Recurrence analysis of turbulent fluctuations in magnetically confined plasmas

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Recurrences in dynamics

- recurrence of states, in the meaning that states are arbitrary close after some time, is a fundamental property of deterministic dynamical systems and is typical for nonlinear or chaotic systems.
- Poincaré recurrence theorem (1890): certain systems will, after a sufficiently long but finite time, return to a state very close to the initial state
- a key element in the formulation of statistical mechanics

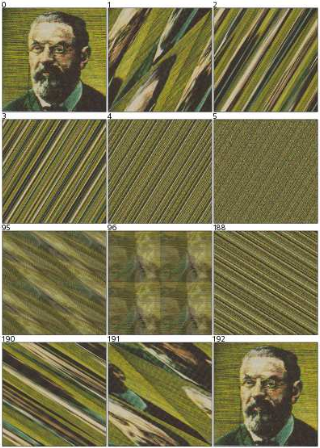
Recurrences in chaos

- Arnold's cat map

$$x \to x + y, \quad \text{mod } 1$$

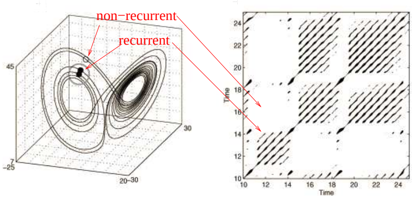
$$y \to x + 2y, \quad \text{mod } 1$$

- strongly chaotic orbit: stretching and folding
- square grid of pixels, to which a color is attributed
- inverse cat map iterated n times



Recurrence plots

- is a graph of a square matrix, in which the matrix elements correspond to those times at which a state of a dynamical system recurs
- recurrence of a state at time i at a different time j is marked within a two-dimensional squared matrix with ones and zeros dots (black and white dots in the plot), where both axes are time axes.
- $R_{ij} = \Theta(\varepsilon - ||\mathbf{x}_i - \mathbf{x}_j||)$



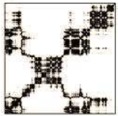
Lorenz chaotic attractor | recurrence plot

Types of recurrence plots


Homogeneous RP (uniform noise)


periodic oscillations


drift (logistic map w. increasing term)


disrupted RP (brownian motion)

- Homogeneous RPs are typical of stationary and autonomous systems in which relaxation times are short in comparison with the time spanned by the RP (e.g. random time series)
- Oscillating systems have RPs with diagonally oriented, periodic recurrent structures For quasi-periodic systems, the distances between the diagonal lines are different.
- The drift is caused by systems with slowly varying parameters. Such slow (adiabatic) change brightens the RP's upper-left and lower-right corners.
- Disruptions: abrupt changes in the dynamics as well as extreme events cause white areas or bands in the RP

Structures in recurrence plots

white noise (mostly isolated points) | sine function (only diagonal lines) | chaotic attractor of Roessler system

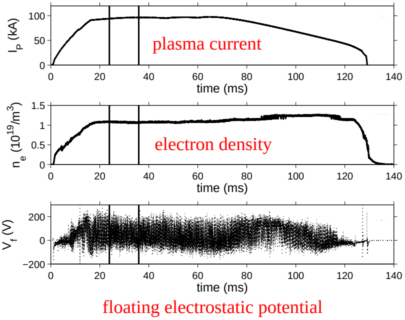
- Single, isolated recurrence points can occur if states are rare, if they do not persist for any time or if they fluctuate heavily
- Diagonal lines occurs when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments.
- Vertical (horizontal) lines mark a time length in which a state does not change or changes very slowly. It seems, that the state is trapped for some time. This is a typical behaviour of laminar states (intermittency)

Recurrence quantification analysis

- recurrence rate (RR): percentage of recurrence points in an RP
- determinism (DET): percentage of recurrence points which form diagonal lines
- laminarity (LAM): percentage of recurrence points which form vertical lines
- averaged diagonal line length (L): average length of the diagonal lines
- trapping time (TT): average length of the vertical lines
- longest diagonal line (Lmax): length of the longest diagonal line (its inverse is the maximum Lyapunov exponent)
- entropy (ENTR): Shannon entropy of the probability distribution of the diagonal line lengths $P(\ell)$

