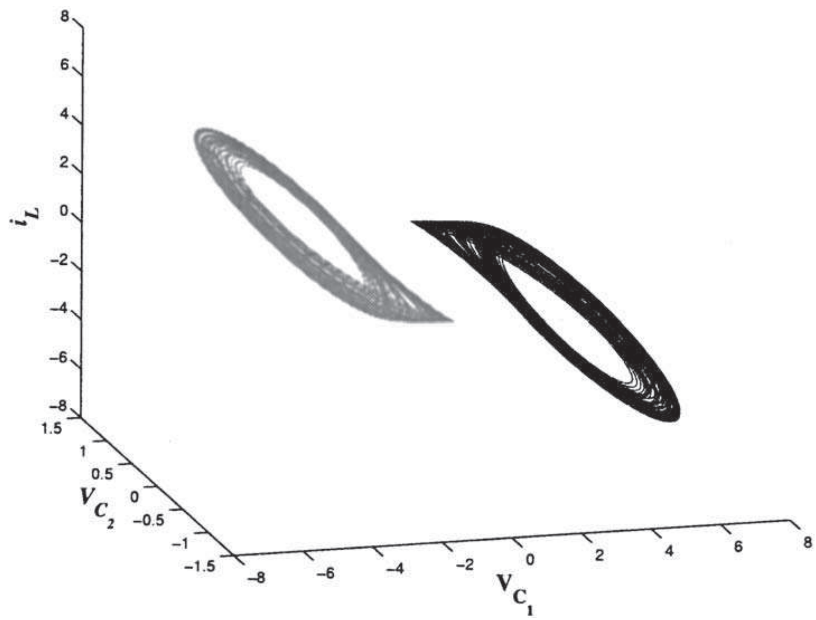

Fig. 1. Two chaotic Rössler-type coexisting attractors of Eq. (1) for $g = 0.575$.

than zero then circuits A and B cannot synchronize. If all the transversal conditional exponents are smaller than zero, the circuits may synchronize depending on the initial condition (as discussed later on). Note that the conditional Lyapunov exponents does not give a sufficient condition to define the final state of the system.

If there is one transversal exponent slightly higher then zero, then there might appear on-off intermittent behavior [20], and thus, nonsynchronized state. This state is characterized by a trajectory which presents regular behavior (synchronized state) and irregular behavior (nonsynchronized state).

If the maximum transversal Lyapunov exponent is slightly smaller than zero, then there might appear a riddled basin, whose characteristics is that the final state cannot be predicted even for higher precision initial condition.

3. Coupling while in the Rössler-type attractor

Eqs. (1) for σ_A=0 and g=0.575, present two coexisting Röslller-type chaotic attractors (left and right attractors seen in Fig. 1), three equilibrium points, and one large unstable limit cycle (not shown in Fig. 1). We are mainly interested in the two coexisting chaotic attractors. The final state of the system, i.e., left or right attractor of Fig. 1, is depicted by the basin of attraction of the noncoupled system. In Fig. 2, one can see the basin of attraction of the attractors of Fig. 1, for $g = 0.575$ and a fixed $i_L(0) = 0$.

These two basins have smooth continuous boundary that spirals-off asymptotically toward the external unstable limit cycle. This type of basin which was experimentally verified in Ref. [26], has no significant final dependence on the initial condition, and the experimental set up of the initial condition on the plan $i_L(0) = 0$ was implemented

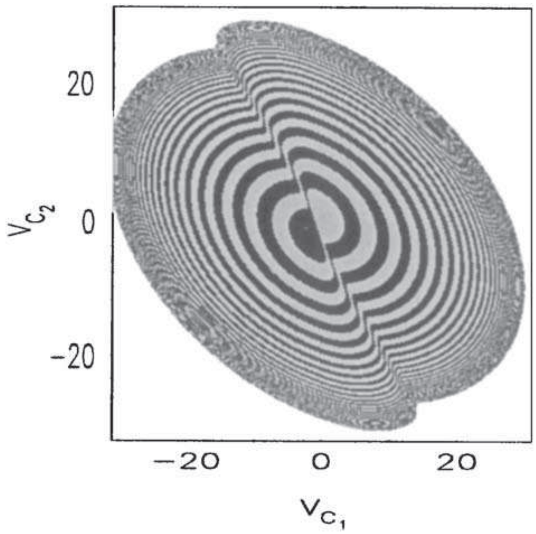

Fig. 2. Basins of attraction of attractors of Fig. 1. Black (gray) points belong to the basin of the right (left) attractor. White points go to the infinity attractor. $g = 0.575$ and $i_L(0) = 0$.

by a special device used in Ref. [26]. This type of basin would be very attracting for a nonlinear communication system, once the initial position would not need to have a high precision to reach some aimed final state.

