# Sculpting Vlasov Phase Space

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# Two Parts

- Undamped electrostatic plasma waves
  described in Phys. Plasmas **19**, 092103 (2012)

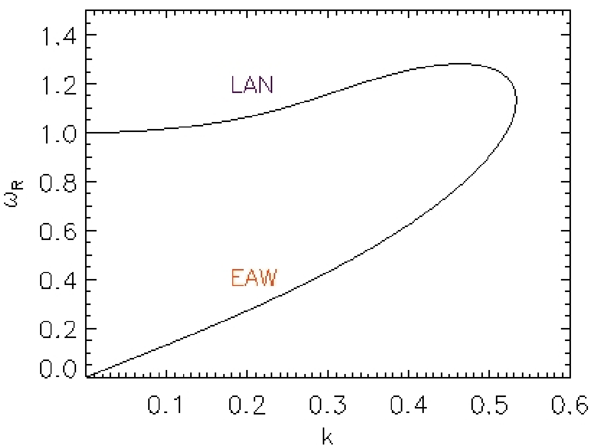
- Nonlinear structures
  preliminary results in J. Sci. Computing **56**, 319–349 (2013)

# Part I:

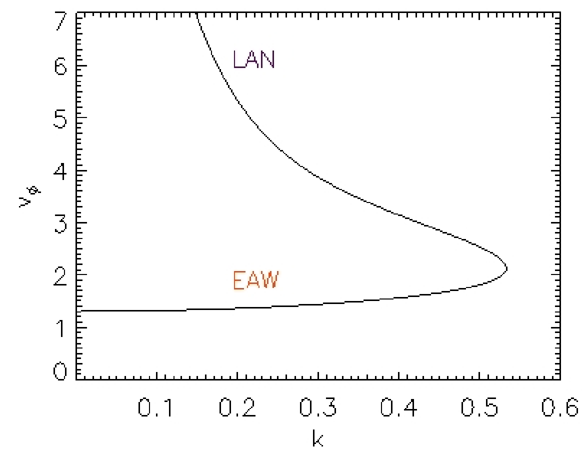
# Undamped Electrostatic Plasma Waves

# Electron Acoustic Waves (EAW)

Holloway and Dorning (1991)
($\nexists$ linear mode)



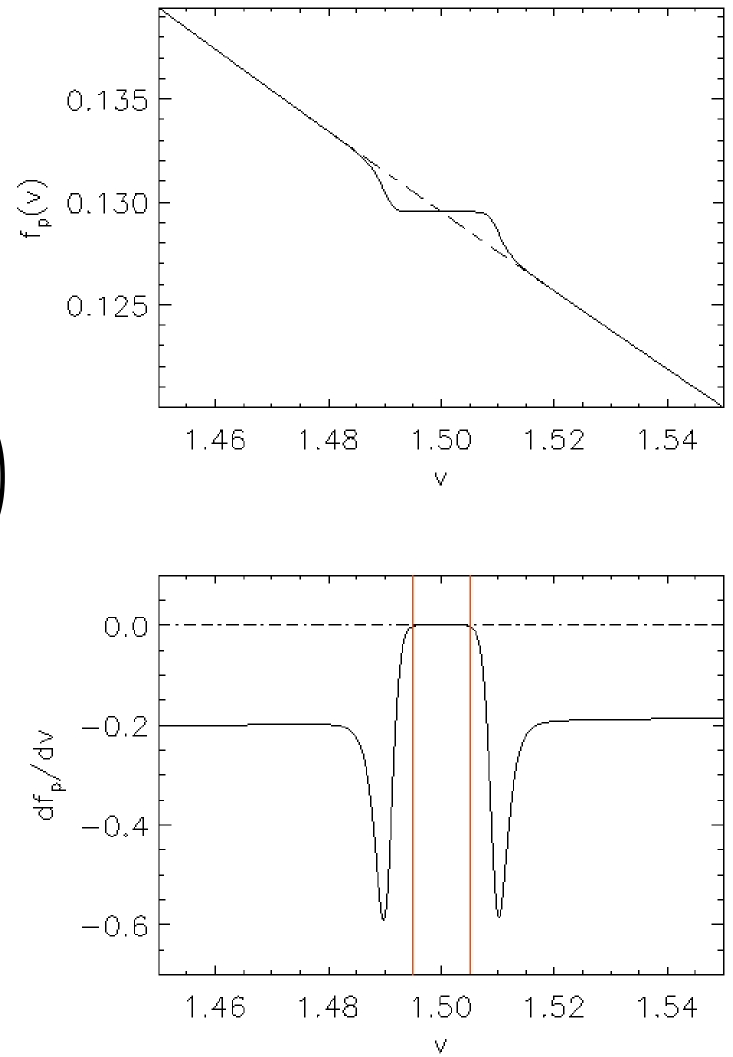

$$
\begin{aligned}
D_r(k,\omega) &= 1 - \frac{1}{k^2} \fint dv \frac{f'_M}{v - v_\phi} \\
D_I(k,\omega) &= -\frac{\pi}{k^2} f'_M(v_\phi) \\
v_\phi &= \omega/k
\end{aligned}
$$




Shadwick and pjm (1994)
($\exists$ linear mode - stationary inflection point)

