Robust attractor of non-twist systems

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OVERVIEW

- **A NON-TWIST 2D-MAP**
- **TWO ISOCHRONOUS RESONANCES**
- **REGION OF MEANDERING TORI**
- **ROBUST SHEARLESS CURVE**
- **DISSIPATION**
- **ATTRACTORS**
- **ROBUST “SHEARLESS” ATTRACTOR**
- **NEW KIND OF ATTRACTOR**

$$\begin{cases} \text{QUASI-PERIODIC} \\ \text{OR} \\ \text{CHAOTIC} \end{cases}$$

- **LYAPUNOV DIAGRAMS**

THE MODEL AND THE SHEARLESS TORUS

The model we will consider is the dissipative *labyrinthic non-twist standard map* (LNSM), which is a perturbation of the non-twist standard map:

$$\begin{cases} y_{n+1} = (1-\gamma)y_n - b\sin(2\pi x_n) - b\sin(\eta\, 2\pi x_n) \\ x_{n+1} = x_n - a\ (y_{n+1} - r_1)(y_{n+1} - r_2) \end{cases}$$

γ : is the dissipation parameter; γ = 0 => No dissipation

b: is the perturbation parameter

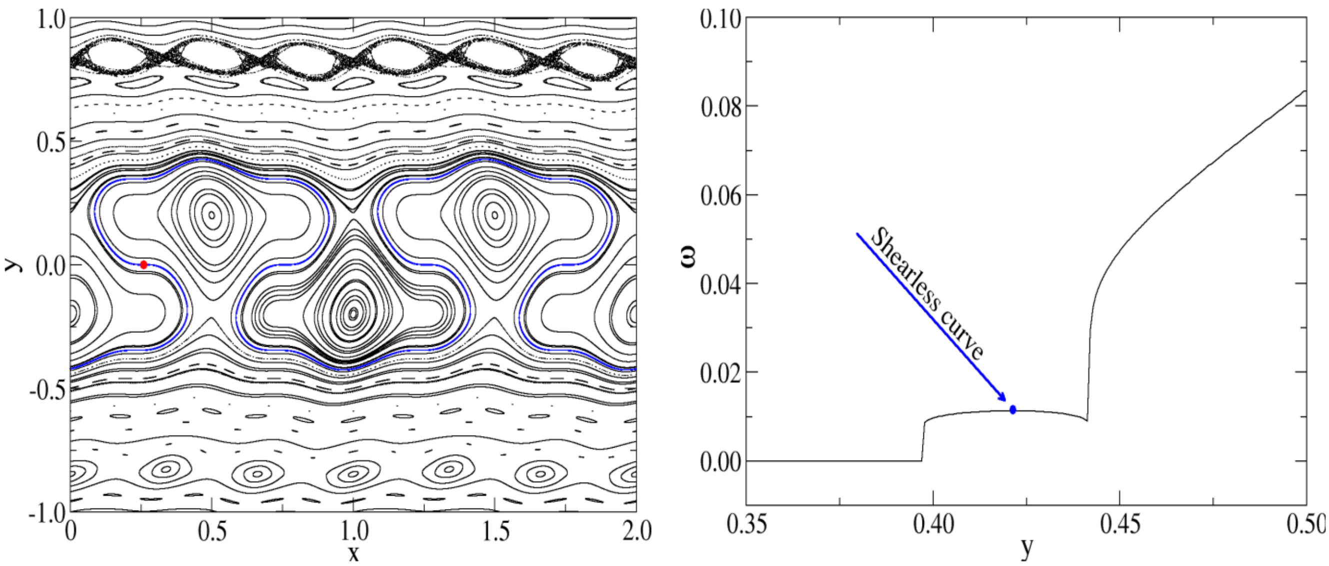
η : induces saddle-node bifurcations inside the islands

