

Rotas para o Caos

Disciplina: Caos em Sistemas Dissipativos

Instituto de Física, Universidade de São Paulo

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- II- Cenário de Ruelle-Takens
- III- Janelas Periódicas

II- Mapa do Círculo

(Unidimensional)

Mapa do Círculo

$$\theta_{n+1} = \theta_n + \Omega - \frac{K}{2\pi} \sin(2\pi\theta_n).$$

K: constante de acoplamento

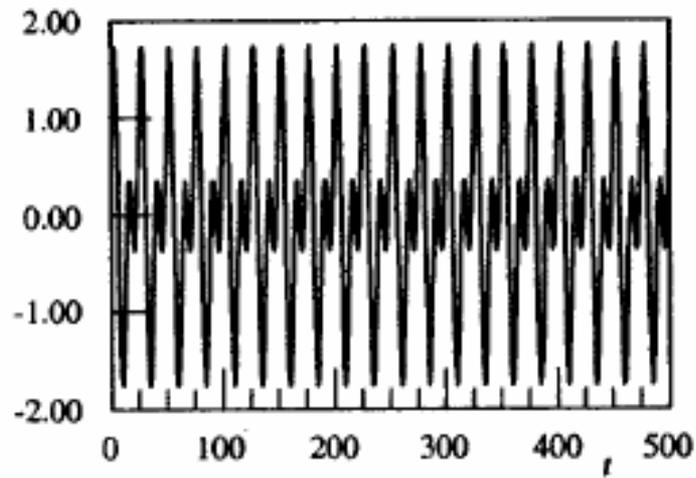
Ω : frequência de forçamento

$K = 0 \rightarrow$ rotação

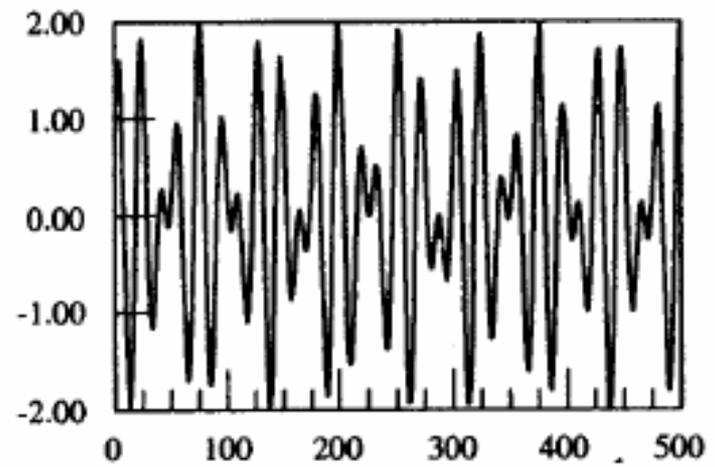
$\Omega = n/m \rightarrow$ órbitas periódicas