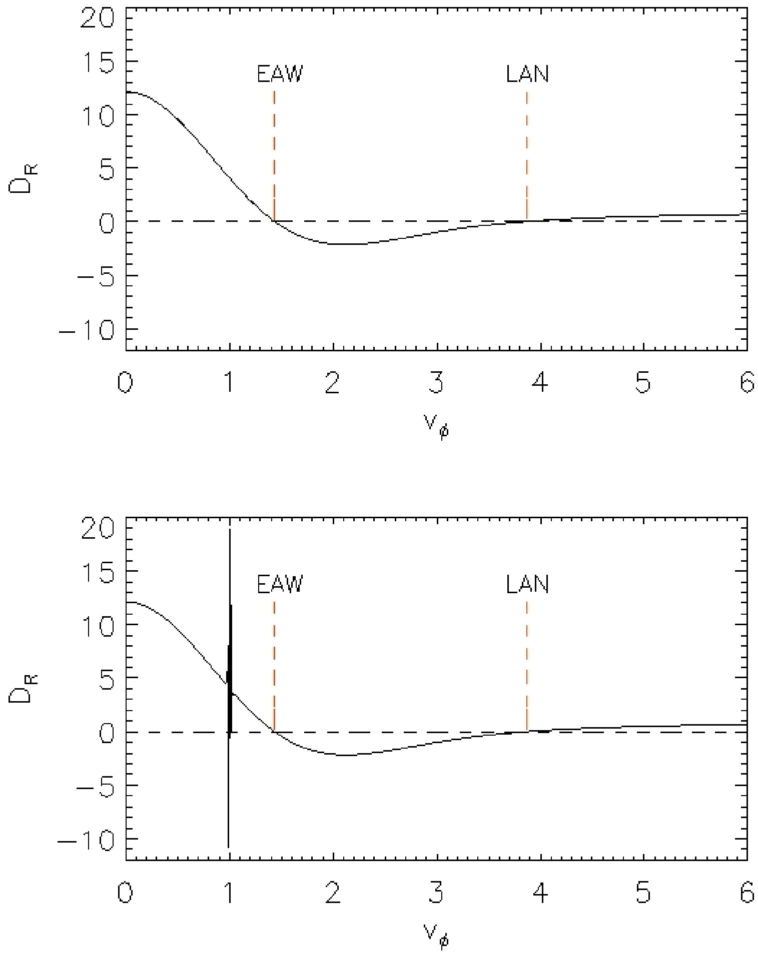
# Plateau Distribution

$$f_p(v) = N\left(f_M(v) - \frac{f_M(v) - f_M(V_0)}{1 + [(v - V_0)/\Delta V_p]^{n_p}}\right)$$

