Nonlinear growth and saturation of large scale MHD modes in presence of current sheets and energetic particles

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Outline

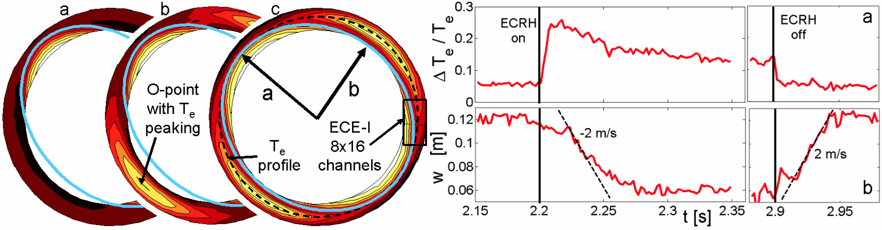
Introduction

- Control of Neoclassical Tearing Modes (NTM) might be of great importance in ITER
- Open questions
  - NTM existence and impact in ITER scenarii,
  - Require auxiliary power to control NTM
  - Scenario performance: Impact of NTM and auxiliary power on the Q factor
- As NTM require pre-NTM or seed islands to develop nonlinearly, identification of mechanisms generating, amplifying and reducing seeds is still an issue:
  - Seeds can be induced by long-period sawtooth. Adding localized heating and current drive can somewhat mitigate this
  - Turbulence
- Successfull dynamical control of island growth needs the development of Rutherford equations and precise diagnostics. To which extent? Do we know all major mechanisms to be included in such equations for precise dynamical control?

Introduction

Dynamical control of magnetic islands

- Electron cyclotron waves can suppress island, or control the island size w by causing a perturbation to the Ohmic plasma current or by replacing missing bootstrap current. It allows to
  - Drive non inductive current in the island region
  - Heat the island



Classen et al (Textor team), PRL 98, 035001 (2007)

Introduction

Dynamical control of magnetic islands and ITER predictions

- Extrapolation of experimental results are required to evaluate the role of islands and/or to control dynamically their evolution.
- Predictions are based on additive Rutherford equation models:

$$\partial_t w = 1.22\eta\left(\Delta' - NL_{tear}\right) + \Gamma_{bs.c.} + \Gamma_{pol} + \Gamma_{GGJ} - \Gamma_{CD} - \Gamma_{Heating}$$

... the validity of which remains still an issue

We aim to clarify the mechanisms at play in island dynamics in the simplest situation:

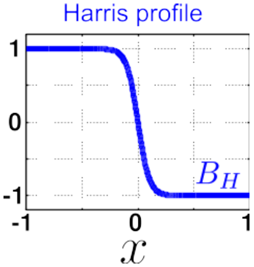
- $|\Gamma_{bs.c.}| + \left|\Gamma_{pol}\right| + |\Gamma_{GGJ}| + |\Gamma_{CD}| + \left|\Gamma_{Heating}\right| = 0$
- moderate Δ'
- 2D slab reduced MHD

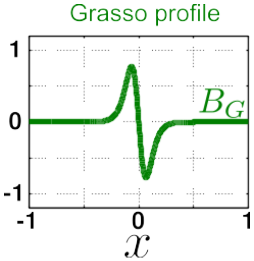
A minimal model

Reduced MHD equations for electrostatic potential ϕ and magnetic flux ψ in 2D slab geometry

$$\partial_t \omega + [\phi, \omega] = [\psi + \psi_{eq}, j + j_{eq}] + \nu \nabla_\perp^2 \omega$$
$$\partial_t \psi + [\phi, \psi + \psi_{eq}] = \eta j$$

where $j := \Delta_\perp \psi$, $\omega = \Delta_\perp \phi$, $\mathbf{B_{eq}} = \partial_x \psi_{eq} \mathbf{e_y}$.





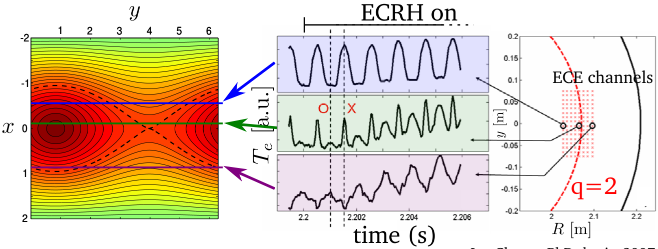
No external current

No external magnetic field

Preliminary question

Can we use diagnostics to control island growth using Rutherford theories?