This smoothly continuous boundary of the basins changes its characteristics when coupling is introduced in the circuits, and what we see is a continuous but not smooth boundary. However, the basins of Fig. 2 help us in understanding the synchronization basin of the coupled circuits for unidirectional and bidirectional coupling.

To see the differences between unidirectional and bidirectional coupling we choose a fixed coupling amplitude $K = 2.0$, and we plot a projection of the synchronization basin on a plane positioned on $V^A_{C_2}(0) = V^B_{C_2}(0) = 0.2334$ and $i^A_L(0) = i^B_L(0) = 0.845$ for the variables $V^A_{C_1}(0)$ (horizontal axis) and $V^B_{C_1}(0)$ (vertical axis), for unidirectional (Fig. 3) and bidirectional couplings (Fig. 4). In these figures, white represents initial conditions that synchronize the circuits, while black and gray regions represent initial conditions that do not synchronize. Black represents $V^A_{C_1}$ and $V^B_{C_1}$ close to the region of the left attractor (noncoupled circuit) of Fig. 2; gray represents $V^A_{C_1}$ and $V^B_{C_1}$ close to the region of the right attractor of this figure.

The squared structures of these basins can be explained through the basins of Fig. 2. In general, for circuits A and B with initial conditions in different basins (in Fig. 2), the final state will not synchronize, regardless the value of the coupling amplitude K. Moreover, in Fig. 3, synchronizing white regions correspond to initial conditions $V^A_{C_1}(0)$ and $V^B_{C_1}(0)$ that would go toward the same attractor, if there were no coupling.

Note that the basins of Fig. 2, for $i^A_L(0) = i^B_L(0) = 0$, are similar to those obtained for $i^A_L(0) = i^B_L(0) = 0.845$. The reason for Fig. 2 to be constructed with $i^A_L(0) = i^B_L(0) = 0$ is that this is a convenient set of initial conditions to experimentally obtain the basin of attraction [26].

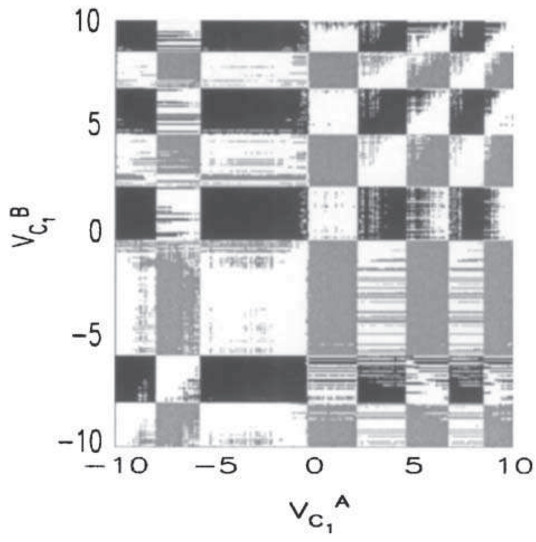

Fig. 3. Synchronization basin (white) and nonsynchronization basin (gray and black) of Eqs. (1) and (2) for unidirectional coupling of amplitude $K = 2.0$. $g = 0.575$.

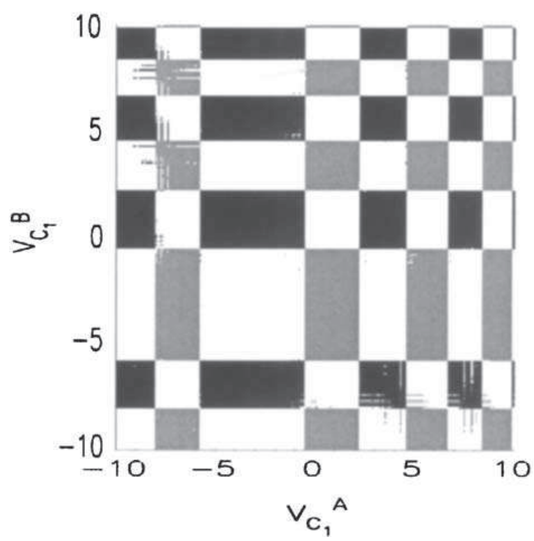

Fig. 4. Synchronization basin (white) and nonsynchronization basin (gray and black) of Eqs. (1) and (2) for bidirectional coupling of amplitude $K = 2.0$. $g = 0.575$.