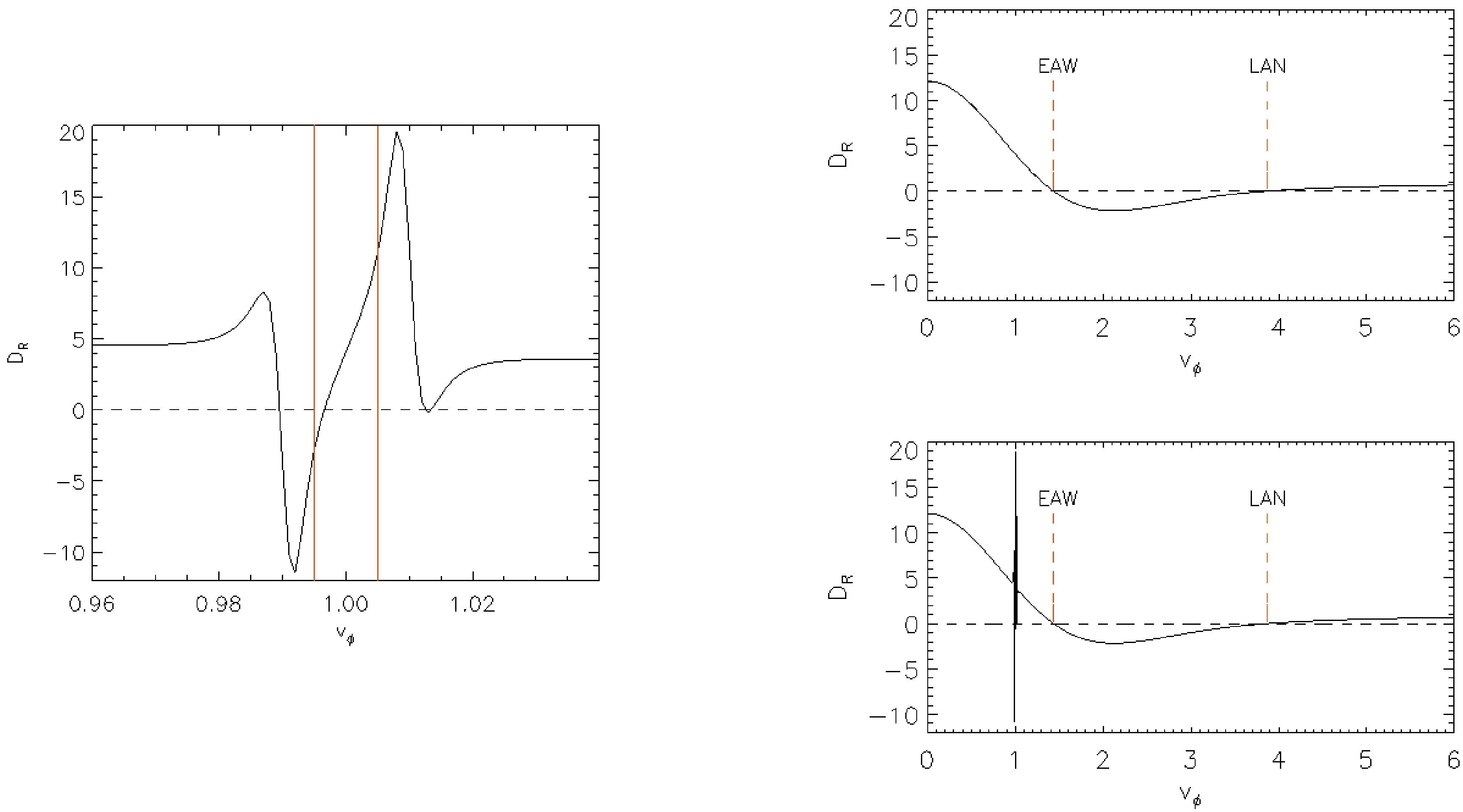


# Corner Modes

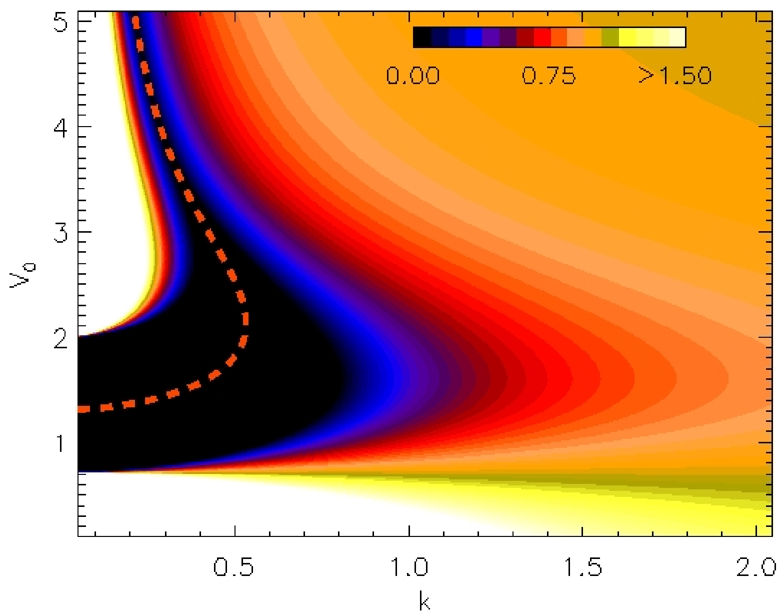
$\exists$ sensitivity to $v_\phi$

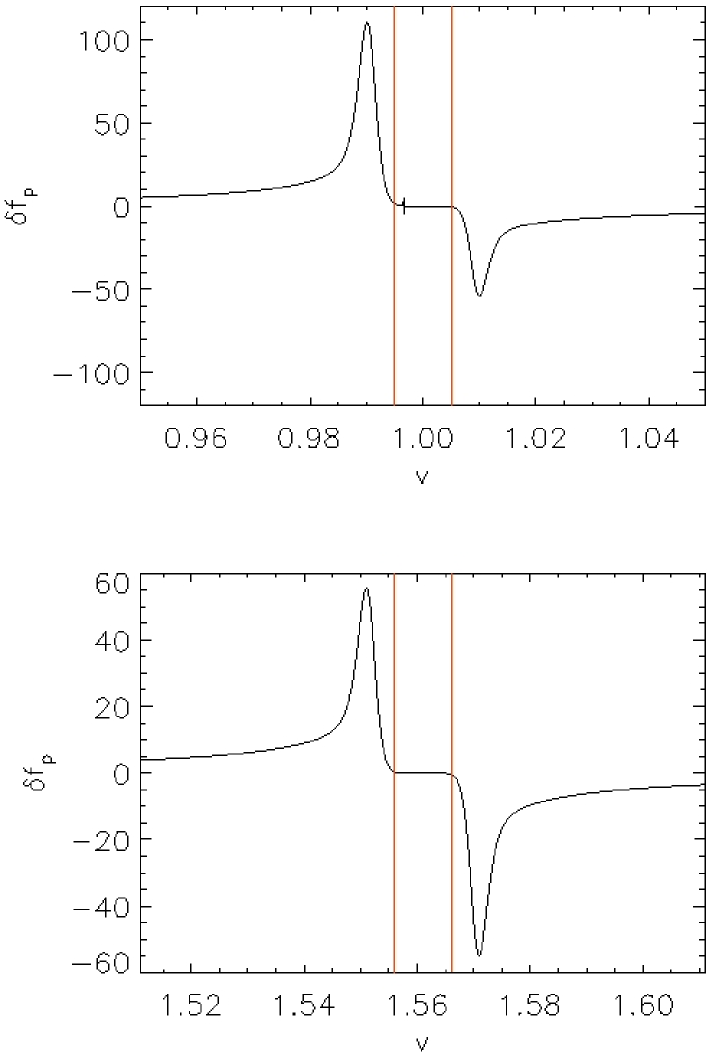


# Corner Modes



# Corner Mode Density



# Chop Chop

Chopped Plateau:

$$
\begin{aligned}
f_{ep} \;=\; & N_M f_M \left[ H(v - v_+) + H(v_- - v) \Big) \right] \\
& + N_p \left[ H(v - v_-) - H(v - v_+) \Big) \right]
\end{aligned}
$$

Rule of Thumb:

$$
k^2 = M(v_\phi) + \frac{(V_0 - v_\phi)\left( f_M^{(+)} - f_M^{(-)} \right)}{(V_0 - v_\phi)^2 - (\Delta V/2)^2},
$$

Determines frequency shifts in non-neutral plasma experiments

# Numerics

# Driving $\rightarrow$ Dynamically Accessible IC

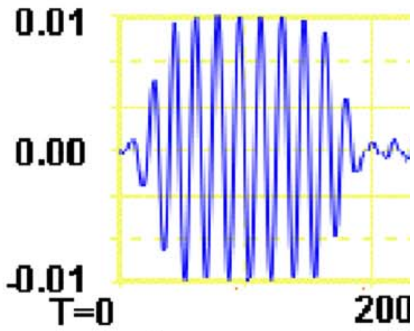
Vlasov with Drive:

$$f_t = -v f_x + \Big(E + E_d(x,t)\Big) f_v \,, \qquad E_x = 1 - \int_{\mathbb{R}} dv\, f$$

External Drive:

$$E_d(x,t) = E_{DA}\, g(t) \cos(kx - \omega t)$$

Drive Created IC:



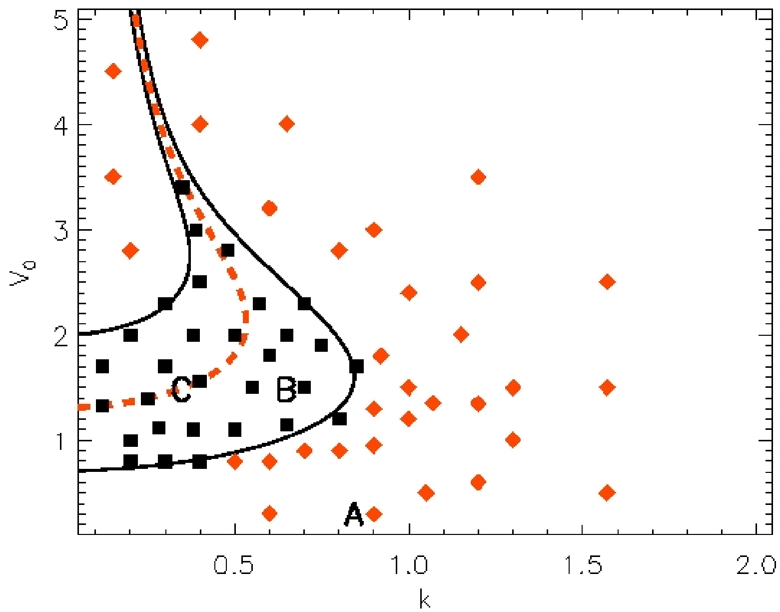

$E_{DA}(t) = .052$ and $T_d = 200$

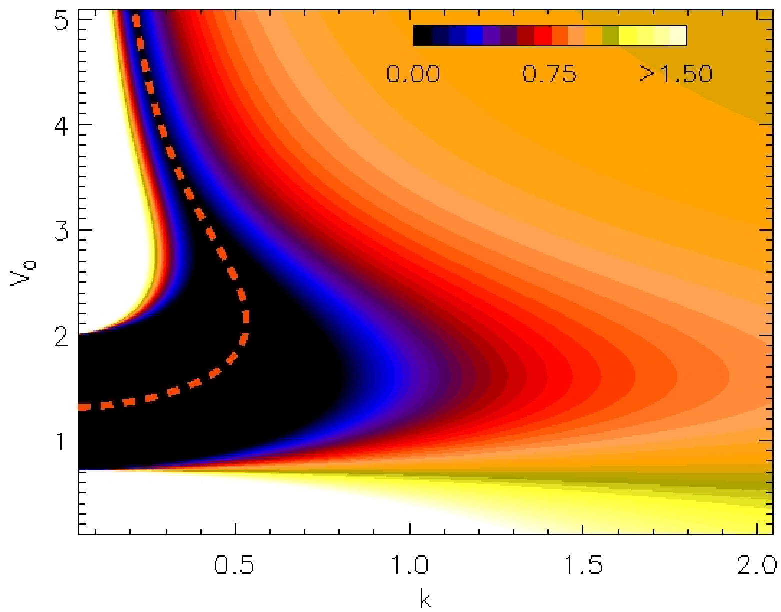
Rose, pjm & Pfrisch, Johnston et al., Afeyan, ... Friedland, ....