- Rutherford theories give prediction of $w_1 = 4\sqrt{\frac{\psi_1}{J_{eq}}}$, i.e. the amplitude of magnetic fluctuations of the tearing mode.
- Experimental measurements : measure of the island width w_s

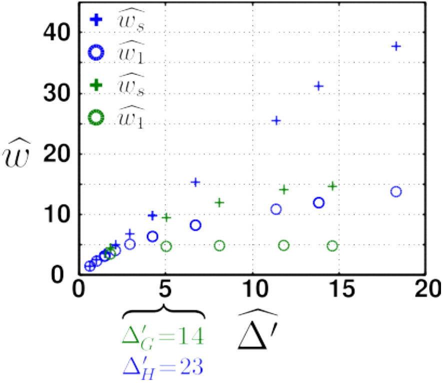


Ivo Classen PhD thesis, 2007

To which extent can we identify w_s and w_1 ?

Preliminary question

Agreement between w_1 and w_s only for small islands (negative or weakly positive Δ')



Question: what determines the saturated island size?

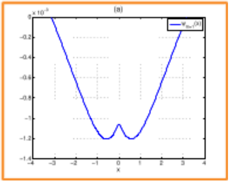
Rutherford equation predicts the evolution of unstable tearing mode:

$$\partial_t w_1 = 1.22\eta \left(\Delta' - 0.41 b_{eq}^{-2} w_1\right)$$

with $w_1 = 4\sqrt{\frac{\psi_1}{J_{eq}}}$ and $b_{eq}^2 = -\frac{\psi_{eq}''(0)}{J_{eq}''(0)}$

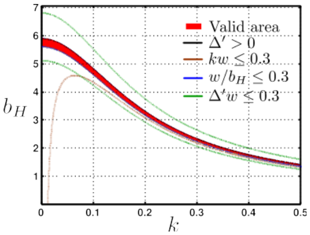
What is the link between island size w_s and w_1?

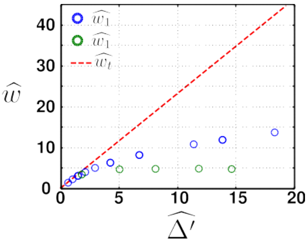
- Well identified phenomena in tearing mode evolution: coalescence, ribbons, plasmoids
- What about nonideal effects? (viscosity, resistivity)?
- To which extent global current profile can be cast into a parameter Δ'?
- Do weakly have resonance mechanisms which affect the island size at saturation w_s?



Limits to the theoretical approach

Many hypotheses involved :
$\hat{w} = w/b_{eq}$, $\hat{\Delta}' = \Delta'/b_{eq}$ and $\hat{w}_t = 2.44\hat{\Delta}'$

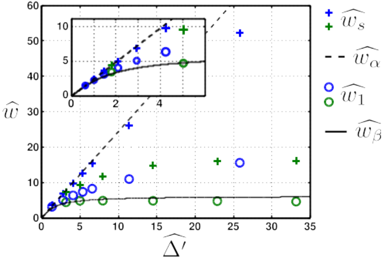




- Theoretically valid area : $\Delta' < 0.2$
- Numerically: much better agreement

$\hat{\Delta}' \leq \hat{\Delta}'_{si} \sim 2.5$: theoretical prediction valid for small islands

Limits of the theoretical approach



$$\hat{w}_\alpha = 2.44\,\hat{\Delta}'$$

$$\hat{w}_\beta = 2.44\,\hat{\Delta}'\left(1 - 0.1\hat{w}_\beta^2\right)$$

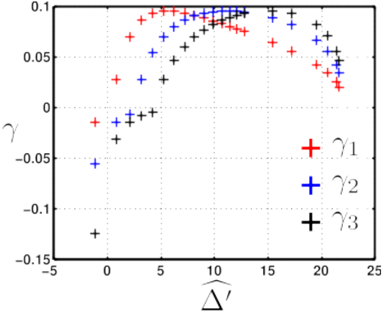
D.F. Escande and M. Ottaviani , Phys. Lett. A. 278, 323 (2004)
A. Smolyakov et al, Phys. Plasma (2013)

- $\hat{w}_1 \sim \hat{w}_\beta$: agreement between theory and experiment with profile G only.
- Results depend on profile at large Δ'
- Strong dicrepancy between w_1 and w_s