$\Omega =$ irracional \rightarrow órbitas quase-periódicas

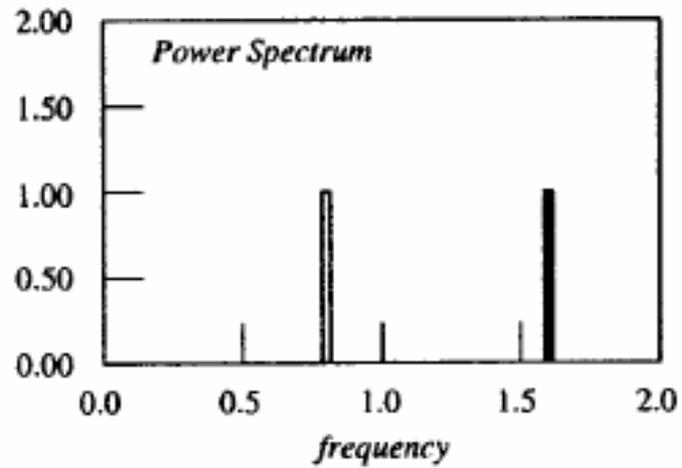
Periodic



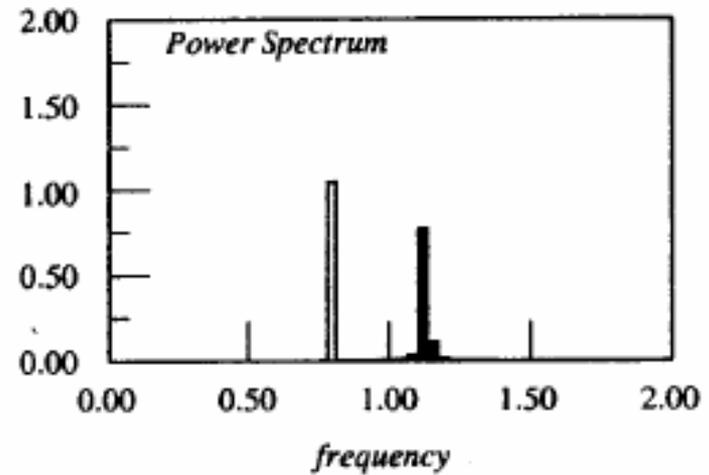
Quasi-periodic



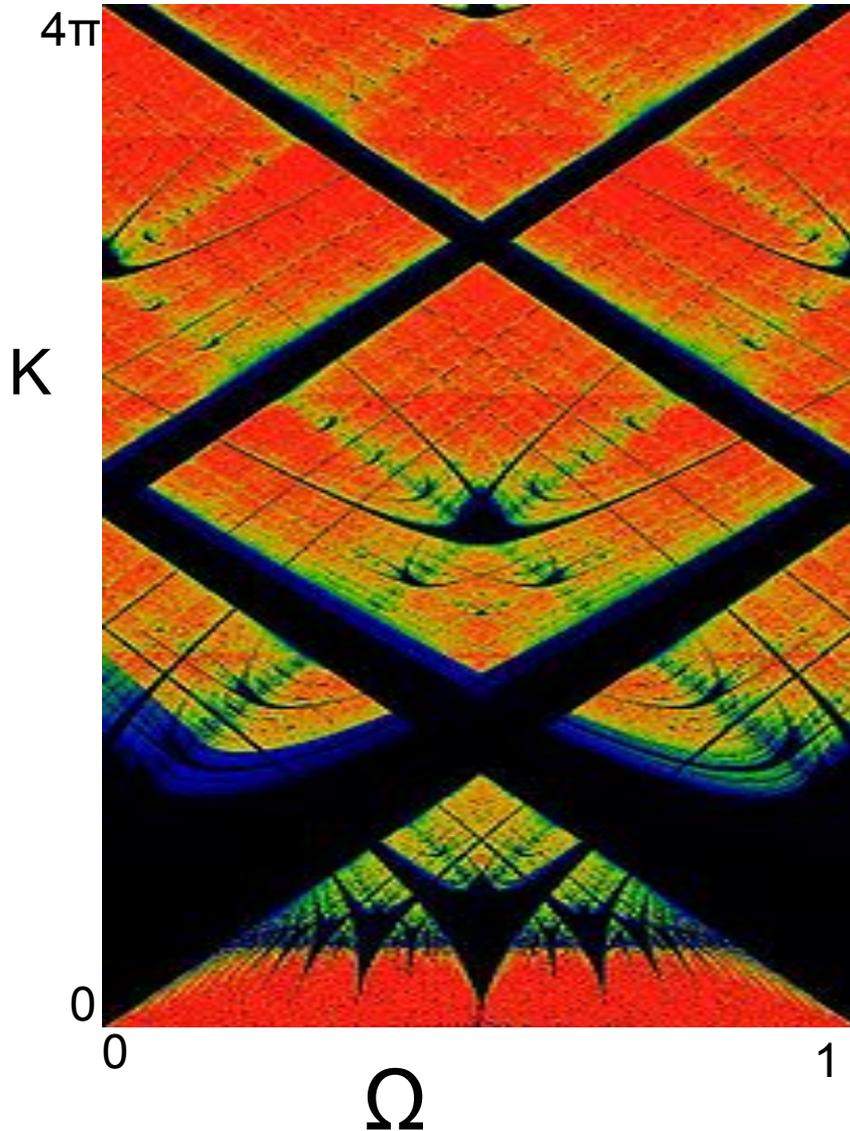
Power Spectrum



Power Spectrum



Espaço dos Parâmetros



Preto: órbitas periódicas

$K \sim 0 \rightarrow$ linguas de Arnold

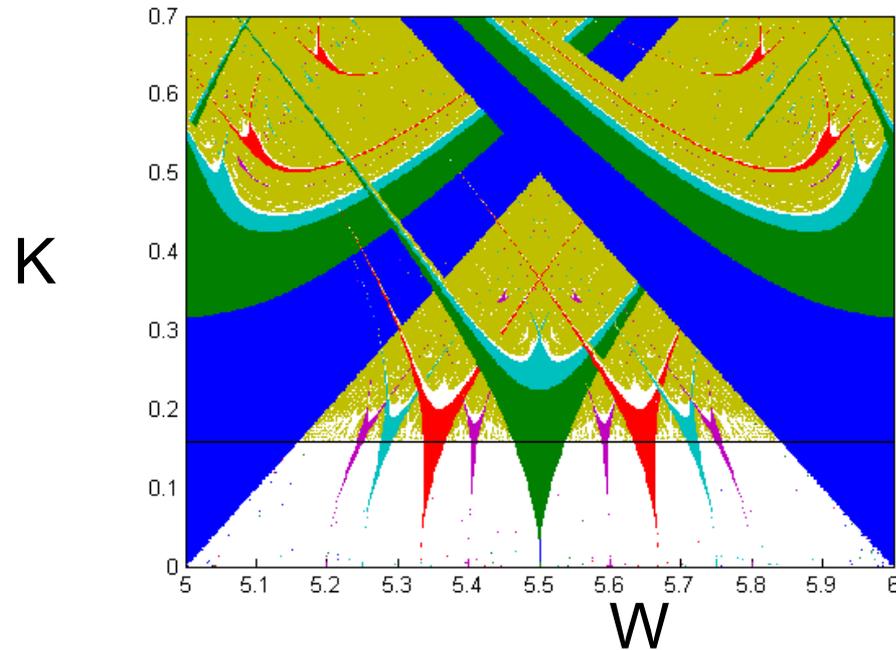
$\Omega = n/m \rightarrow$ órbitas periódicas

$\Omega = \text{irracional} \rightarrow$ órbitas quase-periódicas

Multiestabilidade
(coexistência de atratores)

Seminário da disciplina
Caos em Sistemas Dissipativos (2008)

Felipe A. C. Pereira



A cor azul representa período 1, a cor verde período 2, vermelho período 3, ciano período 4, rosa período 5, amarelo representa regiões caóticas com expoente de Lyapunov maiores que 0,001.