For the bidirectional coupling the basin for the synchronized state is bigger than for the unidirectional case. There is a reason for the white region to be larger in Fig. 4 than in Fig. 3. This can be understood by comparing the variations of the transversal conditional Lyapunov exponent, in respect with the coupling amplitude K, for the unidirectional (full line in Fig. 5) and the bidirectional cases (dashed line in Fig. 5). As we can see, for $K < 3.47$, the value of λ is smaller for the bidirectional case. After this value of K, λ for the unidirectional case is smaller, showing that the bidirectional coupling is benefitted by synchronization if used for lower K.

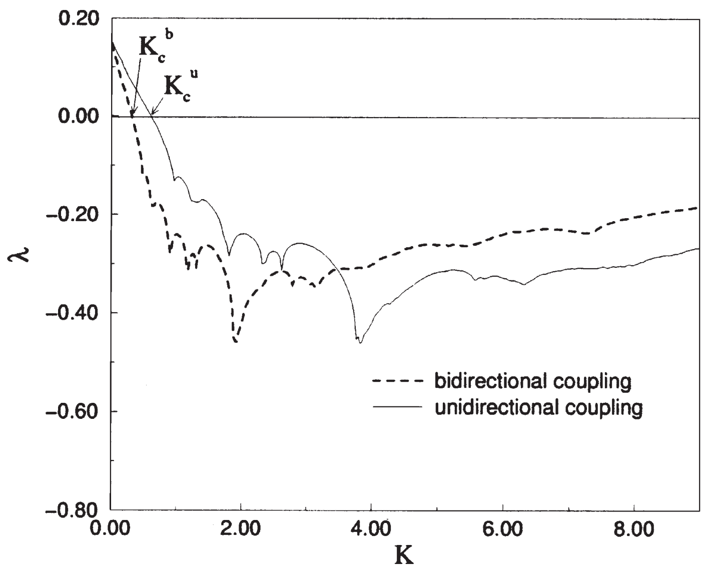

Fig. 5. Transversal Lyapunov exponent versus the coupling amplitude K, for unidirectional coupling (full line) and bidirectional coupling (dashed line). $g = 0.575$. Critical values (for $\lambda = 0$) are indicated.

The values of K for which λ changes its value from positive to negative, i.e., when $\lambda = 0$, are the critical parameters K_c^u (unidirectional coupling) and K_c^b (bidirectional coupling) indicated in Fig. 5. As we see, $K_c^b < K_c^u$. For both couplings these critical values represent the blowout bifurcation, when enough unstable periodic orbits of the 3-D synchronization manifold become transversally unstable. The value of $K = K_r$, for which the first unstable periodic orbit on the 3-D synchronization manifold becomes transversally unstable, is called riddled bifurcation critical point (also called of bubbling bifurcation [27]). Thus, a riddle basin may appear for $K_r < K < K_c$.

Although a riddle basin has no area, it has positive measures. Given two basins, the synchronized and nonsynchronized ones, the synchronized basin is riddled if an sphere with arbitrary small radius centered in a point of the synchronized basin has within it a point of the nonsynchronized basin, forming an open set, which does not contain only points of itself. If the nonsynchronizing basin is also riddled, then it is an intermingled basin [10,11].

For $K < K_c$ and unidirectional coupling, there is no riddled basin on the plane $V_{C_2}^A(0) = V_{C_2}^B(0) = 0.2334$ and $i_L^A(0) = i_L^B(0) = 0.845$ for the variables $V_{C_1}^A$ (horizontal axis) and $V_{C_1}^B$ (vertical axis) as shown in Fig. 6. We see that the boundaries are continuous (but not smooth), and thus, we have neither a fractal nor a riddled basin. The synchronization basin has still considerable size comparable to the nonsynchronization basin. A similar result is obtained for bidirectional coupling.