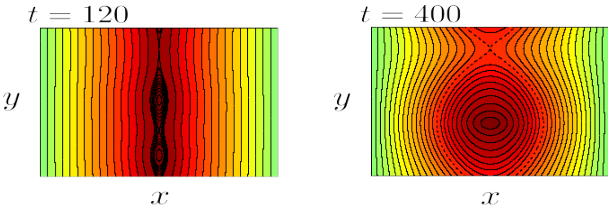
- Need to clarify mechanisms controlling island size at saturation at large $\hat{\Delta}'$

w_s is not sensitive to coalescence

Harris profile, $\hat{\Delta}'_{coal} \simeq 8$



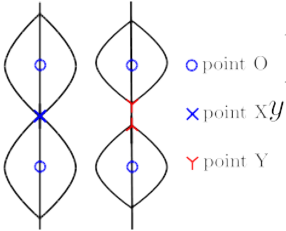
- Δ'_{coal} is defined by $\gamma_2 = \gamma_1$
- For Grasso profile, coalescence occurs for $\hat{\Delta}' > \hat{\Delta}'_{coal} \simeq 15$
- No associated change in behaviour for $\hat{w}_1\left(\hat{\Delta}'\right)$ or $\hat{w}_s\left(\hat{\Delta}'\right)$

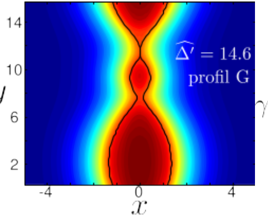


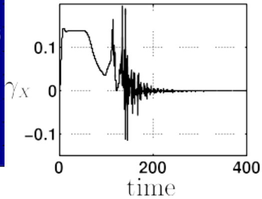
profile H, $\widehat{\Delta'} = 11.5$

Ribbons/plasmoids do not alter w_s

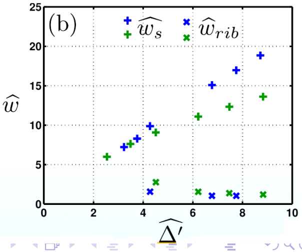
- Ribbon results from collapse of X point into two Y points at $\hat{w}_c$
- Apparition of plasmoids on the ribbon
- They can be detected by computing $\gamma_x(t)$



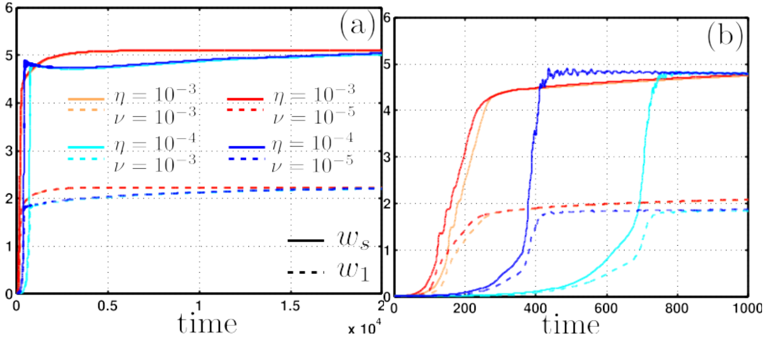




- For both profiles, ribbon formation occurs when $\hat{\Delta}' \geq 4$
- No associated change in behaviour for $\hat{w}_1\left(\hat{\Delta}'\right)$ or $\hat{w}_s\left(\hat{\Delta}'\right)$
- Does not generate discrepancy with theory



Nonideal effects do not alter w_s



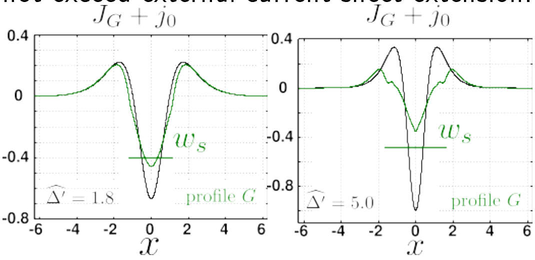
Magnetic island and current far from the resonance

- Global profile properties differ:

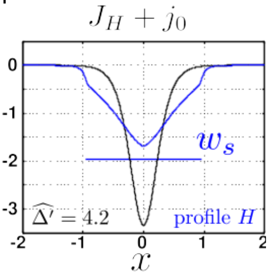
$B_{ext} = 0$ $\langle J_{eq} \rangle = 0$ $\langle B_{eq} \rangle = 0$ $j_{ext} = 0$

- Mean current is dynamically very different but :

$\hat{\Delta}' \gtrsim 5$ larger than central current sheet but does not exceed external current sheet extension.