a: intereferes in the island amplitudes

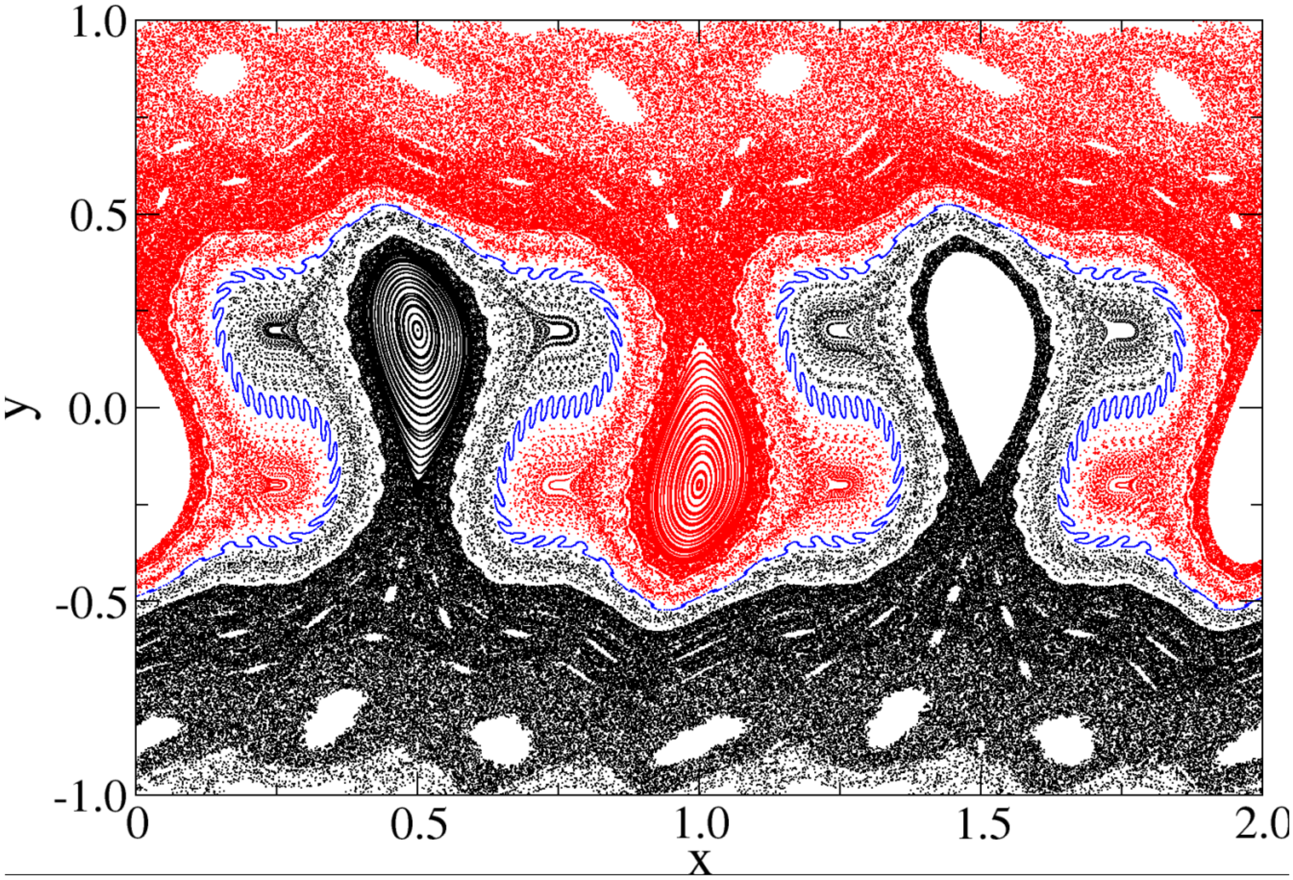
r_1 and r_2 define the positions of the isochronous resonances

Since the condition $\dfrac{\partial x_{n+1}}{\partial y_n} \neq 0$ is violated, the map is non-twist; the rotation number is non-monotonic.

In order to verify the shearless we introduce the winding number $\omega = \lim\limits_{n \to \infty} \dfrac{(x - x_0)}{n}$:



a) Typical phase space of the non-dissipative LNSM for a = 0.5, b = 0.02, $r_1 = -r_2 = 0.2$ and η = 3. The shearless curve is in **blue** and was obtained from iterations of the **red** point. The **red** point is an ***indicator point***; (b) The maximum point of the non-monotonic profile represents the shearless curve.