Dichotomies:

Weak vs. Strong - Adiabatic vs. Slap - Short vs. Long $(\omega, k)$

# Driving $f_p$: Weak-Adiabatic-Shortish




Nonlinear simulations $\rightarrow$ linear theory

# Part II:

# Nonlinear Structures

# The Program

- Vlasov is Hamiltonian wrt noncanonical Poisson Bracket, e.g. Vlasov-Poisson (pjm 1980)

$$\{F,G\} = \int dx dv\, f \left[\frac{\delta F}{\delta f}, \frac{\delta F}{\delta f},\right]$$

- Do for infinite degree-of-freedom Hamiltonian systems that which can be done for finite.

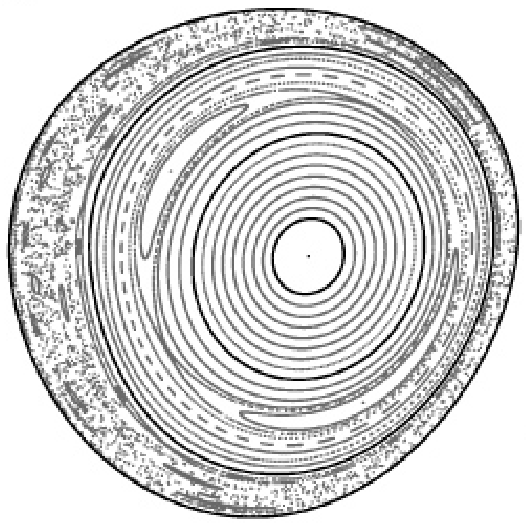
- Example: Krein-Moser theorem. Discrete spectrum pretty easy. Continuous spectrum? Not so easy. Analysis necessary. Signature pjm Pfirsch (1992); Krein's theorem *G. Hagstrom and pjm (2011, 2013).*

- Here: Lyapunov, Weinstein, Moser, ... Theorem about periodic orbits

# LWM Theorem

• Finite-Dimensional Hamiltonian Systems:

▷ $\exists$ other periodic orbits near stable periodic orbit (equilibrium)

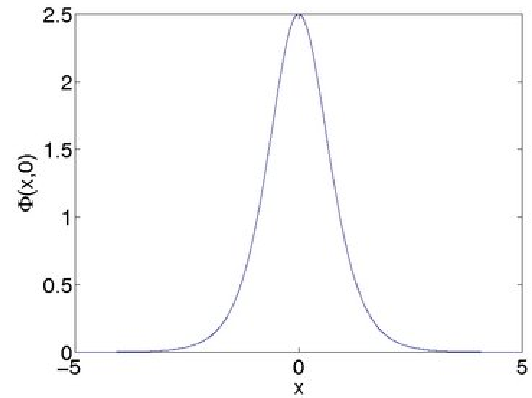


• Infinite-Dimensional Hamiltonian Systems:

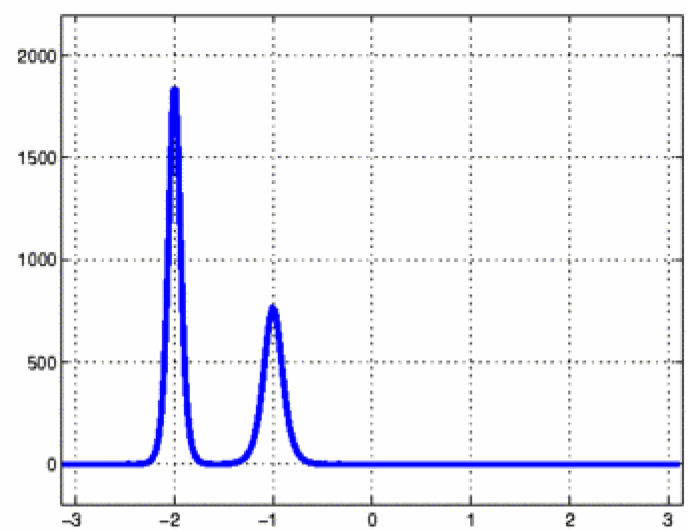
▷ precedent $\rightarrow$ for KdV soliton solution $\exists$ $N$-soliton solution, i.e., motion on an $N$-torus

