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Coupling of modes in RFPs: an analytical approach*

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Abstract. The magnetic structure of a cylindrical RFP plasma in which the equilibrium is perturbed by two resonant modes is determined analytically by using an averaging method to solve the differential field-line equations. The primary modes couple themselves and excite others, also creating magnetic islands in secondary resonance regions. Average magnetic surfaces are compared with Poincaré maps obtained by direct integration of field-line equations.

1. Introduction

After a highly turbulent initial phase, reversed-field pinch (RFP) discharges relax to a quasi-stationary state ('equilibrium'). It has been observed that, by sustaining the plasma current, the 'equilibrium' configuration is maintained well beyond the resistive decay time [1]. According to Taylor's relaxation theory [2], the 'equilibrium' magnetic field profile B_0 is completely characterized in a cylindrical geometry by one parameter Θ, defined as the ratio between the poloidal field measured at the plasma edge and the volume-averaged toroidal field, $\Theta \equiv B_{0\theta}(\text{edge})/\langle B_{0z}\rangle$ (r, θ and z are the cylindrical coordinates). RFP machines normally operate in a low-Θ regime ($\Theta \sim 1.4$), near the minimum energy state.

Magnetic fluctuations, b, are detected during the quasi-stationary stage with levels typically of the order of $b/B_0 \sim 1\%$ [3–7]. The origin of these fluctuations is commonly attributed to MHD resistive instabilities, $(m; n)$ resonant inside the field reversal surface (magnetic surface where $B_{0z} = 0$). The dominant poloidal mode number for all machines is found to be $m = 1$. The toroidal mode number spectrum is broader, usually centred at $n \sim 2R_0/a$ (R_0 is the major radius of the torus and a is the radius of the plasma column). Other global quantities like plasma current, toroidal magnetic flux, and loop voltage exhibit regular oscillations during the quiescent period [8]. Experimental work also shows the presence of large scale modulation (sawteeth) on the x-ray emission [9]. The fluctuation activity is more evident in a high-Θ regime ($\Theta > 1.6$) although it is still observed in low-Θ. These experimental results suggest the existence of some cyclic process inherent to the sustainment of the 'equilibrium' magnetic configuration.

The mechanism of sustainment of this RFP configuration, known as the *dynamo effect*, has been interpreted as a cyclic process [10, 11]: resistive diffusion, growth of instabilities, and reconnection of field lines.

A resistive diffusion occurs naturally in plasmas. Its effect is always destabilizing, leading the configuration far away from the minimum energy state (toward more unstable configurations). In RFPs, MHD stability analysis [12] has shown that the resistive

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instabilities ($m = 1; n$) are usually the first ones to be excited, which is in accordance with the experiments. These resistive tearing modes excited by diffusion may be responsible for relaxation phenomena because they allow the breaking and reconnection of magnetic field lines. Consequently, the magnetic energy can be reduced through changes in the field topological properties. The relaxation that happens via instabilities tends to restore the initial magnetic configuration.

For the relaxation process the $m = 1$ instabilities are dominant. However, $m = 0$ and 2 modes are also detected, mainly after the $m = 1$ signals reach their largest amplitudes. Numerical simulations corroborate these experimental observations [13–16].

Taking into account all this information, the following model has been suggested [14, 15] to explain the *sequence* in which the tearing modes are excited during the cycles of the sustaintment phase.

(i) Since the $m = 1$ resonance surfaces are quite closely spaced (in comparison with a tokamak, as the RFP is a low safety-factor configuration $q(r) \equiv rB_{0z}/R_0B_{0\theta} < 1$), the resistive diffusion may excite two adjacent tearing modes with $m = 1$, i.e. $(1; n_0)$ and $(1; n_0 + 1)$. These resistive modes generate magnetic islands at the corresponding resonance surfaces at r_S ($q(r_{S1}) = 1/n_0$ and $q(r_{S2}) = 1/n_0 + 1$).