Biestabilidade

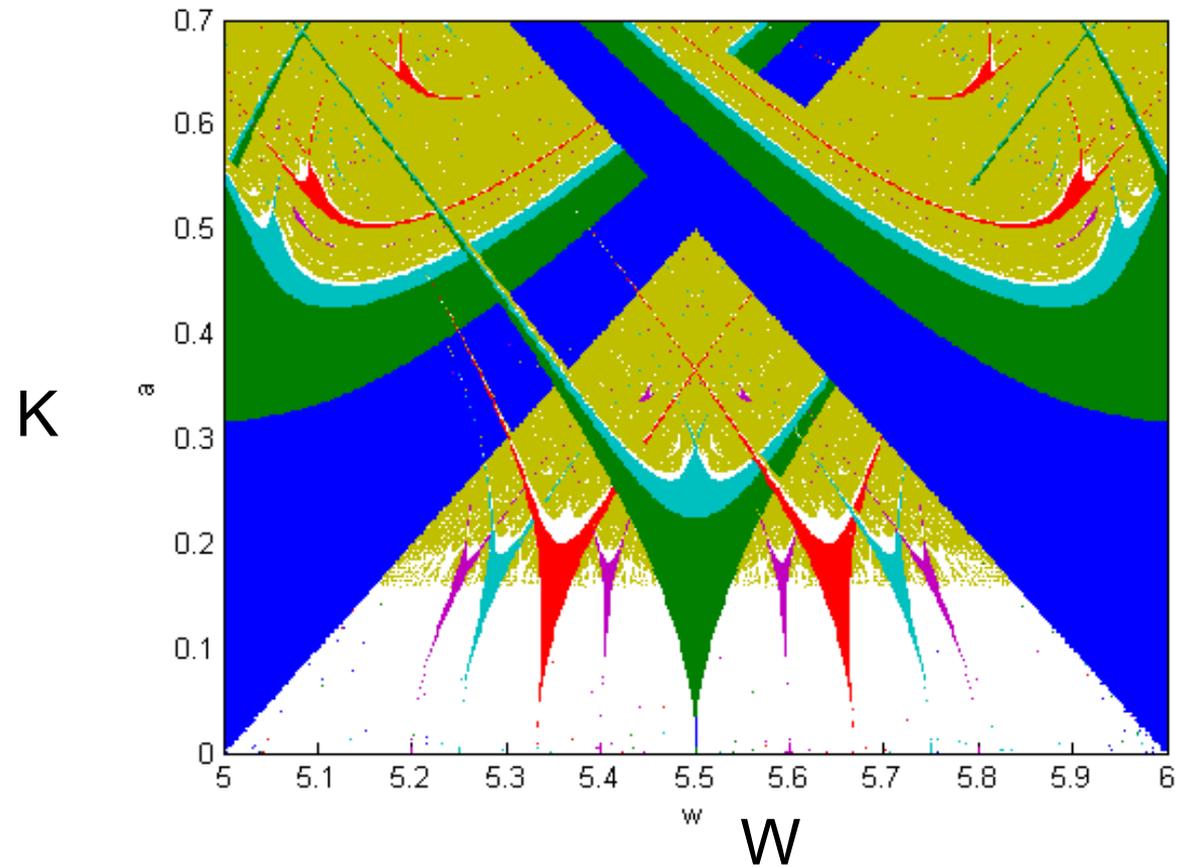


Diagrama de estabilidade para $x_0 = 0,32$