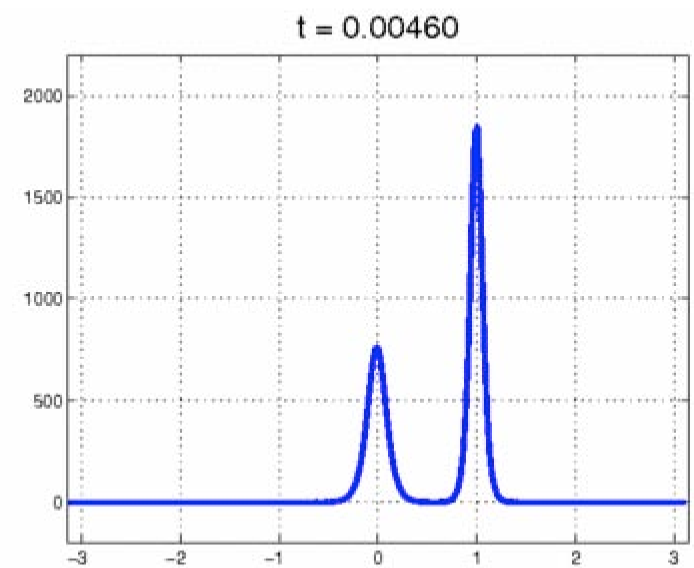
# Solitons on N-Tori

1-soliton $\rightarrow$

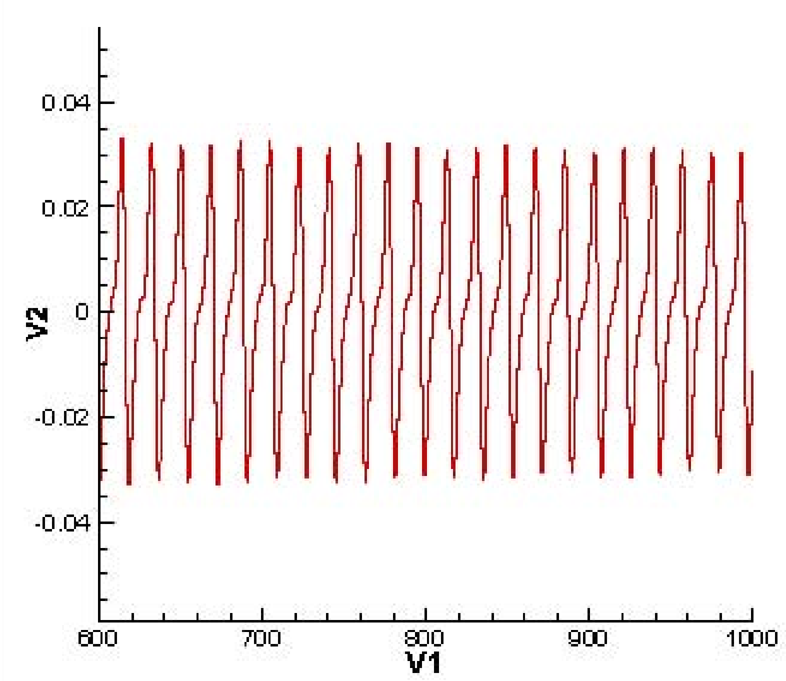


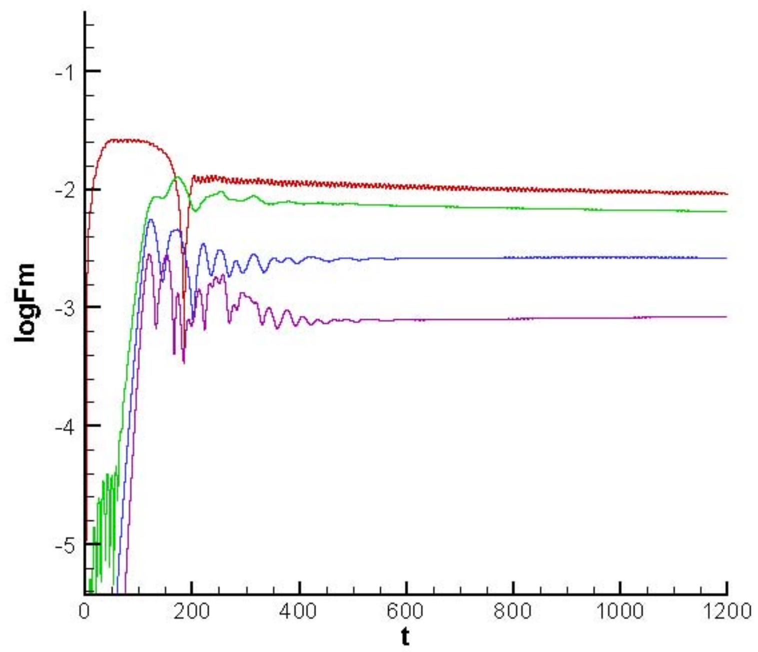

2-soliton $\downarrow$



# Driving $f_M$: Weak - Adiabatic




$$A_d(t) = .052 \text{ and } T_d = 200$$

Appears to settle into periodic orbit – travelling BGK hole.