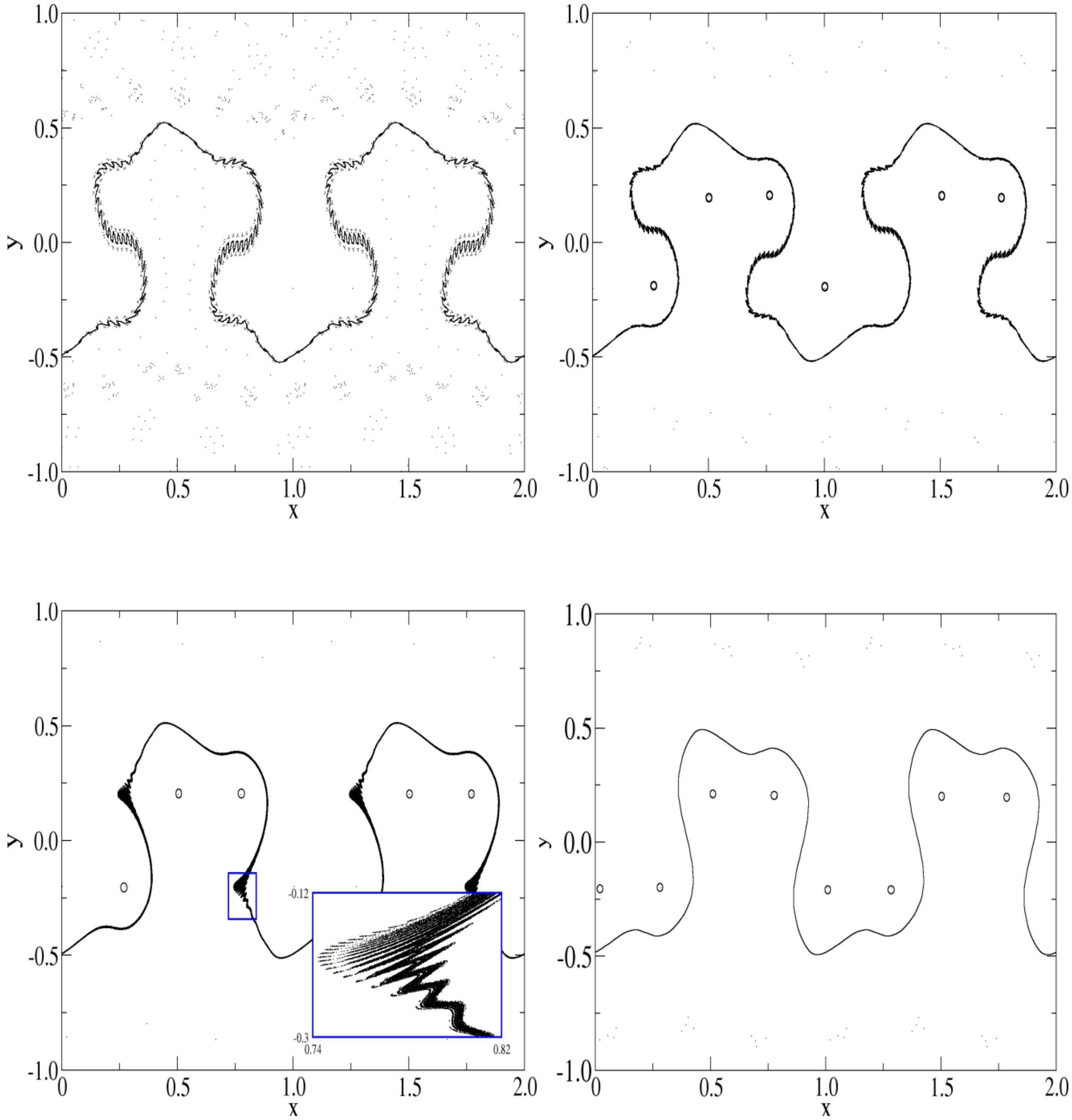
Two disconnected chaotic regions separated by the shearless torus, in blue. The value of the parameter of perturbation is b = 0 .056 and $\gamma = 0$ (non-dissipative).

THE “SHEARLESS” ATTRACTOR

Since a shearless torus survives for relatively strong perturbations and lives together with the chaotic sea, an intriguing question naturally emerges: What will globally happen in the system with the introduction of dissipation when there is at least one shearless torus?