For $K > K_c$ and close to K_c we obtained on-off intermittency. The trajectory alternates its behavior intermittently, changing from a regular phase, expending some time in the vicinities of the synchronization manifold, to irregular bursts, being expelled to the phase space along the transversal manifold. The synchronization basin, on the plane of Fig. 4, for this case of coupling, has nonsmooth continuous boundary, and the

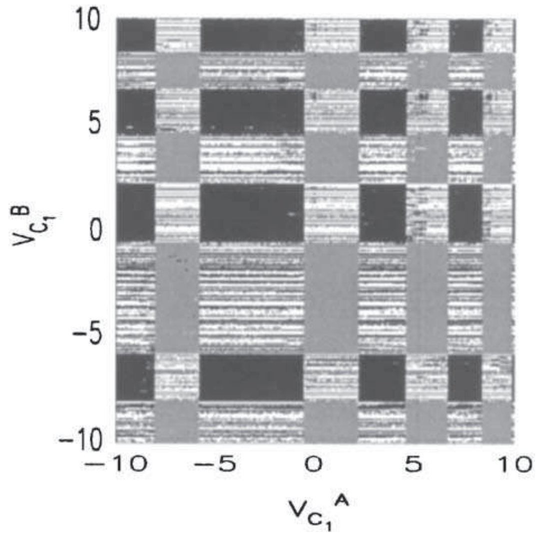

Fig. 6. Synchronization basin (white) and nonsynchronized basin (gray and black) of Eqs. (1) and (2) for unidirectional coupling of amplitude $K = 0.6$. $g = 0.575$.

nonsynchronization manifold fill almost completely the synchronization basin, leaving only a few initial conditions that belong to the other basin.

Synchronization is not always aimed in communication. As suggested in Ref. [23], the two nonsynchronized states of the coupled circuits, when operating in the Rössler-type attractor, can be used to transmit some kind of information. And, once the non-synchronized basins have continuous boundaries, the final nonsynchronized states can be precisely determined, and thus, communicating with the two synchronized states can be performed even for coupling in the range $K_r < K < K_c$.

The fact that riddling is not found for $K < K_c$ (Eqs. (1) and (2), with $g = 0.575$) is certain when the synchronization basin is on the plane defined to make Fig. 4. However, out of this plane, this might not be true. As shown in Ref. [1], the synchronization basin on the 3-D synchronization manifold is intermingled. This means that riddling behavior may depend on the location of the synchronization basin.

4. Coupling while in the Matsumoto-Chua-type attractor

Next, we change to $g = 0.6$ in Eqs. (1) and (2). In this case, for $\sigma_A = \sigma_B = 0$, we have the Double-Scroll attractor. The attractor can be seen in Fig. 7. Comparing this attractor with the one of Fig. 1, we see that it is more complex, in specific its topological entropy is higher, an indication that this Double-Scroll attractor generates more information, and therefore the use of its signal can carry more information [7].

The dependence of the conditional Lyapunov exponents with respect to K, for Eqs. (1) and (2) with unidirectional coupling (full line in Fig. 8) and with bidirectional coupling (dashed line in Fig. 8), is similar to what was shown in

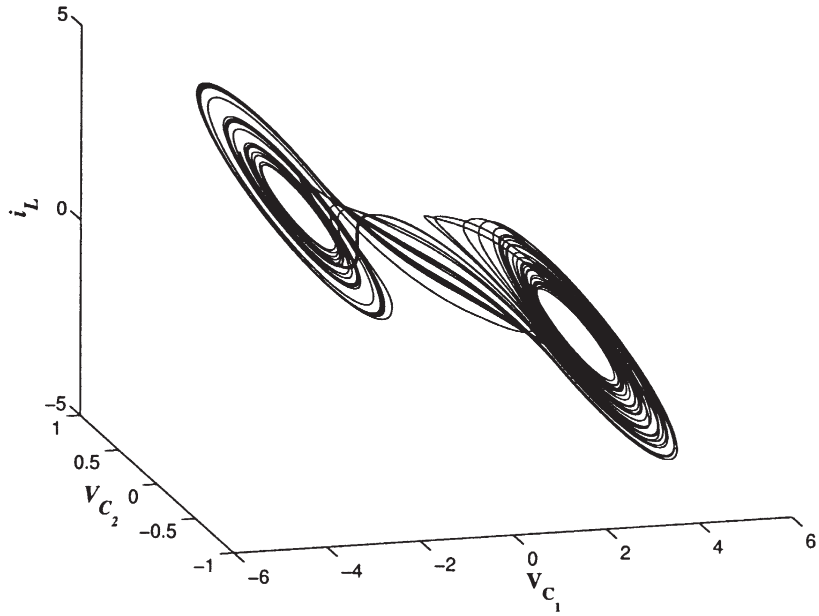

Fig. 7. Double-Scroll attractor obtained making $\sigma_A = 0$ and $g = 0.6$ in Eq. (1).