(ii) As the islands grow a coupling occurs between the primary modes exciting two secondary modes: $(0; 1)$ and $(2; 2n_0 + 1)$. The primary modes couple with $(0; 1)$ producing a broader spectrum. Thus, $(1; n_0 + 1)$ couple with $(0; 1)$ producing $(1; n_0 + 2)$. There is a redistribution of $m = 1$ mode energy and part of it is continually drained to $m \neq 1$ modes. A possible outcome of this process is the saturation of these instabilities. Modes with $m \geqslant 2$ would provide energy sinks. A combined effect of broadening of the spectrum in wavenumber and an enhanced level of the fluctuations is overlapping of magnetic islands, creating a region of stochastic magnetic field that spreads radially.

(iii) The initial configuration is restored by relaxation and again destabilized by diffusion. This cycle is repeated many times until the discharge ends.

In this paper we show analytically that, in a cylindrical plasma with the 'equilibrium' initially perturbed by two resonant modes $(m_1; n_1)$ and $(m_2; n_2)$, magnetic islands are formed *not* only at rational surfaces where $q = m_1/n_1$ and $q = m_2/n_2$, but also at $q = (m_1 + m_2)/(n_1 + n_2)$ and $q = (|m_1 - m_2|)/(|n_1 - n_2|)$. Thus, we explain phase (ii) of the cyclic process model. The primary modes couple themselves exciting other modes. Applying the averaging method developed by Kucinski *et al* [17], we derive an analytical expression for the magnetic surfaces around primary and secondary resonances. We assume that the plasma field is given by the superposition of the Taylor field with a perturbative field associated with the magnetic oscillations. We take the Taylor field model for its simplicity and because the resonance regions are expected to be inside the plasma ($r \leqslant a/2$), where there is good agreement between this model and the experimental profiles. The model adopted for perturbation is the same that as proposed by Bazzani *et al* [18]. Average magnetic surfaces are compared with Poincaré maps obtained by numerical integration of the field-line equations, using typical parameters of RFP ETA-BETA II.

In section 2, the expressions of 'equilibrium' and perturbative fields are given. In section 3, a general formula for magnetic surfaces around resonance regions is derived. In section 4, analytical and numerical maps are presented. In section 5, a summary of our results is related.

2. Typical magnetic fields in the quasi-stationary phase

The magnetic field $\boldsymbol{B}$ of a quiescent RFP plasma can be written as a superposition of a symmetric field $\boldsymbol{B}_0$ with a small perturbative field $\boldsymbol{b}$ which breaks this symmetry:

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}\,. \tag{1}$$

We take $\boldsymbol{B}_0$ as the 'equilibrium' field, and $\boldsymbol{b}$ as the field associated to the resonant modes.

2.1. *Taylor's 'equilibrium' model*

According to Taylor [2], the 'equilibrium' field is obtained by minimizing the total magnetic energy under the constraint that the total magnetic helicity is kept constant.

The solution for a cylindrical RFP ($R_0/a \gg 1$) with circular cross section is the Bessel function model (BFM):

$$B_{0r} = 0 \qquad B_{0\theta} = B_0\, J_1(\mu r) \qquad B_{0z} = B_0\, J_0(\mu r) \tag{2}$$

where J_0 and J_1 are the cylindrical Bessel functions of order 0 and 1, respectively, and B_0 is the field on the magnetic axis ($r = 0$). μ is a constant related to the parameter Θ by $\mu a = 2\Theta$.

One important consequence of Taylor's theory is that the relaxed RFP equilibrium profiles depend only on Θ. The configuration is sustained, while Θ remains constant.

2.2. *Perturbative field*

We adopt the same model as Bazzani *et al* [18]: the poloidal component is neglected ($b_\theta \equiv 0$); the radial component b_r is assumed to be zero on the axis ($r = 0$) and on the plasma edge ($r = a$) and is described by

$$b_r = b_0 \frac{r(a-r)}{a^2} \frac{\partial T}{\partial z}(\theta, z)$$

where b_0 is a constant and $T(\theta, z)$ is a double Fourier series which accounts for the experimental spectra of the magnetic oscillations. This expression fits reasonably the profiles observed by Brotherton-Ratcliffe *et al* [6].