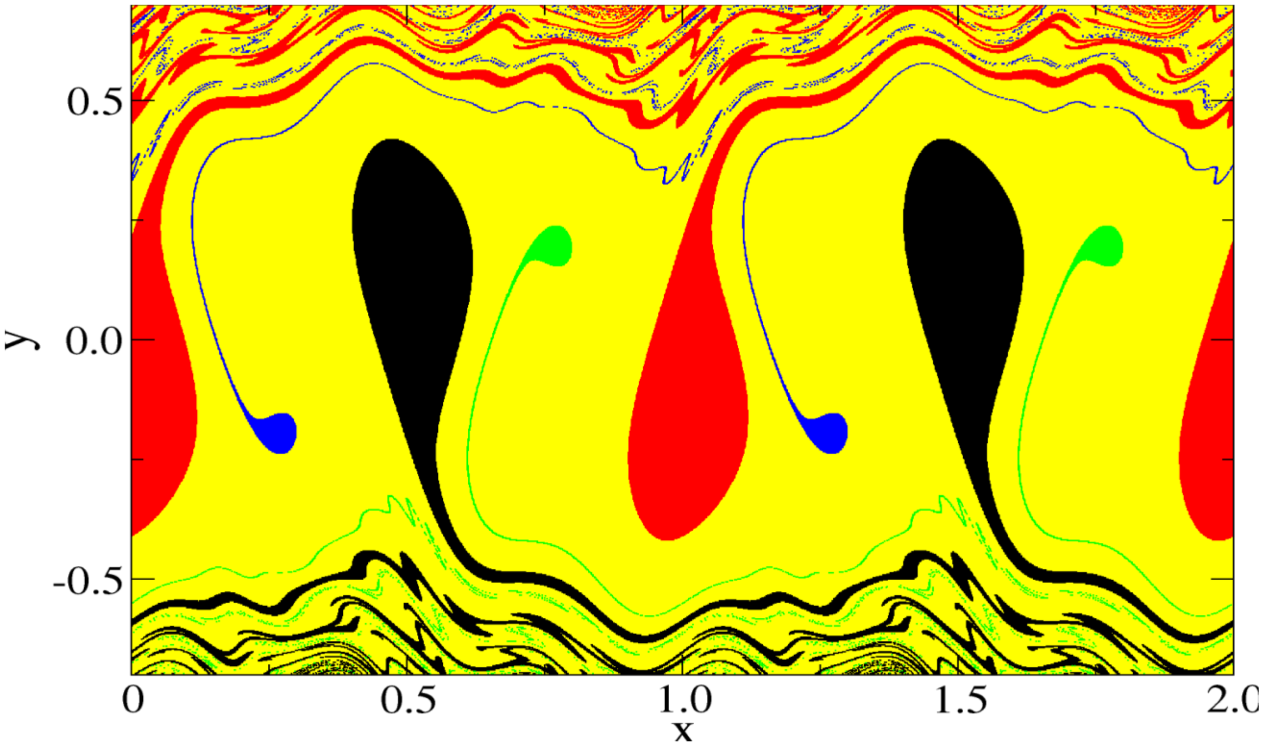
In order to understand this point, we fixed a = 0.5, η = 3, r_1 = 0.2 = – r_2, b = 0.056 and changed γ appropriately.

We show the evolution of the attractors for the dissipative LNSM. The plots have been obtained by iterating 104 times a set of initial conditions and plotting only the last hundred.



Evolution of the shearless attractor, in a) γ = 0.001, b) γ = 0.0022 and d) γ = 0.05 the attractor is quasi-periodic while in c) γ = 0.022 it spreads and looks like a chaotic attractor.