Biestabilidade

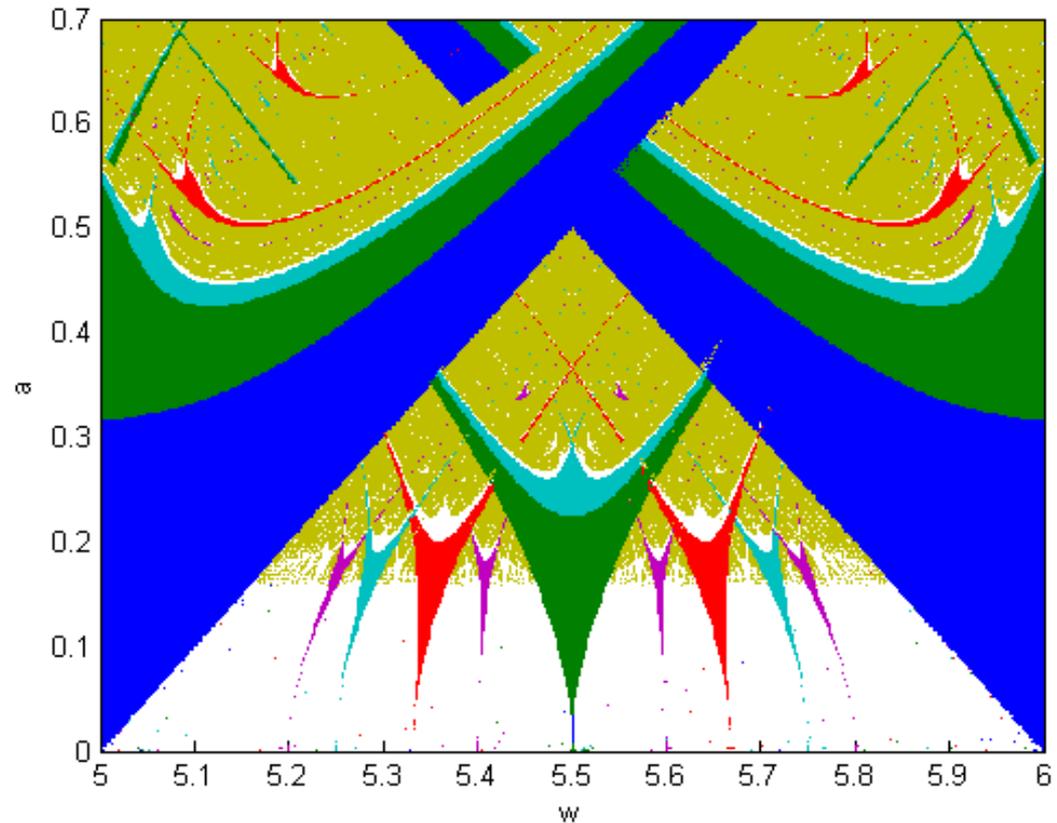
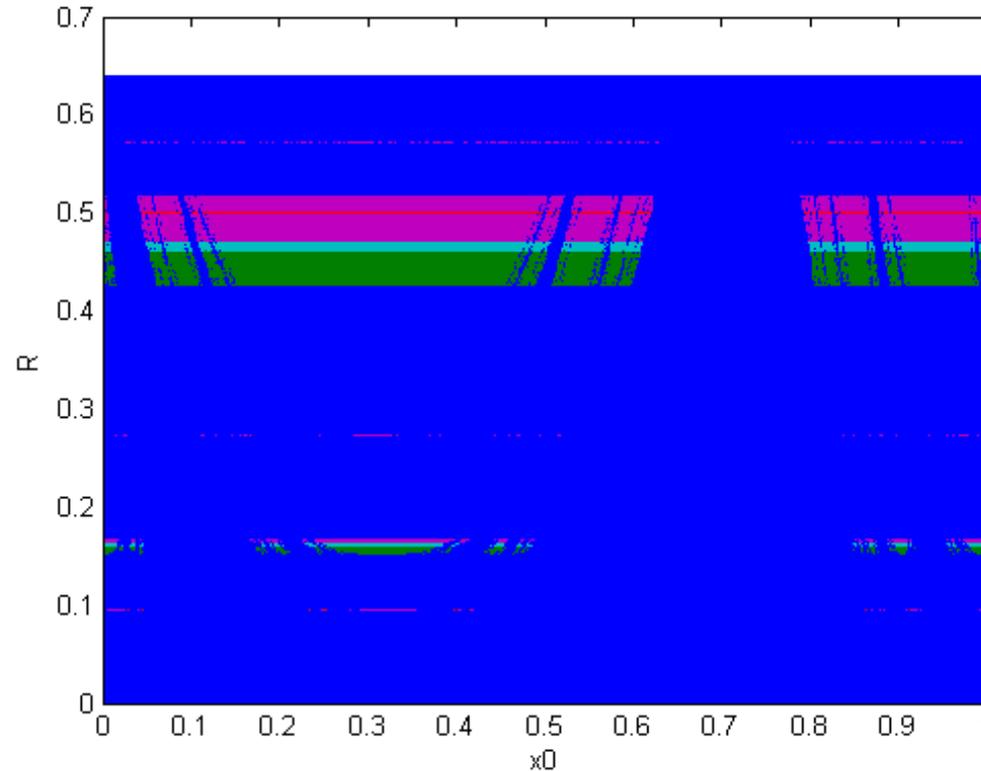


