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Magnetic structure of toroidal helical fields in tokamaks

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Abstract

Using an averaging method to solve the differential field line equations, we present a simple procedure to determine analytically the main magnetic islands structure of toroidal tokamak plasmas perturbed by resonant helical windings Analytical results are compared with Poincaré maps. for small perturbations the sizes and the positions of the islands agree well, even for satellite islands

1. Introduction

Resonant helical windings (RHW) have been widely used in tokamaks to control and to investigate the nature of the disruptive instabilities [1–4]. Perturbations due to RHW just below a critical value can inhibit the Mirnov oscillations. An explanation of this stabilizing effect has been suggested by the Pulsator Team [1]: RHW create a fixed island structure within the plasma that would hinder a rotation of the MHD modes. Increasing the helical field, minor disruptions occur in the resonant surface and neighbouring rational surfaces until the confinement is totally lost, these disruptions can be explained in terms of randomization of the magnetic field lines.

Several authors have used different techniques and approximations in order to determine the magnetic islands structure of a tokamak plasma perturbed by RHW. Finn [5] obtained numerically this structure for a large aspect-ratio tokamak. Elsässer [6] and Cary and Littlejohn [7] employed the Hamiltonian formalism in their analytical analysis. Camargo and Caldas [8] calculated an average vector potential which describes average magnetic surfaces for this same system.

In this work we present an alternative procedure to solve this problem. We wish to stress two points of our work: (1) the toroidal shape of the tokamak is considered: as consequence a single helical perturbation mode (m, n) creates magnetic islands not only at the principal resonance region $q=m/n$, but also at $q=(m\pm 1)/n$ (q is the safety factor), and magnetic islands appear with different sizes on the same rational surface; (2) expressions for the magnetic surfaces around resonances are determined analytically applying an averaging method [9] to solve the differential field line equations. Analytical results are compared with Poincaré maps obtained by numerical integration of the field line equations, using typical parameters of tokamak TBR-1 [3]. These maps are useful in stability analysis and help to find an adequate helical current to control the magnetic oscillations.

Below, we will show how to calculate the equilibrium plasma field, the RHW field and the magnetic

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structure of the perturbed plasma using the analytical method

2. Equilibrium plasma field

A new set of coordinates $(\rho_t, \theta_t, \varphi)$ called "toroidal polar" has been introduced to describe toroidal systems [10] (see Fig. 1). These coordinates can be written in terms of local polar coordinates (ρ, θ, φ),

$$\rho_t = \rho[1-(\rho/R_{0'})\cos\theta + (\rho/2R_{0'})^2]^{1/2},$$
$$\sin\theta_t = \sin\theta\,[1-(\rho/R_{0'})\cos\theta + (\rho/2R_{0'})^2]^{-1/2}, \quad (2.1)$$

where

$$R_{0'} \simeq R_0[1+\tfrac{1}{2}(a/R_0)^2]. \quad (2.2)$$

R_0 and a are the major radius and the radius of limiter, respectively. ρ_t, θ_t and φ have the meaning of radial, poloidal and toroidal coordinates For large aspect ratio $(R_0/a \gg 1)$ ρ_t and θ_t become ρ and θ, respectively.

The Grad–Shafranov equation was written and solved in these new coordinates [10]. The magnetic poloidal flux $(2\pi\Psi_p)$ of the magnetic field $\boldsymbol{B}_0$ of the plasma in static MHD axially symmetric equilibrium is