We consider two resonant modes $(m_1; n_1)$ and $(m_2; n_2)$. In this case, the components of $\boldsymbol{b}$ are given by

$$\begin{aligned} b_r &= b_r^{(m_1;n_1)}(r) \cos\left(m_1\theta - \frac{n_1 z}{R_0}\right) + b_r^{(m_2;n_2)}(r) \cos\left(m_2\theta - \frac{n_2 z}{R_0}\right) \\ \frac{\partial b_z}{\partial z} &= -\frac{1}{r}\frac{\partial (r b_r)}{\partial r} \end{aligned} \tag{3}$$

where

$$b_r^{(m;n)} \equiv b_0 \frac{r(a-r)}{2a^2} c_m a_n$$

and c_m and a_n are the coefficients of the poloidal and toroidal modes, respectively.

3. Magnetic structures in RFP plasmas

For the magnetic field

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}_{m_1;n_1} + \boldsymbol{b}_{m_2;n_2} \tag{4}$$

the field-line equation

$$B \times d\ell = 0$$

can be rewritten as a two-dimensional system of equations in terms of curvilinear coordinates x^1, x^2 and x^3:

$$\frac{dx^1}{dx^3} = \frac{B^1}{B^3} \qquad \frac{dx^2}{dx^3} = \frac{B^2}{B^3} \tag{5}$$

where $B^i \equiv B \cdot \nabla x^i$ are the contravariant components. This is a non-autonomous system in the sense that the B^i's depend explicitly upon three coordinates. Consequently, near resonances there are no magnetic surfaces; the field lines wander around some average surfaces.

An averaging method has been developed by Kucinski *et al* [17] to find analytical expression for approximate magnetic surfaces without directly integrating and mapping these equations.

The method requires that the total field B be written as a sum of a symmetric field and a perturbation that breaks this symmetry. Here, we write

$$B = B_{m_1;n_1} + b_{m_2;n_2}$$

where $B_{m_1;n_1}$ is the superposition of the Taylor 'equilibrium' field with the field associated with the resonant mode $(m_1; n_1)$:

$$B_{m_1;n_1} \equiv B_0 + b_{m_1;n_1} \tag{6}$$

where $B_{m_1;n_1}$ is a function of r and $m_1\theta - n_1 z/R_0$. It has helical symmetry and the corresponding lines form magnetic surfaces with two basic kinds of structures: (i) islands, around the resonance region $q = m_1/n_1$, and (ii) circular cylinders, far from the resonance.

The perturbation $b_{m_2;n_2}$ corresponds to the other mode $(m_2; n_2)$.

In the application of the averaging method, the choice of the coordinates is fundamental because some details of the magnetic structure may disappear in the averaging process, according to the choice of variables.

x^1 and x^3 must be related to the helically symmetric field $B_{m_1;n_1}$.

It is appropriate to take x^1 as a magnetic surface quantity in order to have $B_{m_1;n_1} \cdot \nabla x^1 = B^1_{m_1;n_1} = 0$, and

$$x^3 \equiv m_1\theta - n_1\frac{z}{R_0}. \tag{7}$$

The symmetry of this system is destroyed by the dependence of $b^i_{m_2;n_2}$ upon x^2.

All the physical quantities are periodic functions of x^3 with periodicity L. Averages are taken over a period of x^3.

3.1. The symmetric system—the choice of x^1

The magnetic structure of the symmetric system can be described in terms of a magnetic flux function $\psi_0(r, x^3)$. Most conventionally, $2\pi\psi_0$ is the flux of $B_{m_1;n_1}(r, x^3)$ through a cylindrical helical ribbon of width r, specified by a fixed value of $x^3 \equiv m_1\theta - n_1 z/R_0$.

We obtain

$$\psi_0(r, x^3) = G_0(r) + G_1(r)\sin x^3$$

with

$$G_0(r) = \int_0^r \frac{r\,R_0}{m_1}\left(B_0 \cdot \nabla x^3\right) \mathrm{d}r$$

$$G_1(r)\sin x^3 = \int_0^r \frac{r\,R_0}{m_1}\left(b_{m_1;n_1} \cdot \nabla x^3\right) \mathrm{d}r\,.$$

ψ_0 must have an extremum around

$$B_0 \cdot \nabla x^3 = m_1 \frac{B_{0\theta}}{r} - n_1 \frac{B_{0z}}{R_0} \simeq 0\,.$$

This is represented schematically in figure 1(*a*). The magnetic surfaces $\psi_0(r, x^3) =$ constant are shown in figure 1(*b*).

