Entropy growth in billiards

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Outline

1. Entropy Ansatz
2. Classical Billiards
3. Quantum Billiards
4. Future work

Entropy Ansatz.

$$\rho\left(x,t\right)=\frac{1}{t^{\delta}}F\left(\frac{x}{t^{\delta}}\right) \tag{1}$$

- δ Diffusion exponent.

$$S=-\int\rho\ln\left(\rho\right)dx \tag{2}$$

- Is coordinate invariant (in phase space).

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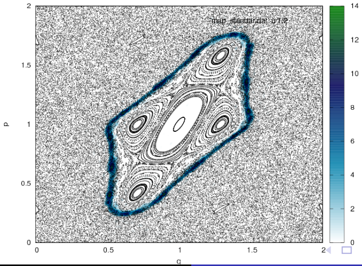
Standar Map.

$$p_{n+1} = \left[p_n + \frac{k}{2\pi} \sin\left(\pi q_n\right) \right] mod\left(2\right) \tag{3}$$
$$q_{n+1} = \left[q_n + p_{n+1} \right] mod\left(2\right)$$

Figure: Initial conditions along KAM illand $k = 2.31$



Entropy Growth.

Figure: Entropy Growth for $k = 2.21$ causes $\delta = 0.2484$

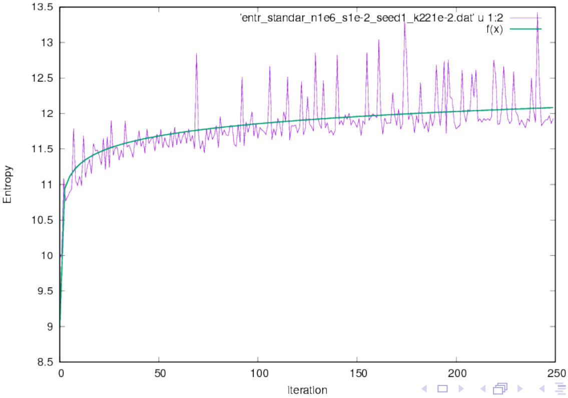
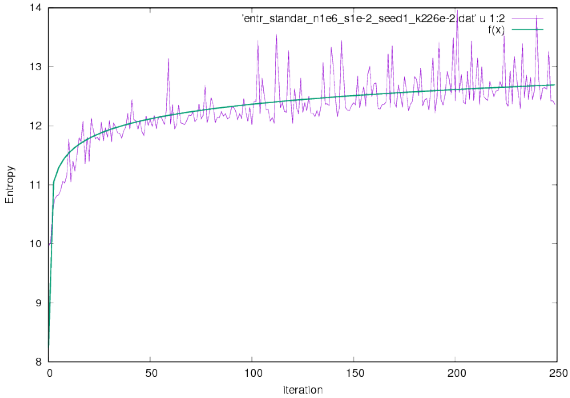


Figure: Entropy Growth for $k = 2.26$ causes $\delta = 0.3586$



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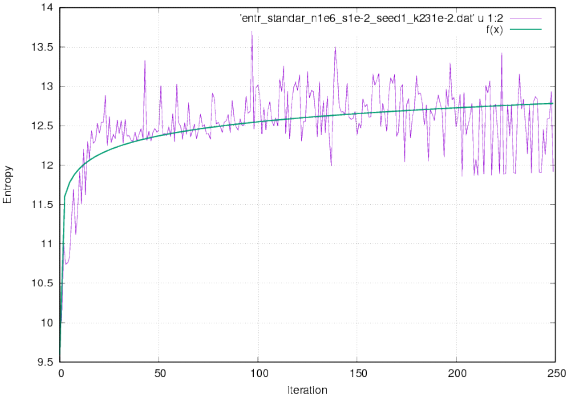

Schrödinger Equation.