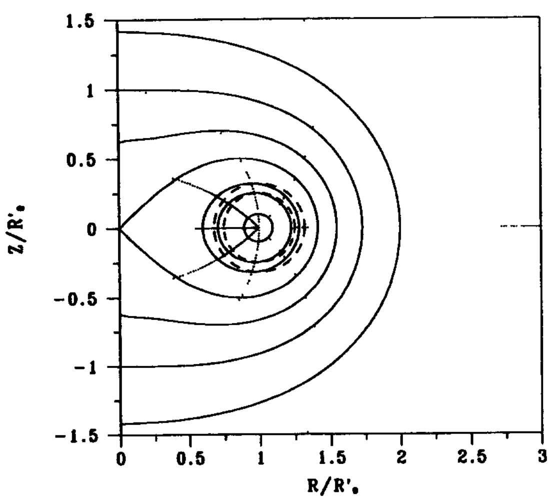

Fig. 1. Coordinate surfaces ( ) constant-θ_t surfaces, (- - -) constant-ρ surfaces, (——) constant-ρ_t, surfaces

$$\Psi_p(\rho_t, \theta_t) \simeq \Psi_c(\rho_t) + \Psi_{c'}(\rho_t)\left(\int_{\rho_t}^{a} \frac{\rho}{R_0} A(\rho)\,d\rho\right)\cos\theta_t \quad (2.3)$$

$\Psi_c(\rho)$ is the poloidal flux function of a straight cylindrical plasma with an arbitrary current density distribution J_z. $A(\rho)$ is obtained from the expression

$$A(\rho) = \frac{1}{\rho^2 \Psi_c'^2}\int_0^{\rho} \Psi_c'^2\,d\rho + \beta_p - 1, \quad (2.4)$$

where β_p is the poloidal beta. It is important to note that the surfaces with constant Ψ_p contain the field lines of $\boldsymbol{B}_0$ $(\boldsymbol{B}_0\cdot\nabla\Psi_p = 0)$

The physical components of $\boldsymbol{B}_0$ are

$$B_{0\rho_t} = -\frac{1}{R_{0'}}\frac{\partial\Psi_p}{\partial\theta_t}, \quad B_{0\theta_t} = \frac{1}{R_{0'}}\frac{\partial\Psi_p}{\partial\rho_t},$$
$$B_{0\varphi} = -\frac{\mu_0 I(\Psi_p)}{R}, \quad (2.5)$$

where

$$R = R_{0'} - \rho\cos\theta \quad (2.6)$$

and $I(\Psi_p)$ is the poloidal current function

3. Perturbed plasma field

The RHW are pairs of conductors carrying currents $\pm I_H$ wound on the tokamak vessel of radius $\rho_t = a_v$, as shown in Fig. 2. After m toroidal and n poloidal turns a conductor returns to the same position.

Due to the toroidal geometry, different m modes

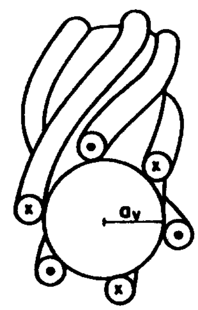

Fig 2 Three pairs of conductor carrying currents $\pm I_H$ wound on the tokamak vessel of radius $\rho_t = a_v$

with the same number n are created, each one described by a scalar potential Φ [11],

$$b=\nabla\Phi\,,$$

$$\Phi=-\frac{\mu_0 I_H}{\pi}\left(\frac{\rho_t}{a_v}\right)^m \sin(m\theta_t-n\varphi)\,, \tag{3.1}$$

where b is the field corresponding to the resonant mode (m, n).

The magnetic field B of the perturbed plasma is taken as a simple superposition of the equilibrium field B_0 with the small field b associated to one resonant mode $(m; n)$,

$$B=B_0+b\,. \tag{3.2}$$

For marginally stable states, when the plasma response is considerable, this approximation may not be valid. Otherwise, b can be thought of as the resultant perturbation

4. Averaging method

The averaging method requires that the total field B must be written as a sum of a symmetric field $B_0(\rho_t, \theta_t)$ and a small perturbation $b(\rho_t, \theta_t, \varphi)$ that breaks this symmetry.