Outside the islands, each magnetic surface can be specified by r_0 defined as the value of r at $x^3 = 0$. Thus, the magnetic surfaces are described by the equation

$$\psi_0(r, x^3) = \psi_0(r_0, 0) = G_0(r_0)\,.$$

For each value of x^3, we have approximately

$$r \cong r_0 - \frac{G_1(r_0)}{G_0'(r_0)} \sin x^3\,. \tag{8}$$

x^1 is chosen to be r_0:

$$x^1 \equiv r_0\,. \tag{9}$$

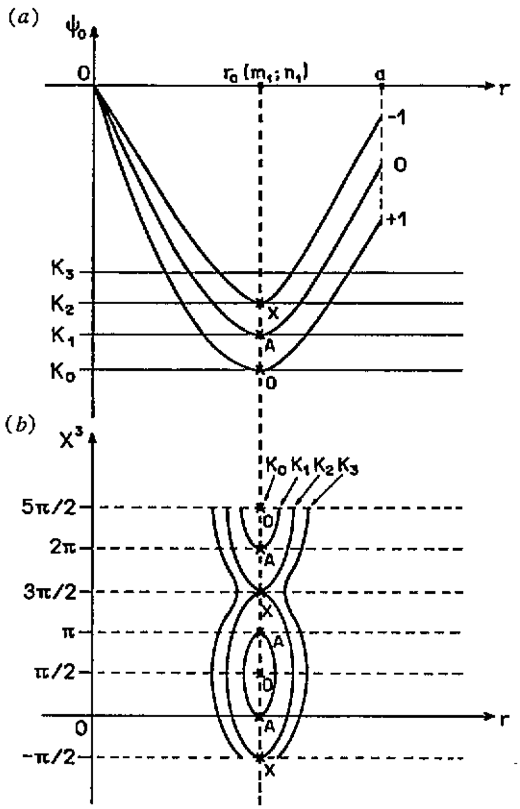

Figure 1. (*a*) Magnetic flux function $\psi_0(r, x^3)$ of the helically symmetric field $B_{m_1;n_1}$. The labels $+1, 0$ and -1 correspond to $\sin x^3 = +1, 0$ and -1, respectively. (*b*) Magnetic surfaces $\psi_0 =$ constant $= K_i$ of $B_{m_1;n_1}(r, x^3)$. Islands are formed at $q(r_0) = m_1/n_1$. $\psi_0 = K_0$ corresponds to the magnetic island axis and $\psi_0 = K_2$ to the separatrix.

3.2. The perturbed system—the choice of x^2

As the symmetry is broken, the field lines no longer lie exactly on surfaces. Although approximate surfaces can be determined in terms of average coordinates $\overline{x^1}$ and $\overline{x^2}$. If $b_{m_2;n_2}$ is not strong enough, we may expect an island structure at $q = m_2/n_2$ and chains of islands in some other rational q surfaces resultant from coupling between this mode and the $(m_1; n_1)$ mode.

It is appropriate to take

$$x^2 \equiv M\theta - N\frac{z}{R_0} \tag{10}$$

if we want to analyse the magnetic structure around the rational surface:

$$q(r_{0(M;N)}) = \frac{M}{N}.$$

In order to write $b_{m_2;n_2}(r, m_2\theta - n_2 z/R_0)$ as a function of the chosen coordinates, we define

$$\alpha x^2 - \beta x^3 \equiv m_2\theta - n_2\frac{z}{R_0}$$

so that the rational numbers α and β are given by:

$$\alpha = \frac{m_1 n_2 - n_1 m_2}{m_1 N - n_1 M} \qquad \beta = \frac{M n_2 - N m_2}{m_1 N - n_1 M}.$$

A major difficulty in the application of an averaging method lies upon an adequate choice of variables. Once this is overcome it remains a lengthy algebra, although without tricks.