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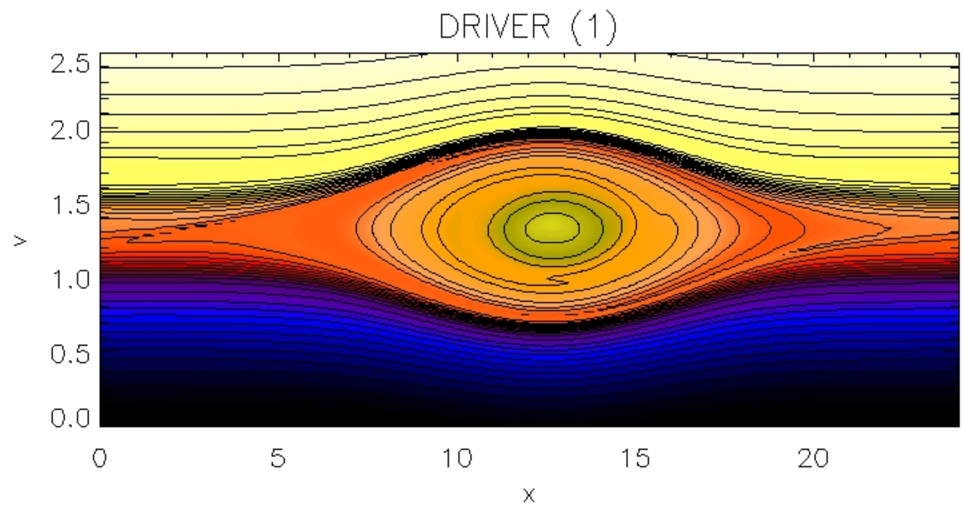
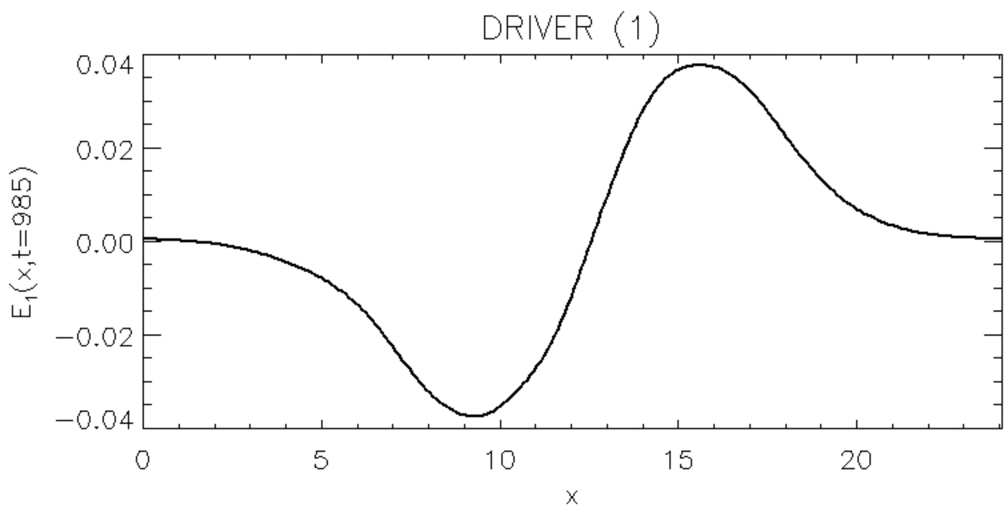
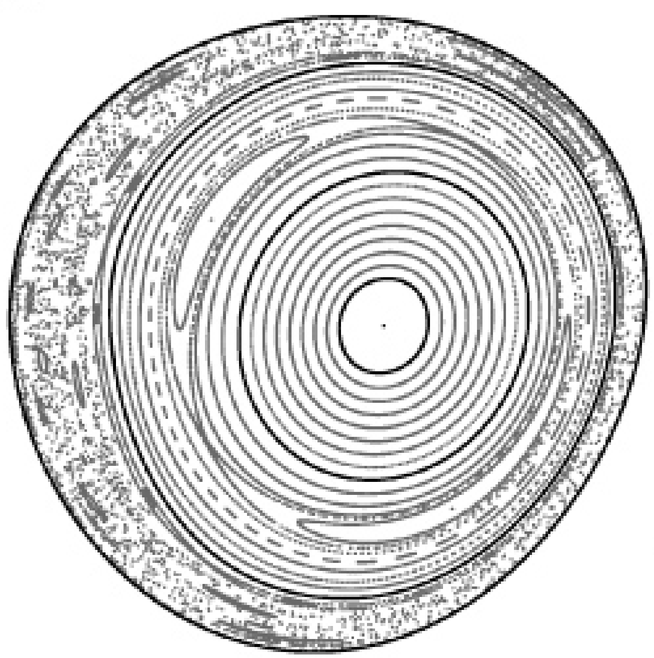
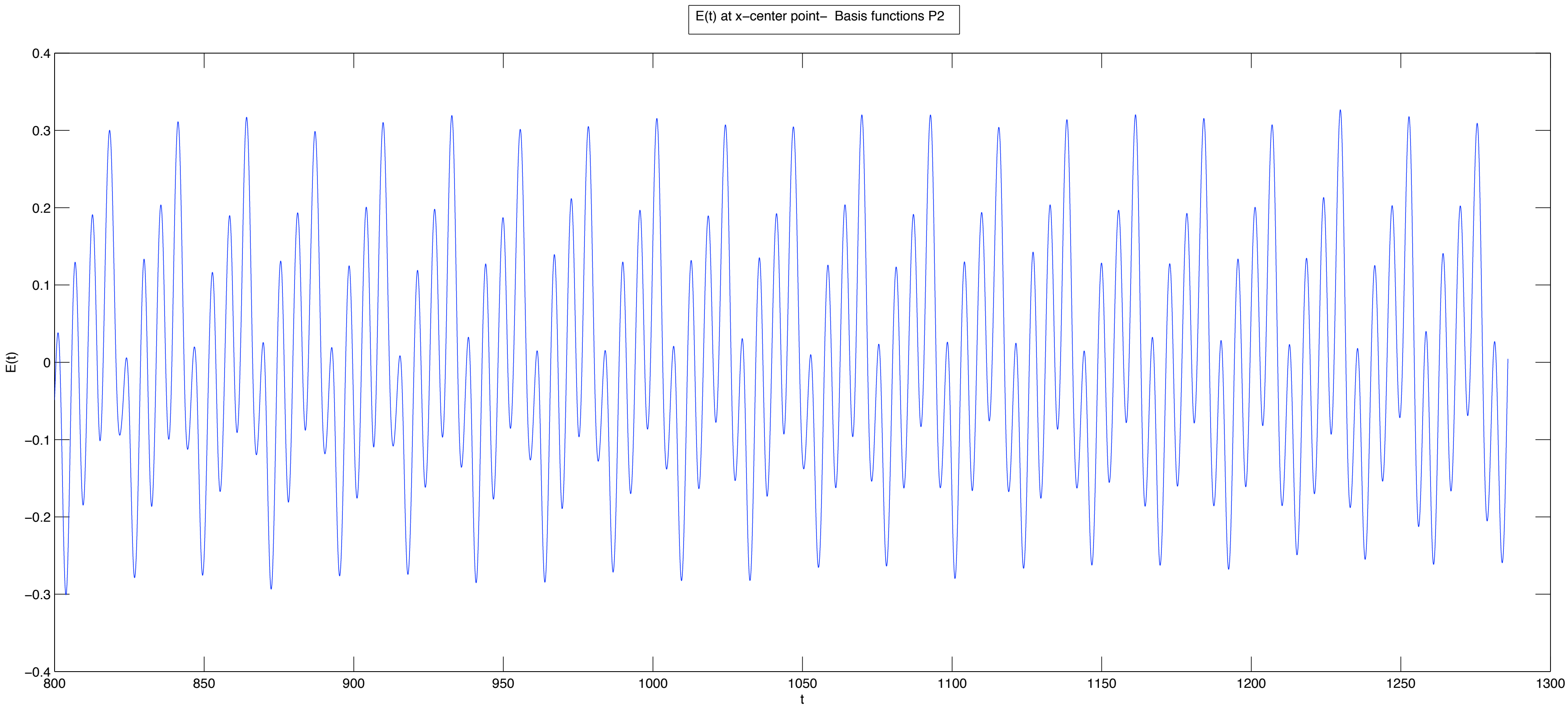