The field line equation

$$B\times dl=0 \tag{4 1}$$

is written in terms of the coordinates

$$x^1\equiv\Psi_p(\rho_t, \theta_t)\,, \tag{4.2}$$

$$x^2\equiv u=m\theta_t-n\varphi \quad (m, n \text{ integer numbers})\,,$$

$$x^3\equiv\theta_t\,, \tag{4.2}$$

as a two-dimensional system of equations,

$$\frac{d\Psi_p}{d\theta_t}=\frac{b\ \nabla\Psi_p}{(B_0+b)\cdot\nabla\theta_t}\,,$$

$$\frac{du}{d\theta_t}=\frac{(B_0+b)\cdot\nabla u}{(B_0+b)\ \nabla\theta_t}\,, \tag{4 3}$$

where $B_0\cdot\nabla\Psi_p=0$. This is a non-autonomous system in the sense that the right hand sides depend explicitly upon θ_t. All the physical quantities are periodic functions of θ_t with periodicity $L=2\pi$.

In the averaging method developed by Bogolyubov et al. [9,12] the field line equations are written in terms of average coordinates x^{-1} and x^{-2} instead of x^1 and x^2

We use the same notation as Morozov and Solov'ev [9] to define $\bar{f}, \tilde{f}$ and $\hat{f}$,

$$\bar{f}(\bar{x}^1, \bar{x}^2)=\frac{1}{2\pi}\int_0^{2\pi} f(\bar{x}^1, \bar{x}^2, \theta_t)\, d\theta_t\,,$$

$$\tilde{f}(\bar{x}^1, \bar{x}^2, \theta)=f-\bar{f},$$

$$\hat{f}(\bar{x}^1, \bar{x}^2, \theta_t)=\int_0^{\theta_t}\tilde{f}\, d\theta_t\,, \tag{4.4}$$

where the integrations are carried out with fixed $\bar{x}^1$ and $\bar{x}^2$.

By applying this method to solve Eqs. (4 3), we obtain the following average equations, valid around the resonant (equilibrium) surface corresponding to safety factor $q=m/n$,

$$\frac{d\bar{\Psi}_p}{d\theta_t}\simeq -n\frac{\partial\Psi}{a\bar{u}}\,,\quad \frac{d\bar{u}}{d\theta_t}\simeq n\frac{\partial\Psi}{\partial\bar{\Psi}_p}\,,$$

with the function Ψ defined as

$$\Psi(\bar{\Psi}_p, \bar{u})\equiv\frac{1}{2\pi}\int B\, d\sigma$$

$$-\frac{1}{2\pi n}\int_0^{2\pi}\left(\frac{b\cdot\nabla\Psi_p}{B_0\ \nabla\theta_t}\right)\left(\frac{B_0\cdot\nabla u}{B_0\ \nabla\theta_t}\right)d\theta_t\,. \tag{4.6}$$

$d\sigma$ is an element of the toroidal helical surface $x^2=$const bounded by a magnetic surface of the unperturbed system ($\Psi_p=$const) Terms of order of $(b/B_0)^2$ were not considered. For the TBR-1 [3] the perturbations on the poloidal equilibrium field component due to helical currents are of the order of 1%.

The system of equations (4 5) is equivalent to

$$\nabla\Psi(\bar{\Psi}_p, \bar{u})\ dl\simeq 0 \tag{4.7}$$

Thus, the surfaces

$$\Psi(\bar{\Psi}_p, \bar{u})=\text{const} \tag{4.8}$$

contain the average field lines of B around $q=m/n$.

The "real" (approximate) positions of magnetic field lines are obtained from average positions by [9,12]

$$\Psi_p \simeq \bar{\Psi}_p + O(b/B_0)\,,$$

$$u \simeq \bar{u} + \frac{\boldsymbol{B}_0 \cdot \nabla u}{\boldsymbol{B}_0 \cdot \nabla \theta_t} + O(b/B_0) \qquad (4.9)$$

Only dominant terms are maintained in each expression.