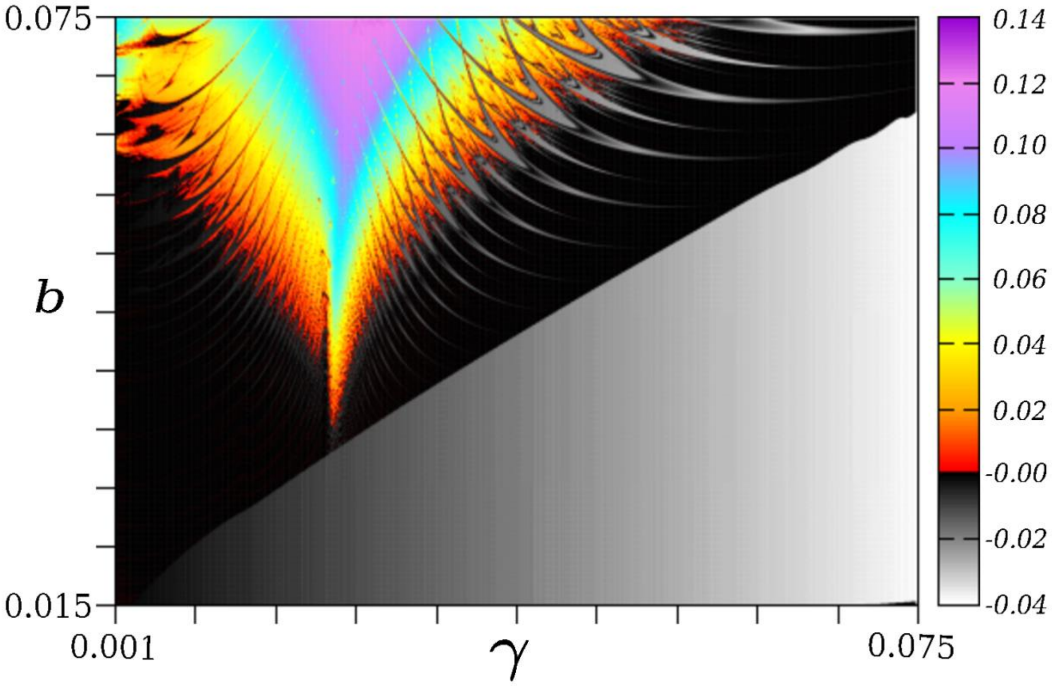
BASINS OF ATTRACTION OF EACH ATTRACTOR



This plot qualitatively suggests that the shearless attractor divides the basins of attraction of the stable foci in two distinct groups. Furthermore, the shearless attractor presents a basin of attraction bigger than the one of each stable focus.

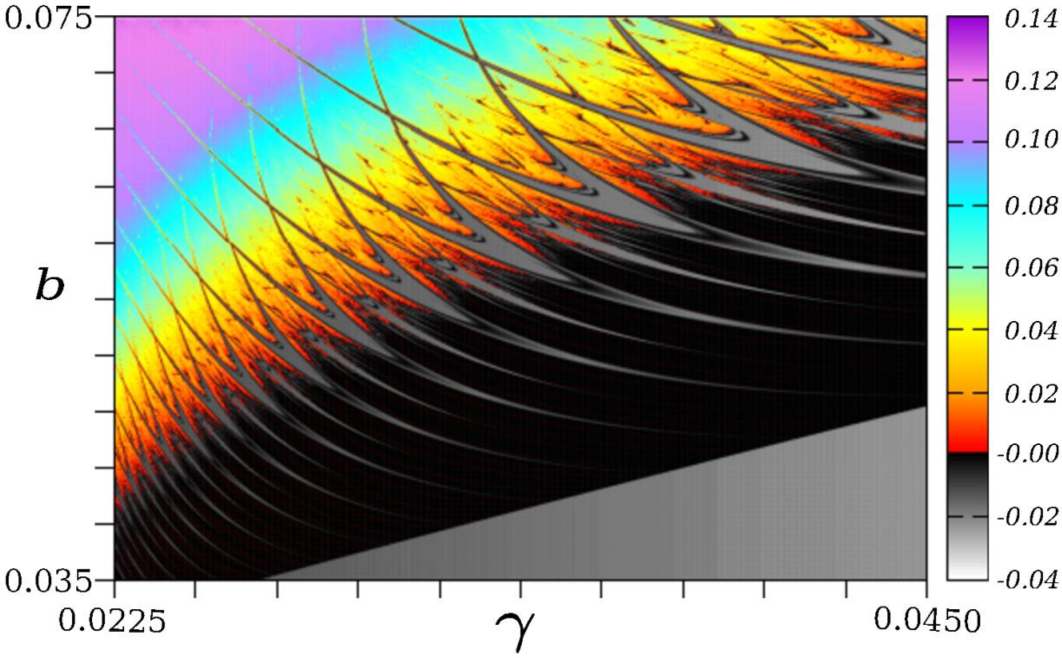
LYAPUNOV DIAGRAMS

To obtain the Lyapunov diagram for the shearless attractor from the dissipative LNMS we fixed a = 0.5 and we divide the space parameter (b, γ) in a grid of 1000 × 1000.



Lyapunov phase diagram characterizing the shearless attractor for the LNSM. White and gray tones means that it does not exist, the black region characterizes it as quasi-periodic and colored regions identify it as chaotic.

Amplification of previous figure in the space of parameters (b, γ).



Another point that should be mentioned is the presence of the so-called periodic-windows, also known as shrimps, in the Lyapunov diagram. In our case the shrimps correspond to regions where the shearless attractor does not exist and only the stable foci are present, or when it is quasi-periodic.

CONCLUSIONS

- We report the occurrence of a new kind of attractor.

- When dissipation is present in the system, the sturdy barrier (shearless) becomes a powerful attractor, called ***shearless attractor***, which can be quasi-periodic or chaotic depending on the set of parameters.

- We have also gotten shrimp structures in the Lyapunov diagram.

- We evaluate that these results are a new non-twist manifestation and correspond to a new fundamental for the non-linear dynamics theory.