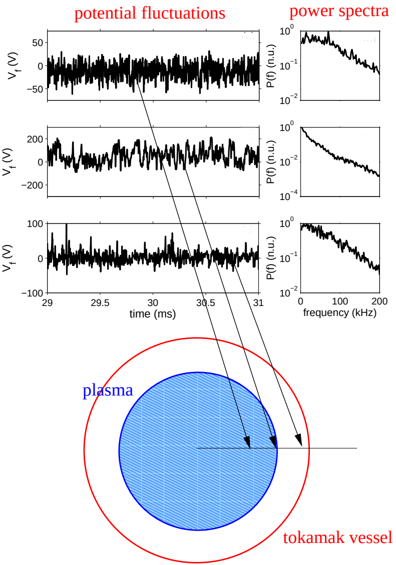
Turbulent fluctuations in TCABR

- Langmuir probes measure mean density (ion saturation current) and floating electrostatic potential
- probes mounted on a movable shaft (radial profiles)
- floating electrostatic potential exhibits large-amplitude fluctuations



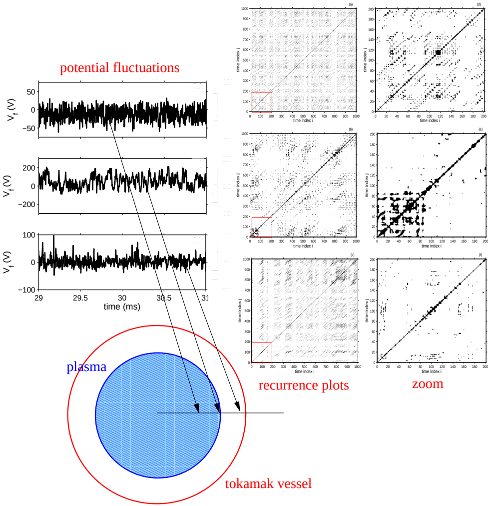
Radial dependence of turbulent fluctuations

- amplitudes of floating potential fluctuations at different radial positions
- plasma edge and scrape-off region
- broadband spectra of fluctuations: difficult to distinguish radial dependence of fluctuating behavior
- suggests the use of recurrence-based techniques



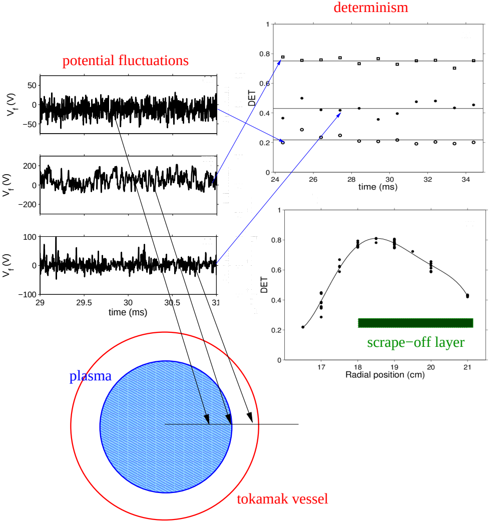
Recurrence plots of turbulent fluctuations

- embedding dimension 4 and time delay is the minimum of autocorrelation function
- diagonal and vertical structures at plasma edge: larger deterministic content?
- scattered plots inside and outside plasma edge: more stochastic behavior?



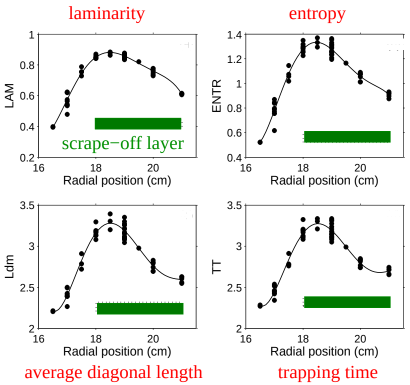
Recurrence quantification analysis of turbulence I

- moving window of length 1*ms* corresponding to 1000 points
- determinism (DET) computed for the moving window has a nearly constant value along the time series (stationarity)
- DET mean value is higher at plasma edge than otherwise
- radial profile of DET obtained for 57 different tokamak discharges (consistency against parameter changes)



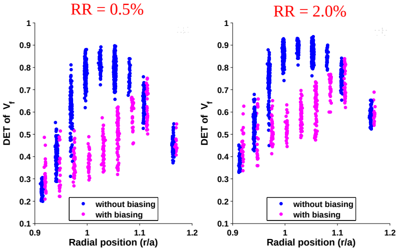
Recurrence quantification analysis of turbulence II