Using (4.6) we can derive an explicit expression for Ψ with $\boldsymbol{B}_0$ and $\boldsymbol{b}$ already given, taking into account the corrections to the average lines given by (4.9). The magnetic structure of $\boldsymbol{B}$ around $q=m/n$ can be analytically determined without directly integrating of Eqs. (4.3).

Satellite islands are found in the neighbouring rational surfaces (with $q=(m\pm1)/n$) by choosing the coordinate x^2 to be

$$x^2 \equiv (m\pm1)\theta_t - n\varphi$$

and following a similar procedure.

5. Numerical method

The field line equation $\boldsymbol{B}\times d\boldsymbol{l}=0$ is numerically integrated and mapped. Each point is the intersection of the magnetic line with a transversal plane after one toroidal turn.

The maps are made using typical values of TBR-1 [3]: $I_p=9$ kA (plasma current), $R_0=30$ cm (major radius), $a=8$ cm (radius of the limiter), $a_v=11$ cm (radius of the vessel), $B_{0\varphi}=0.5$ T, $\Lambda(a)=0.28$. The current density distribution J_z chosen to describe the TBR-1 equilibrium is

$$J_z = \text{const}\times[1-(\rho/a)^2]^3 \qquad (5.1)$$

With this choice $q(0)\simeq1$ and $q(a)\simeq4$.

In Figs. 3 and 4, the map (A) is numerical and the (B) is analytical. These maps are made for helical current $I_H=90$ A in a transversal plane $\varphi=\pi$. In Figs. 3A and 3B, the principal mode is $m/n=2/1$ and in Figs. 4A and 4B, $m/n=3/1$. Note that the positions of the 2/1 islands are similar in Figs. 3 and 4, but not the positions of 3/1 islands! This fact is general for any plane φ and for any helical current intensity.

The stochastic behaviour of the field lines around the separatrices that are observed on the numerical maps was analysed analytically in Ref. [13]. One difference between the analytical method used in this work and that one used in Ref. [13] is that in our case the function Ψ (which specifies the average magnetic surfaces) can be interpreted as a magnetic flux. Another difference is that the method of Ref. [13] is more precise but a little more complex than the method presented here.

As we can observe, there is a good agreement between the average magnetic surfaces and the Poincaré maps for the sizes and positions of the islands.