Diagrama de estabilidade para $x_0 = 0,78$

Biestabilidade 2



Azul indica período 1, verde período 2, vermelho período 3, ciano período 4, rosa caos com expoente de Lyapunov maior que 0,001

Diagramas de Bifurcação

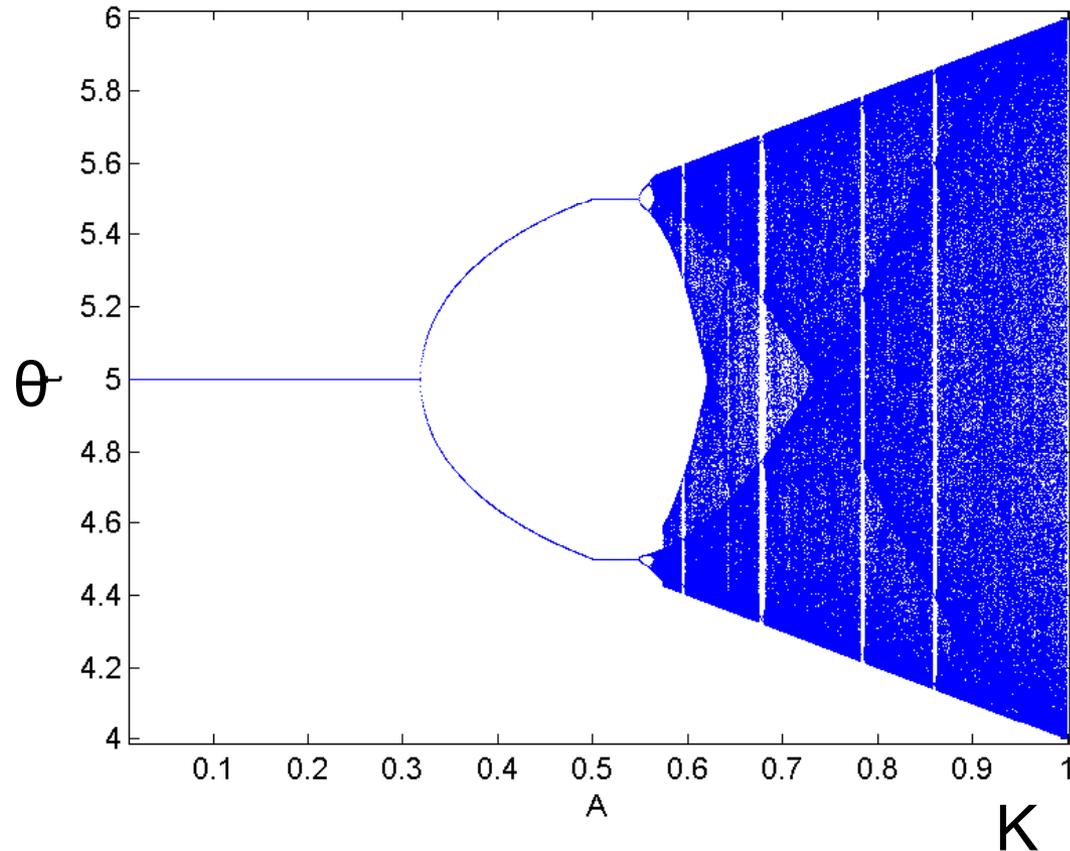
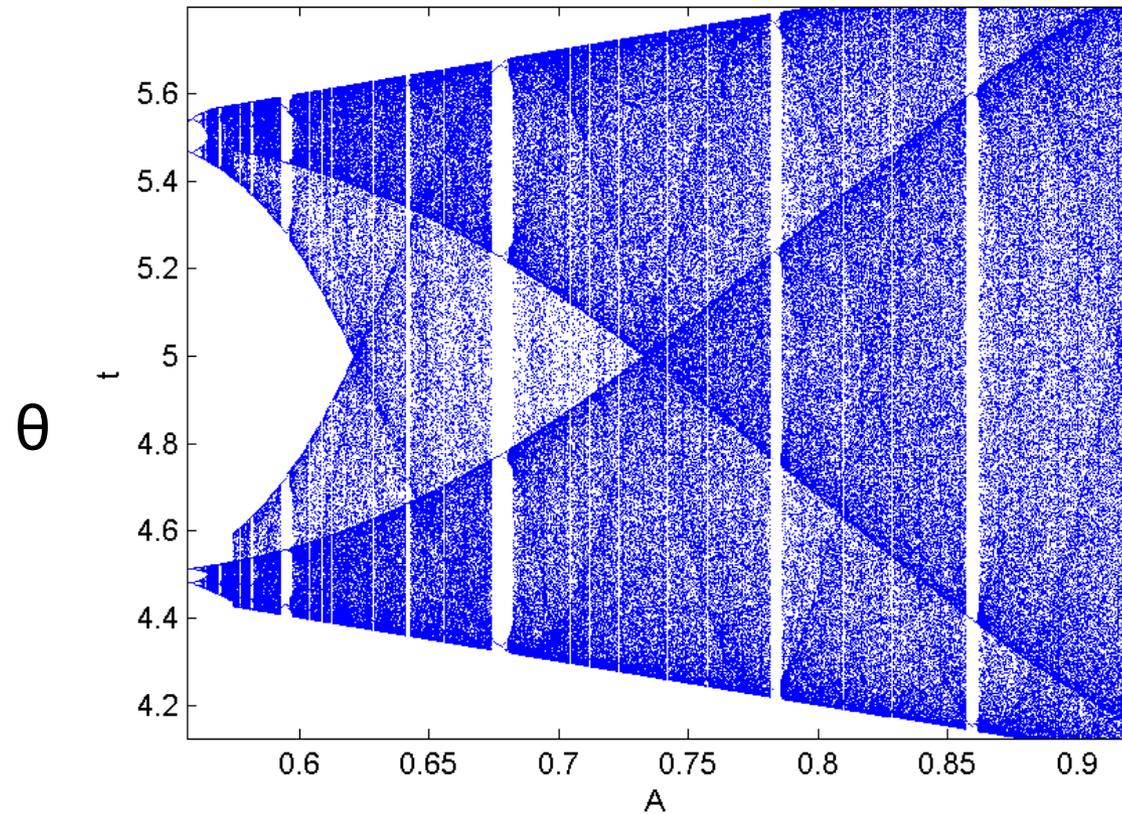


Diagrama de bifurcação para $w = 5$ e $x_0 = 0,235621$.

Diagramas de Bifurcação



Ampliação do Diagrama anterior.