- used surrogate data (shuffling the original data) to destroy time correlations and compare DET for different radii
- laminarity (LAM), Shannon entropy (ENTR), average diagonal length (Ldm) and trapping time (TT) obey the same trend with respect to radial variations
- results suggest a larger deterministic content (and smaller contribution of stochastic effects) at the plasma edge



Effect of a bias radial electric field

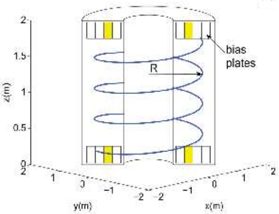
- application of bias radial electric field contributes to a major improvement in plasma confinement
- related to the creating of internal transport barriers that decrease radial transport of particles and energy
- DET with and without electric biasing
- DET decreases the most where it is more intense (at plasma edge)
- breakup of coherent structures



Texas Helimak

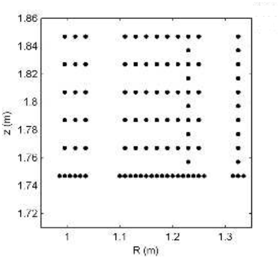
- plasma toroidal device operating at the Fusion Research Center (University of Texas at Austin)
- toroidal vacuum vessel with 16 toroidal field coils and 3 vertical field coils
- combination of a toroidal field and a vertical field produces helical field lines
- plasma density $n \leq 10^{11} cm^{-3}$, electron temperature $T_e = 10eV$
- simulates the scrape-off layer of a tokamak

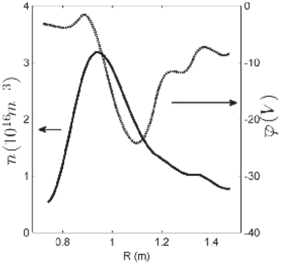
Texas Helimak



Radial profiles in Helimak

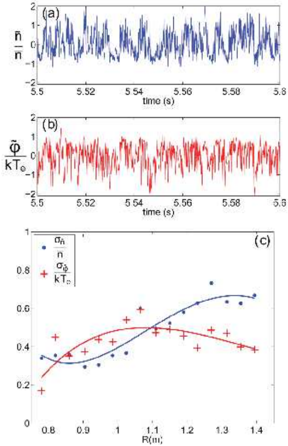
- 700 Langmuir probes mounted at four sets of bias plates at the top and the bottom of the vessel
- field lines have long connection length $\sim 40m$: neglect end effects (sheared cylindrical slab with one-dimensional equilibrium
- ion saturation current: gives the mean radial electron density profile $\bar{n}(R)$
- mean floating electrostatic potential $\bar{\varphi}(R)$





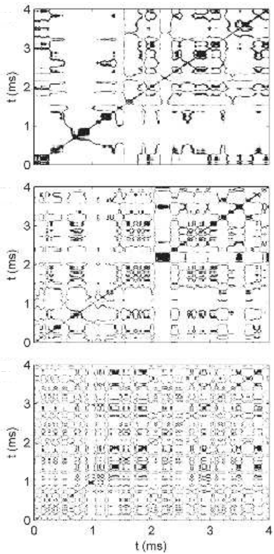
Turbulence fluctuations at Helimak

- plasma density at a fixed position R: $n(t) = \bar{n} + \tilde{n}(t)$
- floating potential: $\varphi(t) = \bar{\varphi} + \tilde{\varphi}(t)$
- turbulent fluctuations of both quantities (tildes)
- standard deviations of $\tilde{n}$ and $\tilde{\varphi}$ have different radial profiles
- turbulence levels depend on the radial position R



Recurrence plots of turbulence at Texas Helimak

- data series show the fluctuations of ion saturation current
- moving window 4 ms, corresponding to 2.000 points, with and without bias
- for negative bias the size of the recurrence structures in the RPs is bigger than for positive bias
- DET decreases from negative to positive bias
- the regularity is lower at positive bias than at negative one: increase of the turbulence



bias = − 30 V

DET = 0.93

bias = 0 V

DET = 0.90

bias = + 15 V

DET = 0.76

Conclusions

- recurrence plots are useful tools to quantify the deterministic content of turbulent fluctuations
- deterministic content is higher at the plasma edge
- bias radial electric field: more deterministic content at negative bias than at positive bias
- observations made in TCABR (São Paulo) and Texas Helimak

References

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- specific
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