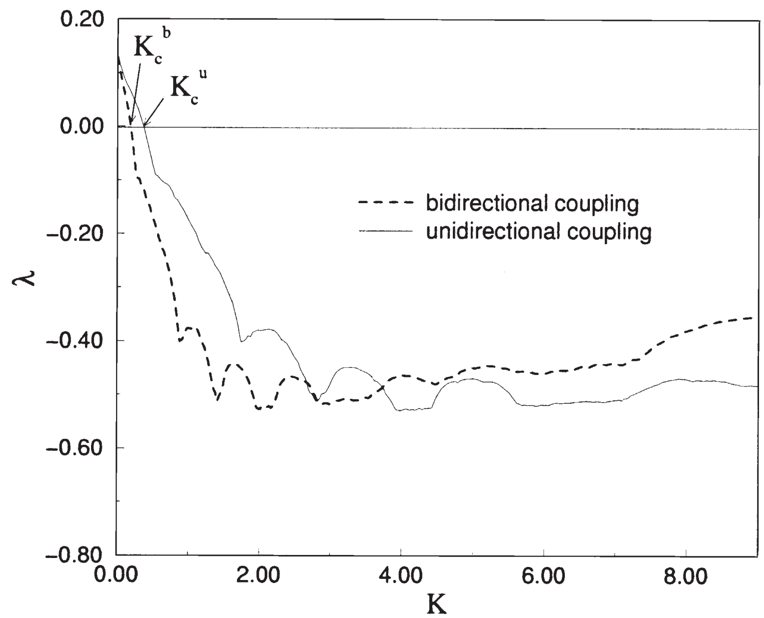

Fig. 8. Transversal Lyapunov exponent versus the coupling amplitude K, for unidirectional coupling (full line) and bidirectional coupling (dashed line). $g = 0.6$.

Fig. 5, that is, bidirectional coupling reduces instabilities for K smaller than the value $K \cong 3.73$.

Considering K_r the parameter for which the riddled bifurcation comes about, the synchronization basin of this coupled circuits for $K < K_r$ should have only two basins, the infinity basin and the synchronized basin.

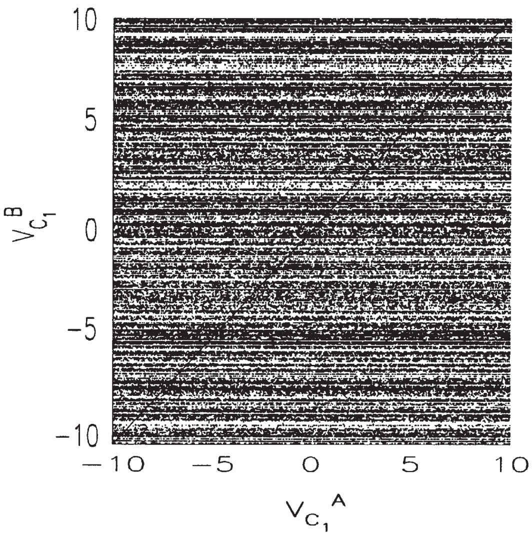

Fig. 9. Synchronization riddled basin of the circuits with unidirectional coupling of amplitude $K = 0.2$, and $g = 0.6$.

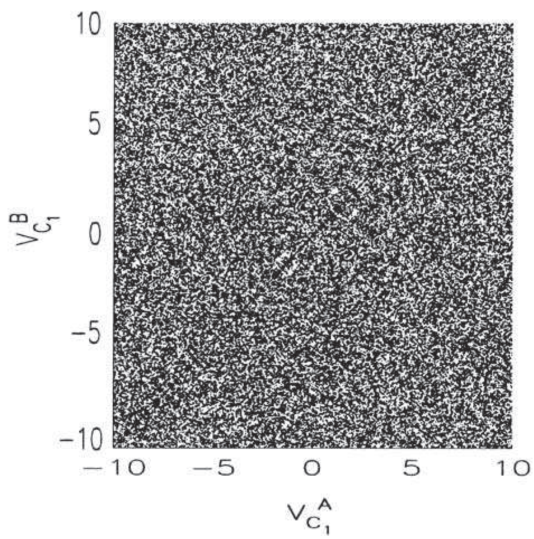

Fig. 10. Synchronization intermingled basin of the circuits with bidirectional coupling of amplitude $K = 0.2$, and $g = 0.6$.