K

Número de Rotação

$$\omega = \lim_{n \rightarrow \infty} \frac{\theta_n}{n}.$$

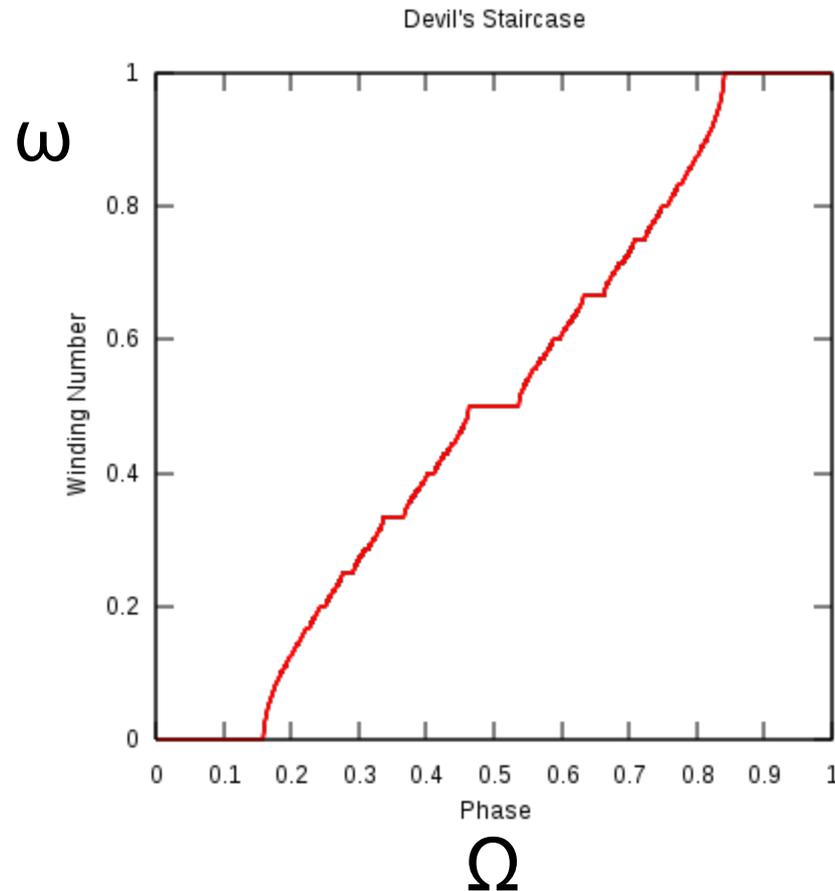
Órbitas regulares

$\omega = \text{racional} = n/m \rightarrow$ órbitas periódicas

$\omega = \text{irracional} \rightarrow$ órbitas quase-periódicas

Órbitas caóticas $\rightarrow \omega$ não converge