Central periodic orbit is BGK mode



# Strong Drive



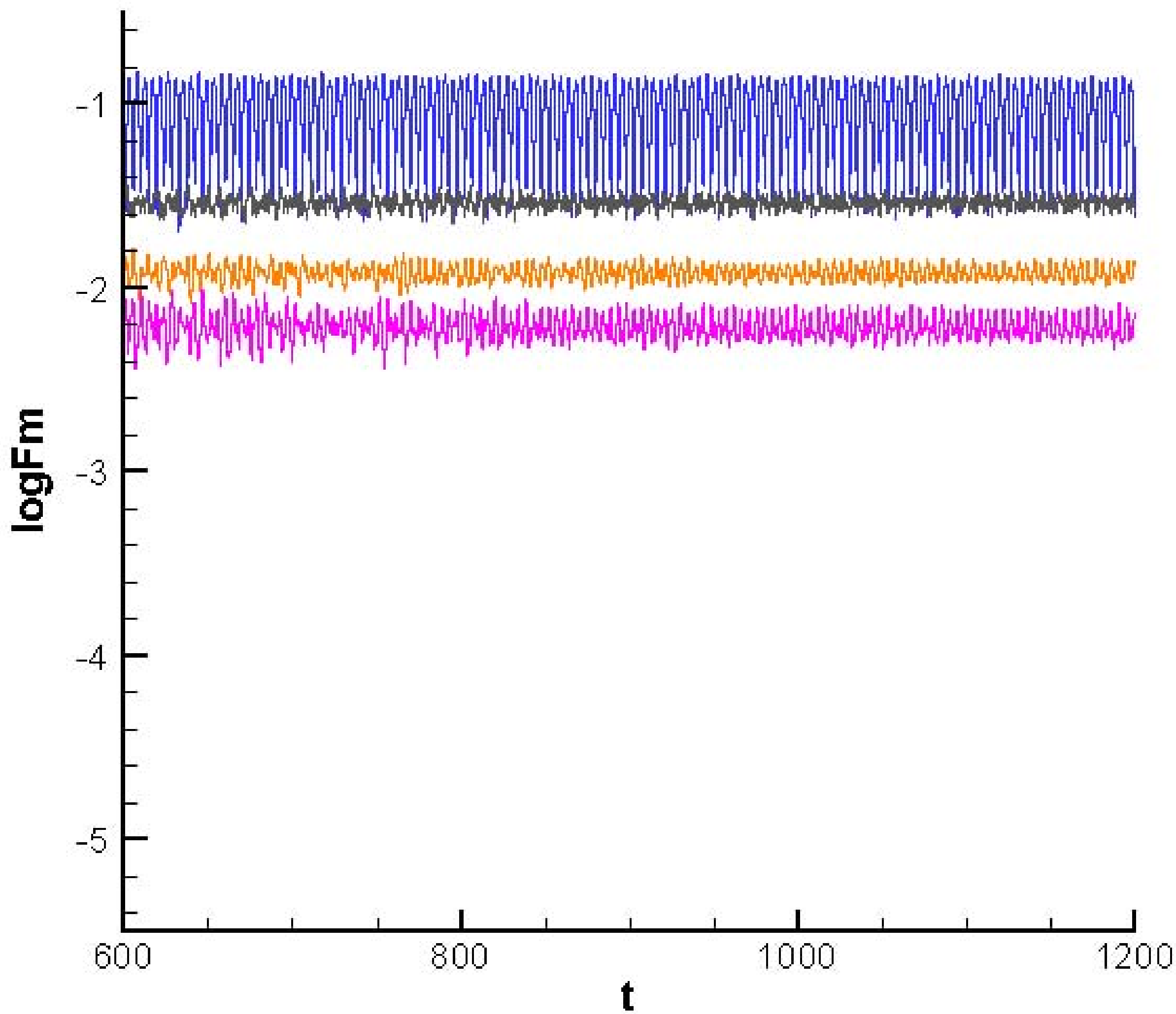

$A_d(t) = .4$ and $T_d = 200$

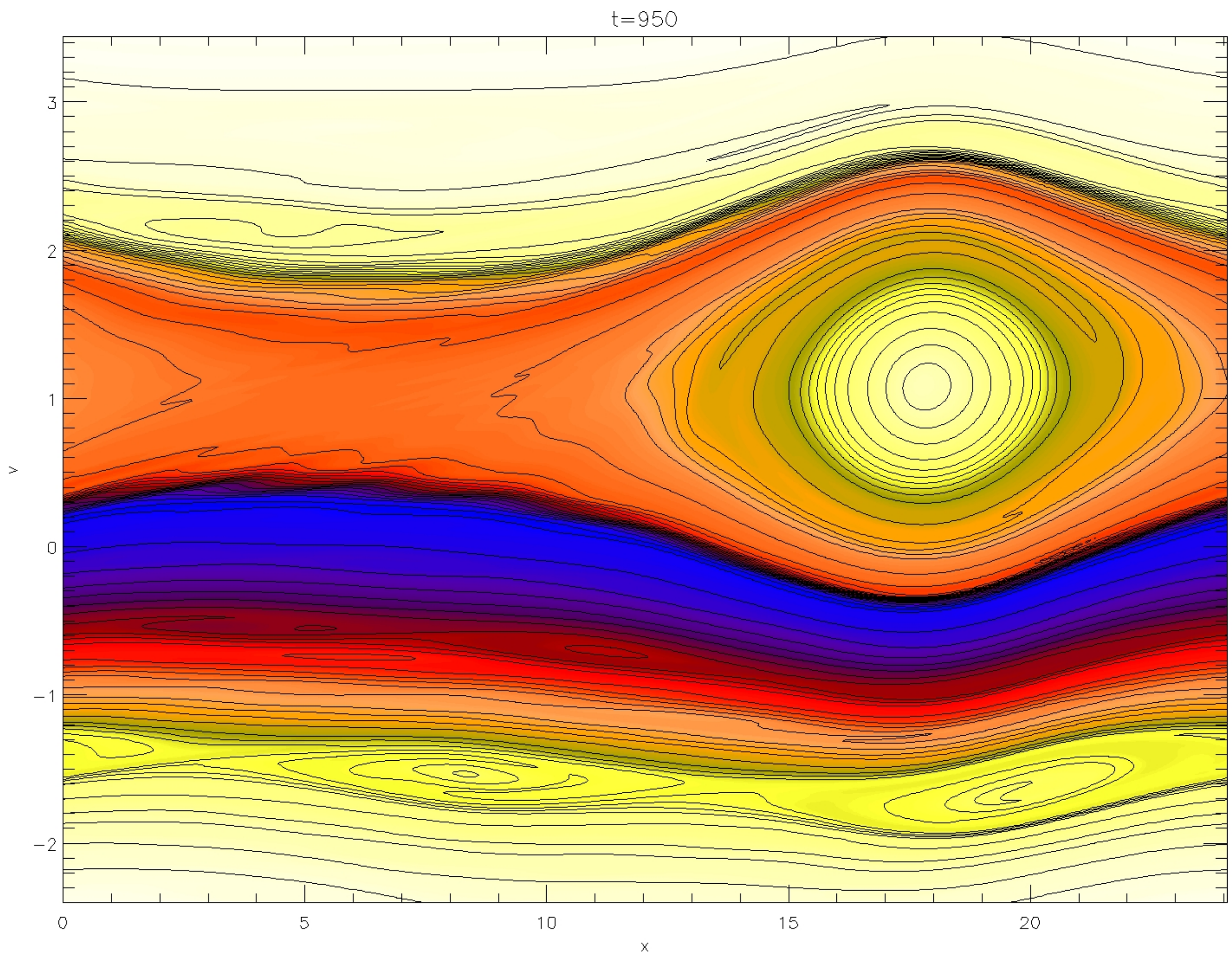
Higher Order Periodic/Quasiperiodic Orbit: $E(t) = A(t)E_0(t)$