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2} \nabla^2 + V(x) \right] \psi(x,t) \quad (4)$$

- “Comfortable” equation.
- Partial Differential Equation.
- Small numbers (many)-multiplications.
- Wave particle duality.
- Quantum to Classical transition.
- Breaks when are many particles

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Feynman QM.

$$A = \sum_{All\ Paths} \exp\left(\frac{i}{\hbar}\int L(x, \dot{x}, t)\, dt\right) \tag{5}$$

- Generalizes to Relativistic Quantum Mechanics.
- Small number division.
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$$\frac{d^2 Q_i}{dt^2} = -\nabla \left(V(x) - \frac{\hbar^2}{2} \frac{\nabla^2 \sqrt{\rho(x)}}{\sqrt{\rho(x)}} \right) \Bigg|_{x=Q_i} \tag{6}$$

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- Quantum potential is "difficult" to calculate.
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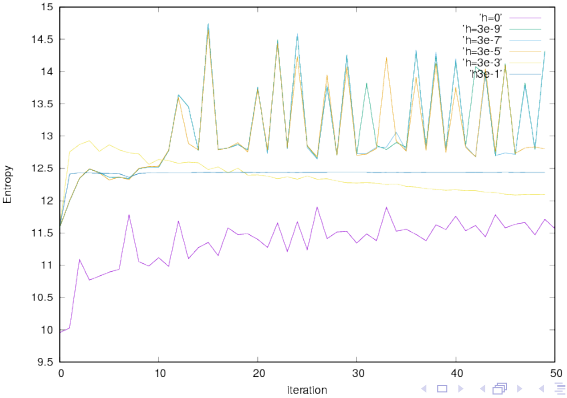
Entropy growth in quantum billiards.

Figure: Entropy Growth for $k = 2.21$ and different values of $\hbar$.



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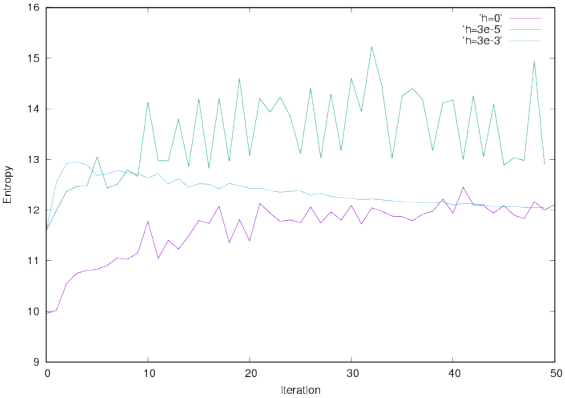

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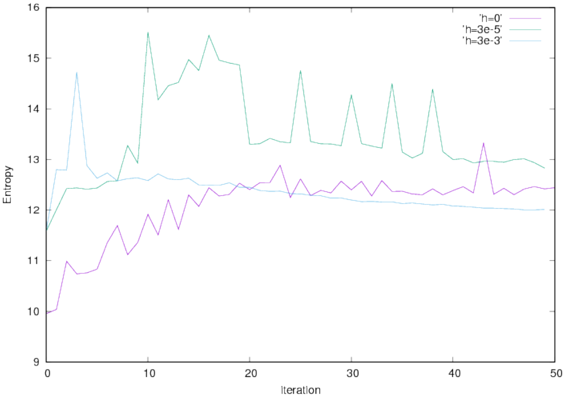

Future work.

- Entropy ansatz can be used.
- QB $\leftrightarrow$ CB relations.
  - Difussion exponent $\leftrightarrow$ Transition rate.
- Simulations to find thermodynamic limit.

$$\frac{d^2 Q_i}{dt^2} = -\nabla\left(\ln\left(\rho\right)\right)\big|_{x=Q_i} \tag{8}$$

Summary

- The Entropy Ansatz gives δ in an easy way.
- Bohmian Mechanics has an easy numerical simulation for the transition QM-CM.

- Left stuff
  - Hussimi representation.
  - Von Neuman equation.
  - Ergodic and Regular Quantum Billiards.