3.2.1. Average magnetic surfaces. We use the notation

$$\bar{f}(\overline{x^1}, \overline{x^2}) \equiv \frac{1}{L}\int_0^L f(\overline{x^1}, \overline{x^2}, x^3)\mathrm{d}x^3$$

$$\tilde{P}(f) \equiv \tilde{f}(\overline{x^1}, \overline{x^2}, x^3) = f - \bar{f}$$

$$\hat{P}(f) \equiv \hat{f}(\overline{x^1}, \overline{x^2}, x^3) = \tilde{P}\left(\int_0^{x^3} \tilde{f}\,\mathrm{d}x^3\right)$$

where the integrations are carried out with fixed $\overline{x^1}$ and $\overline{x^2}$.

An approximate magnetic surface is described by [17]:

$$\Psi(\overline{x^1}, \overline{x^2}) = \Psi(x^1 - \delta\overline{x^1}, x^2 - \tilde{x^2} - \delta\overline{x^2}) = \text{constant} \equiv K$$

where the function Ψ is obtained by

$$\Psi(\overline{x^1}, \overline{x^2}) = \overline{\int_0^{\overline{x^1}} \sqrt{g}\, B^2_{m_1;n_1}(\overline{x^1}, x^3)\,\mathrm{d}\overline{x^1}} - \overline{\int_c^{\overline{x^2}+\tilde{x^2}} \sqrt{g}\, b^1_{m_2;n_2}(\overline{x^1}, x^2, x^3)\,\mathrm{d}x^2} \tag{11}$$

with $\tilde{x^2}$ the first-order correction to $\overline{x^2}$:

$$\tilde{x^2} = \frac{\hat{B}^2_{m_1;n_1}}{B^3_{m_1;n_1}}$$

and g the determinant of the covariant metric tensor of the set of coordinates $x^1 \equiv r_0$, $x^2 \equiv M\theta - Nz/R_0$ and $x^3 \equiv m_1\theta - n_1 z/R_0$:

$$\sqrt{g} = (\nabla x^1 \cdot \nabla x^2 \times \nabla x^3)^{-1}$$

and c a constant which satisfies the condition

$$\overline{\sqrt{g}\, b^2_{m_2;n_2}(\overline{x^1}, c, x^3)} = 0\,. \tag{12}$$

$\delta\overline{x^1}$ and $\delta\overline{x^2}$ are terms of the order of $(b_0/B_0)^2$. In our case, the first-order solution $(\delta\overline{x^i} = 0)$ is satisfactory because the fluctuation level is very small $(b_0/B_0 \sim 1\%)$.

Following the same procedure as in Kucinski *et al* [17], we derive an analytic expression for Ψ, using (2) and (3) for $B_{m_1;n_1}(r, x^3)$ and $b_{m_2;n_2}(r, x^2, x^3)$, and the expression (8) which relates r to $x^1 \equiv r_0$.

Around the rational region $q = M/N$, the constancy of Ψ determines the relation

$$r_0 \cong r_{0(M;N)} \pm \tfrac{1}{4} W_{M;N} \sqrt{2\ (K \pm \sin(\alpha\overline{x^2} + \beta\pi/2))} \tag{13}$$

where $W_{M;N}$ is the width of the separatrix

$$W_{M;N} = 4\left[\frac{r b_r^{(m_2;n_2)}(r)|J_\beta(\alpha f(r))|}{\alpha N(-q'(r)) B_{0\theta}(r)}\right]^{1/2}_{r=r_{0(M:N)}} \tag{14}$$

and $f(r)$ is given by

$$f = \frac{m_1 N - n_1 M}{(m_1 - n_1 q)^2} \frac{1}{B_{0\theta}} \left(\frac{1}{n_1} \frac{\partial\, r b_r^{(m_1;n_1)}}{\partial r} + r b_r^{(m_1;n_2)} \frac{\mathrm{d}q/\mathrm{d}r}{(m_1 - n_1 q)^2}\right).$$

Each value of the constant K individualizes one magnetic surface of $\boldsymbol{B}$. $K = 1$ corresponds to the separatrix, $K < 1$ to the inner surfaces of the separatrix and $K > 1$ to the outer surfaces.