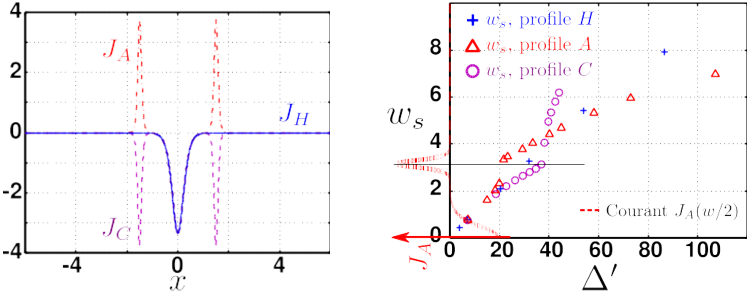
No such constraint with profile H



Role of external current sheets in island size evolution?

Current sheets and island size

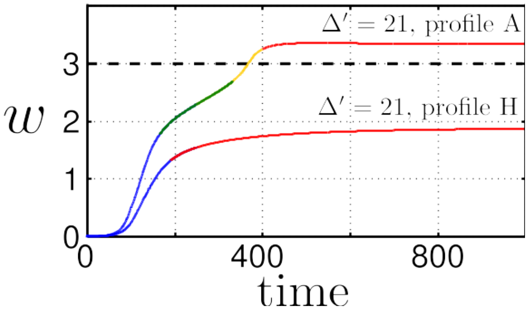
- Profiles A and C : same as H but with *external* current sheets
- Δ' fixed, different saturation widths if current sheets are crossed by the separatrix



1. Current sheets do modify island size.
2. Physics far from resonance cannot be cast into Δ' parameter

A. Poyé et al, Phys. Plasmas 20, 020702 (2013)

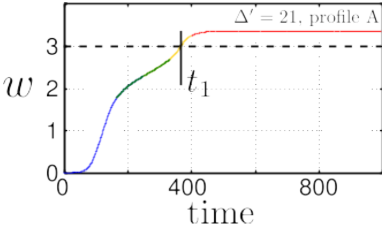
Current sheets and island size

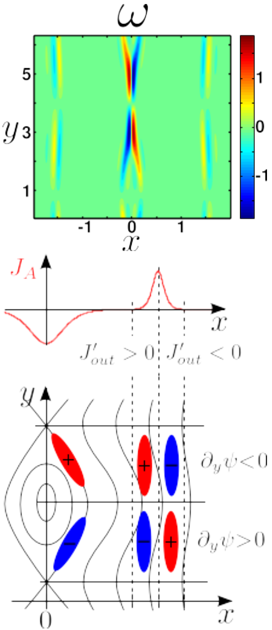


1. Initial growth/Rutherford
2. Long distance effect of current sheet
3. Direct interaction with current sheet
4. Saturation

Nature of the interaction between island and current sheets ?

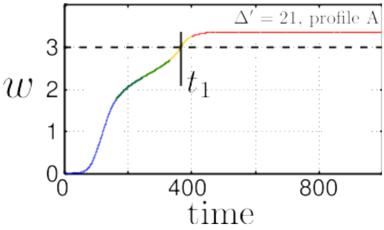
Mechanisms of interaction





1. Generation of vorticity on the current sheets
2. Attraction of vorticity cells
3. Creation of vorticity along separatrix

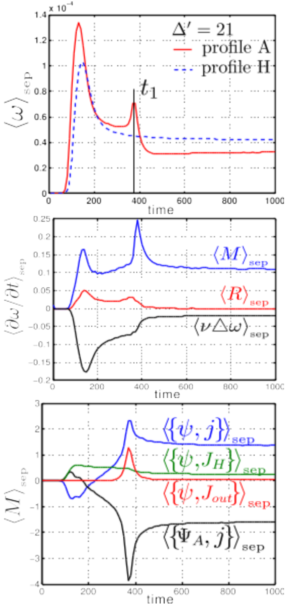
Mechanisms of interaction



1. Generation of vorticity on the current sheets
2. Attraction of vorticity cells
3. Creation of vorticity along separatrix :

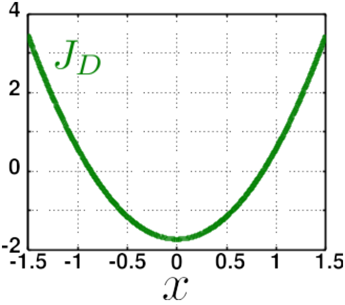
$$\partial_t w = \mathcal{R} + \mathcal{M} + \nu \Delta_\perp \omega$$

$$\mathcal{M} = [\psi_A, j] + [\psi, j] + [\psi, J_H] - \partial_x J_{out} \partial_y \psi$$



Could this lead to diverging islands?