However, contrary to what we saw in the previous section, for K in the critical interval, $K_r < K < K_c$, the basin becomes riddled for unidirectional coupling, Fig. 9, and intermingled for bidirectional coupling, Fig. 10.

The characterization of such basin is done by calculating the uncertainty exponent α (defined in Ref. [28]), for the basin of Fig. 9, obtaining $\alpha_u = 0.0006 \pm 0.0006$ (riddled basin), and for the basin of Fig. 9, obtaining $\alpha_b = 0.0005 \pm 0.0003$. To understand what this exponent means, suppose we can specify an initial condition with precision ε. The uncertain probability $f(\varepsilon)$ of predicting incorrectly the future state of the system (synchronized or not) is given by $f(\varepsilon) \approx \varepsilon^{\alpha}$. So, for an initial condition with precision of 10^{-16}, $f(\varepsilon) = 0.98$, thus, we have 98% of chance to predict incorrectly the final state. In fact, theoretically for riddled basins this exponent should be null. Even though the difference is very small, $\alpha_u < \alpha_b$, showing that systems with intermingled basin are, at least by the numerical point of view, more sensible to the final state determination.

To calculate this exponent, we randomly pick 10000 initial conditions on the space represented by Figs. 9 and 10. Each point generates a pair of δ-perturbed points given by $(V^A_{C_1}, V^B_{C_1} \pm \delta)$, which are iterated for an interval of time $\Delta t = 400$. The final state of the three (the randomly chosen initial condition and its perturbed pair of points) points are then verified. If any one of the perturbed points have a final state different than the nonperturbed one, we say that this point is uncertain.

The transient $\Delta t = 400$ represents an interval of time within which the trajectory visits a given Poincaré section 100 times. To choose this transient we observe that the synchronization basins (Figs. 2–4, 6, 9, 10) does not present perceptible changes for a transient time within the interval [200,10000]. In the view of this result, we find appropriate to choose the transient to be Δt=400. In order to verify if the synchronized state is indeed stationary we check if the distance $V^A_{C1} - V^B_{C1}$ remains bounded to 0.005 for an integration time of $t = 80$.

The fraction of uncertain points, $f(\varepsilon)$ over the whole amount (10000) is then calculated for 11 different values of ε, ranging from 10^{-2} to 10^{-10}. The uncertainty exponent is the slope of the log–log plot of $f(\varepsilon)$ versus ε.

For $K > K_c$ we can have on-off intermmitency, depending on the initial condition chosen. Otherwise the only other final state allowed is the nonsynchronized state.

5. Conclusions

In the unidirectional coupled Matsumoto-Chua system, riddled basin can be avoided by choosing special initial conditions. So, $g = 0.575$ (Rössler-type attractor for the uncoupled circuit), the final state has no dependence on these sets for different ranges of values of K, even for critical values, when K is close to K_c (for which the conditional Lyapunov exponent becomes positive).

In general, when the initial condition of circuit A is in a different basin than the circuit B, the final state will not be the synchronized one, regardless the value of the coupling amplitude K, observation already discussed in previous works as [29]. Thus, in cases like this, if the final full synchronized state is required, we suggest the use of the OPCL method [30,31]. In this method, there are ways to obtain the set of initial conditions which will drive the coupled circuits to the synchronized state (the synchronization basin), even if the circuits have their initial conditions set in different basins.

The absence of coexisting chaotic attractors for the Double-Scroll regime ($g = 0.6$), at first glance, can let us speculate that the synchronization basin of the coupled circuits is less complex than those obtained for the Rössler system. But that is not the case. As we show, due to a higher Lyapunov exponent of the Double-Scroll attractor, the synchronization basin for this regime is highly more complex than that for coupled circuits independently operating in the Rössler-type attractor regime ($g = 0.575$).

The trajectory of the Double-Scroll attractor is able to carry more information than the trajectory of the Rössler system, but it is more difficult for synchronizing circuits to operate in the Double-Scroll regime.

So, when using in the coupled system $g = 0.6$, the critical interval ($K_r < K < K_c$) is to be avoided, if the coupling is a negative feedback of the type used in this work.

We characterized the existence of the riddled and intermingled basins through the uncertainty exponent. This calculation demonstrated that the final state of a system with riddled basin is undetermined.

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