The signal $\pm\sin(\alpha\overline{x^2} + \beta\pi/2)$ must be taken according to the signal of $J_\beta(\alpha f(r))$. If $J \geqslant 0$, the signal of sin is positive.

To first order the average coordinate $\overline{x^2}$ is obtained by

$$\overline{x^2} = x^2 - \tilde{x^2} = x^2 - f(r_{0(M;N)}) \cos x^3\,.$$

The condition (12) imposes a unique restriction in the choice of M and N: β must result in an *integer number*.

3.2.2. Coupling of the primary modes. Secondary islands arise at rational surfaces

$$q = \frac{M}{N} = \frac{m_2 + \beta m_1}{n_2 + \beta n_1} \qquad \beta = \pm 1\,.$$

Higher-order islands would be formed at $q = (m_2 + \beta m_1)/(n_2 + \beta n_1)$, $\beta = \pm 2, \pm 3, \ldots$ if the perturbations are strong as enough but not so strong as to destroy the magnetic surfaces. The size of these islands decreases with increasing values of $|\beta|$.

To determine which couplings result from the initial modes $(m; n)$ and $(m; n+1)$, we take

$$m_1 = m \qquad n_1 = n \qquad m_2 = m \qquad n_2 = n + 1\,.$$

With these choices, the resonance rational surfaces are localized at

$$q = \frac{m(\beta + 1)}{n(\beta + 1) + 1} \qquad \beta = \pm 1, \pm 2, \ldots.$$

The two first secondary resonances are given by $\beta = +1$ and $\beta = -1$, which correspond to the regions $q = 2m/(2n + 1)$ and $q = 0/1$, respectively. Therefore, we show anallytically that the primary modes $(m; n)$ and $(m; n + 1)$ couple themselves and excite the secondary modes $(2m; 2n + 1)$ and $(0; 1)$. This result helps to confirm the picture that explains the *sequence* in which the resistive modes are excited during the cycles of the sustainment phase in a RFP discharge.

The width of the islands (14) differs by the factor $[|J_\beta(\alpha f)|/\alpha N]^{1/2}$ from the 'traditional' expression [8, 10]:

$$W_{m;n} = 4\left[\frac{rb_r^{(m;n)}(r)}{n(-q'(r))B_{0\theta}(r)}\right]^{1/2}_{r=r_{0(m;n)}}$$

which does not take into account the coupling effects. Due to the coupling, islands on the same resonant surface have different shapes. For the primary islands, the difference between our formula for $W_{M;N}$ and the traditional one is found to be of the order of ~20% for the cases treated in this work.

4. Analytical and numerical maps

Maps of the magnetic structure of B are drawn using typical values in RFP ETA-BETA II experiments [18]:

radius of plasma column: $a = 0.125$ m
major radius: $R_0 = 0.65$ m
pinch parameter: $\Theta = 1.5$
ratio between the maximum amplitudes of b and B_0: $b_0/B_0 = 0.3\%$
primary modes: $(m_1; n_1) = (1; 9)$, $(m_2; n_2) = (1; 10)$.

The Fourier coefficients c_m and a_n are taken in agreement with experimental measurements:

$$c_0 = 0.5 \qquad c_1 = 1.0 \qquad c_2 = 0.25$$

$$0 \quad \text{for} \quad n < 8, \quad n > 19$$
$$a_n = n/10 \quad \text{for} \quad 8 \leqslant n \leqslant 10$$
$$(20 - n)/10 \quad \text{for} \quad 10 \leqslant n \leqslant 19$$

Figures 2(a) and (b) are maps of B in the $\theta = 0$ toroidal plane. We take the value of b_0/B_0 in order that there is no overlapping of islands. The outer chain of islands corresponds to the primary resonance $q = \frac{1}{9}$ and the inner one to $q = \frac{1}{10}$. The intermediate chain at $q = \frac{2}{19}$ results from the coupling between the primary modes.

We call the numerical map the one obtained by numerically integrating the field-line equation $B \times d\ell = 0$ for ~1000 poloidal circuits. Each point of a chain of islands is the intersection of the field line with a transversal plane after one poloidal circuit.