$A(t) = A(t + T/4)$ with $E_0(t) = E_0(t + T)$

$E_0(t)$ like weak drive

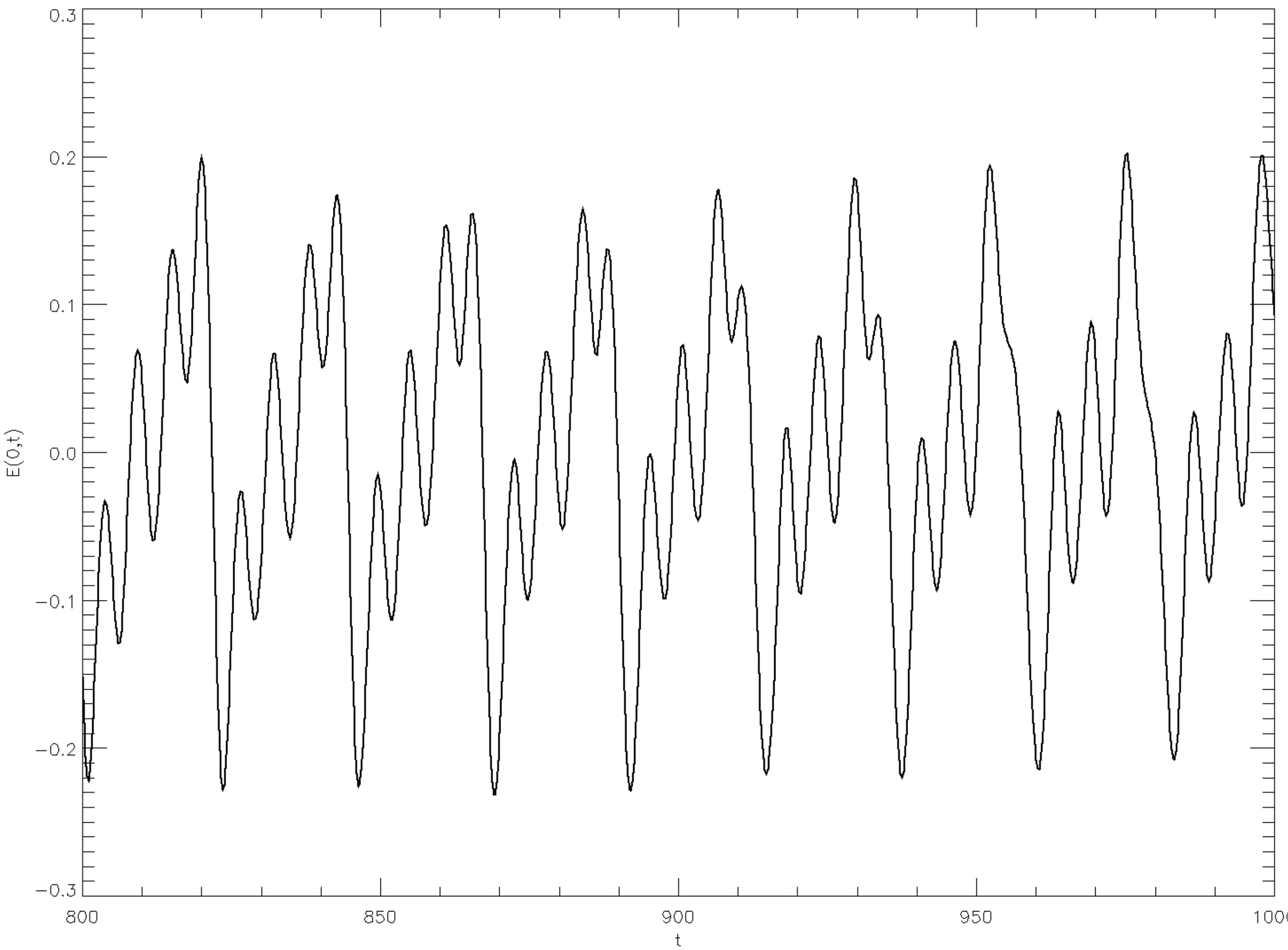
# Strong Drive Fourier





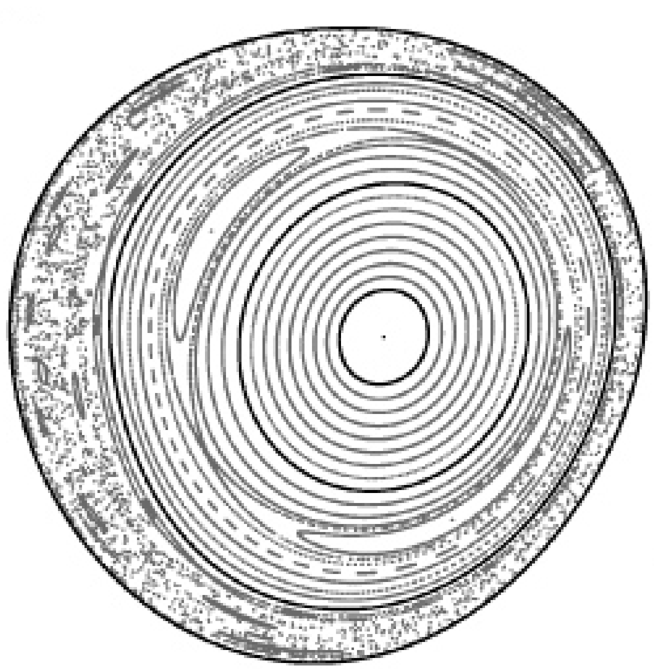

2 interacting BGK modes [Demeio and Zweifel (1990)]

mpg1(phase_space_2.mpg,shade_surf.mpg)



0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
E(0,t)
800
850
900
950
1000
t

# Periodic and Quasiperiodic orbits



# Multimode Drive

$$E_d(x,t) = E_{DA}^{(1)} g_1(t) \cos(k_1 x - \omega_1 t) + E_{DA}^{(2)} g_2(t) \cos(k_2 x + \omega_2 t)$$

mpg2(movie_1)

Recalcitrance

# Strong Slap

$$E_d(x,t) = E_{DA}^{(1)} g_1(t) \cos(k_1 x - \omega_1 t)$$

mpg2(movie_2)

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