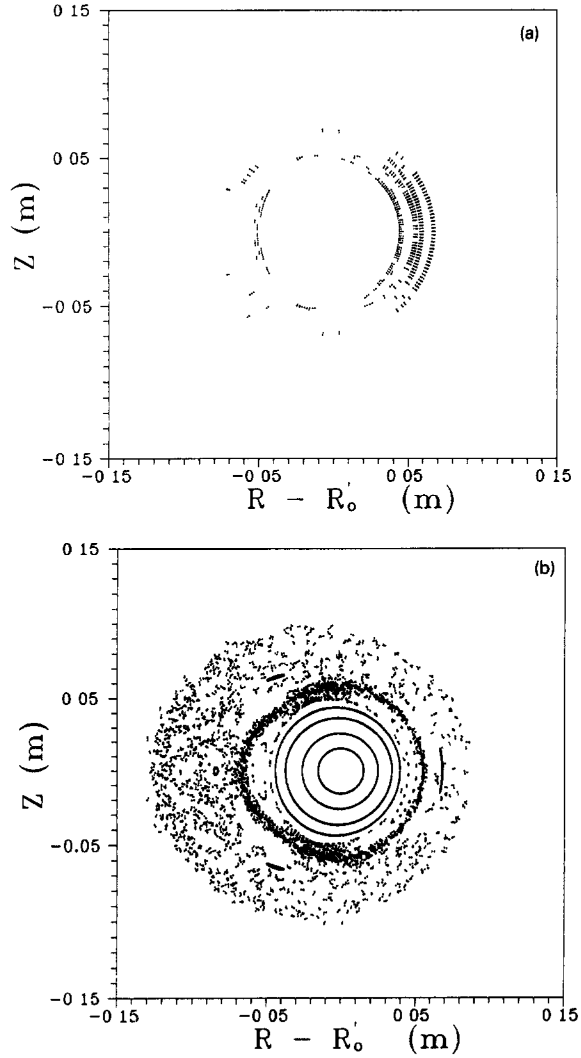

Fig. 3. (A) analytical map. (B) Numerical map. Principal mode $m/n=2/1$. Helical current $I_H=90$ A. Transversal plane $\varphi=\pi$. All other parameters have typical values of tokamak TBR-1.

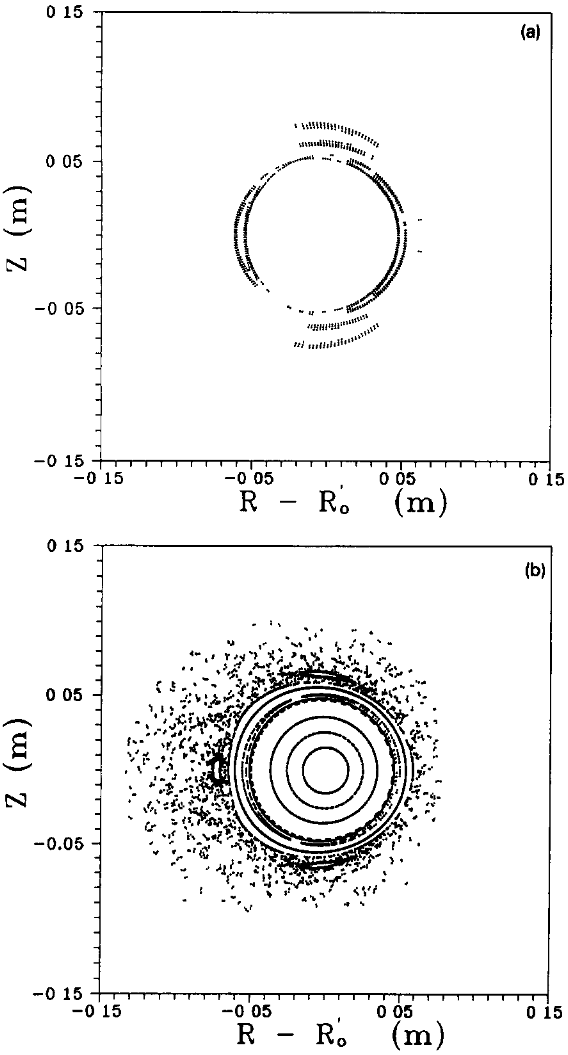

Fig 4 The same as in Figs 3 for principal mode $m/n=3/1$

6. Conclusions

In toroidal tokamak plasmas a single perturbative helical mode $(m; n)$ causes the formation of islands at rational surfaces with $q=m/n$ (principal islands) and $q=(m\pm1)/n$ (satellite islands). Our theory predicts this phenomenon as a consequence of toroidicity Also, due to the toroidal geometry, the islands on the same resonant surface do not have equal widths These results are shown in our analytical maps and agree quite well (specially for small helical current amplitudes) with those obtained by numerically integrating the field line equations.

For small values of the ratio between the external helical current and the plasma current $I_H/I_p=1\%$ the principal and satellite islands already almost overlap and, therefore, higher current amplitudes would induce plasma disruptions.

The analytical method used to calculate the magnetic structure of toroidal axially symmetric plasma under influence of small perturbation is adequate to further theoretical analysis of stability [14] and transport [15]. The application of the averaging method turned out to be simple due to the convenient choice of the coordinates. Our procedure could be generalized to include plasma response by writing the total magnetic field as a sum of a symmetric field and a small term that breaks this symmetry, this small term would contain the perturbation and the plasma response (which is of the same order of magnitude as the perturbation).

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