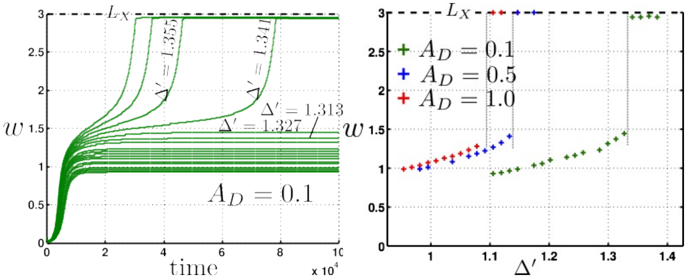
1. Competition between
   - tearing instability (which feeds the growth of the island)
   - attraction of the separatrix by external current sheets
2. If the island becomes large enough, it will be fed by externally generated vorticity



$$J_D' = 6A_D x$$

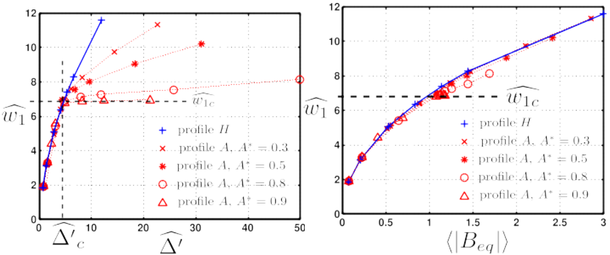
Is there a critical width above which no saturation occurs?

Could this lead to diverging islands?



- There is a critical island width above which there is no saturation
- Critical island width independant of the amplitude

Towards island size prediction



$$\langle B_{eq} \rangle = L_y \int_{-w_s/2}^{w_s/2} dx \, |B_{eq}(x)|$$

$$\hat{w}_1 = \hat{w}_1{}^H \left(\langle B_{eq} \rangle\right)$$

Impact of external current sheets: univocal link with the amount of equilibrium magnetic field inside the island

Conclusion

- It has been shown that island size at saturation does not depend on known phenomena developping in the vicinity of the singular surface (coalescence, ribbons, plasmoids), but also viscosity and resistivity
- New mechanisms linked to the presence of the current sheets and the vorticity generation have been identied. It limits the validity of boundary layer theories and the idea that current far from resonance can be casted into a Δ' parameter.
- Role of global profile effects on island size at saturation has been linked with the width at saturation of the tearing unstable mode in a univocal way

A reduced model for precessional Fishbones

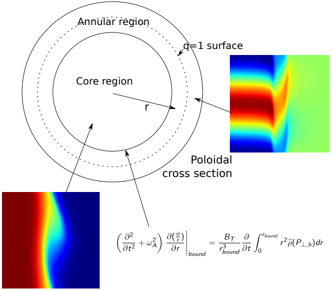
Hypotheses

- Only deeply trapped particles taken into account, with a single value for the magnetic moment
  - $\Rightarrow$ Reduction of the phase space from 6D to 2D (α, P_α)
  - $\Rightarrow$ Analytic calculation for the precession frequency
- All fast particles contained well inside the $q = 1$ surface
- Kink-like shaped dominant mode (mode numbers $m = n = 1$, electric potential $\phi \propto r$ well inside of the $q = 1$ surface)
- Fluid description for the bulk of the plasma, neglecting the thermal pressure effects

Numerical code structure

Domain separation

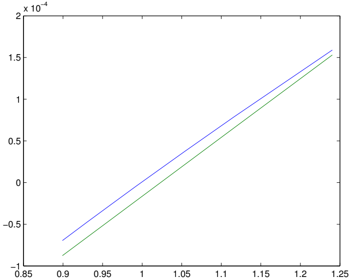
- Annular region (AMON code):
  - Nonlinear reduced MHD description for the bulk plasma
  - No energetic particles
  - Simplified to a 2D slab description
- Core region (VLAP code):
  - Linearized MHD response for the bulk plasma (with kink-like profile)
  - Nonlinear kinetic description of energetic particles
  - Simplification to 2D phase space (gyro- and bounce-averaging, only deeply trapped particles)



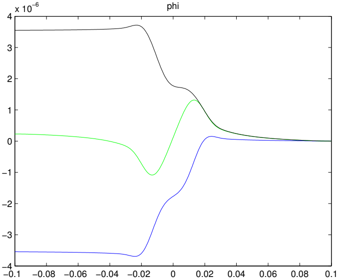
- Time dependent boundary conditions couple the dynamics of the two regions

Linear results

- Linear analytic results are obtained for the growth rate, resonant frequency, and mode profiles
- Good agreement is found with numerical simulations



Growth rate as a function of K/K_0. In blue, the numerical values, in green, the analytic prediction.



ϕ profile in the annular layer. In black modulus, in blue real part, in green imaginary part. The profiles are the expected ones.

Nonlinear simulations: work in progress