The analytical map is derived from (8) and (13). The choice of $(M; N)$ determines the resonance region to be mapped. For example, we take $(M; N) = (2; 19)$ to plot the (2; 19) magnetic structure.

The average magnetic surfaces and the Poincaré maps agree perfectly as to the sizes and positions of the primary mode islands. The agreement is still good for the (2; 19) structure. The other secondary island (0; 1) was not plotted because its size is very small (one order of magnitude smaller than the (2; 19) size).

5. Conclusions

Using an averaging method to investigate the magnetic structure in RFPs analytically, we show that

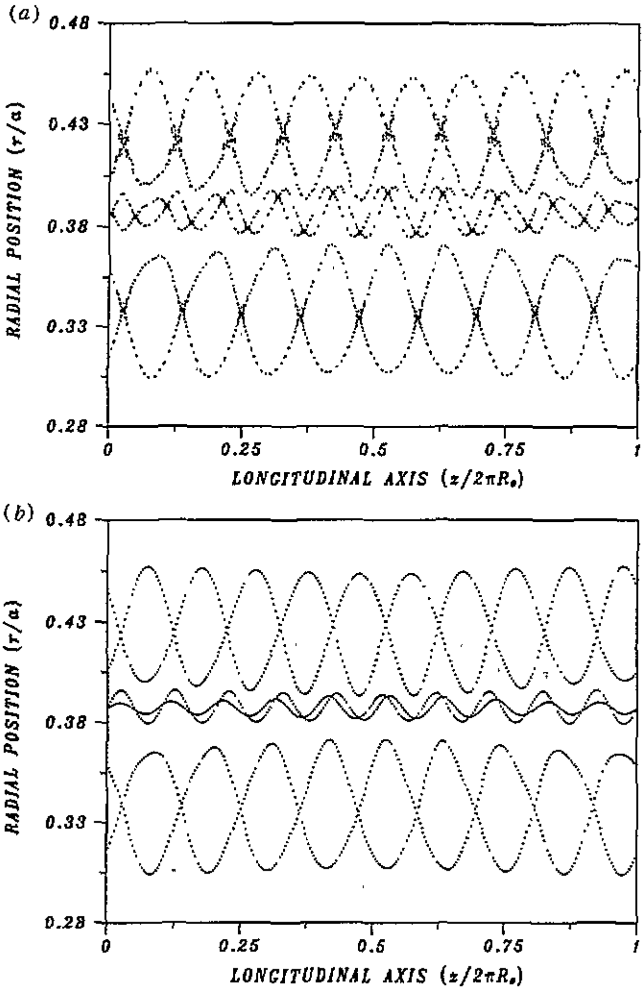

Figure 2. Magnetic islands produced by the primary modes (1; 9) and (1; 10) of 0.3% relative amplitude in the $\theta = 0$ plane. (*a*) Numerical map: each chain of islands is obtained by integrating the field-line equations for ~1000 poloidal cycles. (*b*) Analytical map: the separatrices at $q = \frac{1}{9}$, $\frac{1}{10}$ and $\frac{2}{19}$ are obtained by taking $K = 1$ in the average surface equation (13).

(i) two adjacent resonant modes with $m = 1$ can couple themselves and excite the secondary modes $(2m; 2n + 1)$ and (0; 1);

(ii) the magnetic structure of a (primary or secondary) resonance region $q = M/N$ is influenced by both primary modes (islands on the same resonant surface have different shapes);

(iii) (2; 19) secondary islands is one order of magnitude narrower than the primary islands;

(iv) for a small fluctuation level $b_0/B_0 = 0.3\%$ the islands are about to overlap. As the experimental value of b_0/B_0 is typically 1%, the interior of the RFP plasma must be characterized by a stochastic magnetic field.

Our analytical results agree quite well with those obtained by numerically integrating the field-line equations. The application of the averaging method became simple due to the adequate choice of coordinates. Our analytical technique is computationally more efficient than numerical tracing of a field line. The CPU time is ~100 times smaller.

A similar, but a slightly simpler, averaging method has been applied successfully in the determination of the magnetic structure of a tokamak plasma in which the toroidal equilibrium is destroyed by resonant helical fields [19].

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