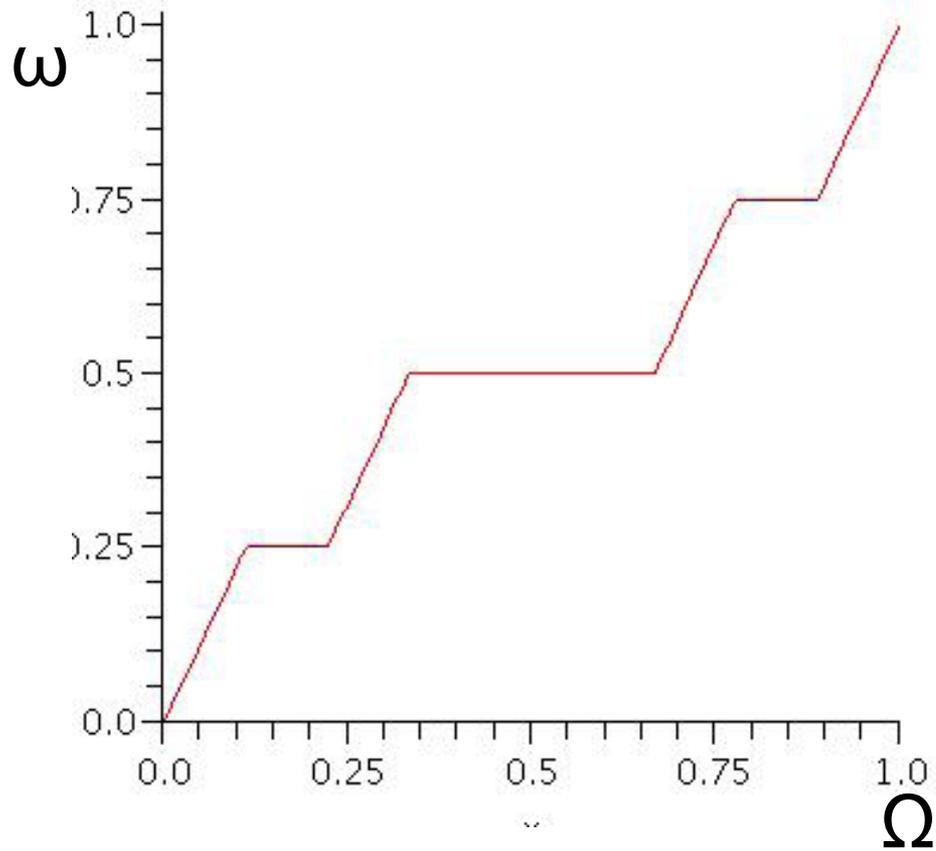
A Escada do Diabo



Órbitas periódicas
→ $\omega = m/n = \text{cte.}$

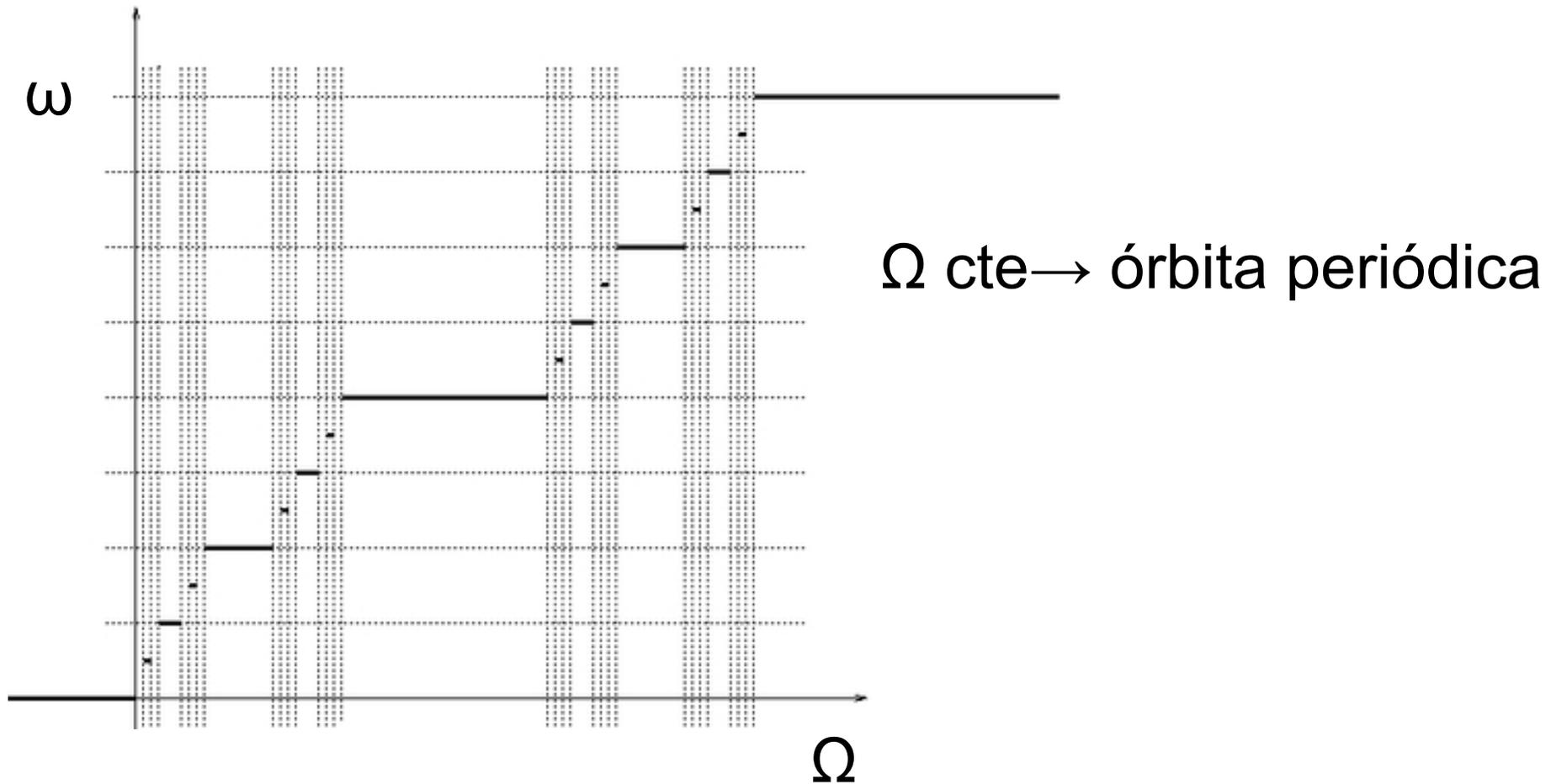
Número de rotação ω x frequência Ω
para $K = 1$

Escada do Diabo (Fractal)



Número de rotação X frequência
(amplitude fixa)

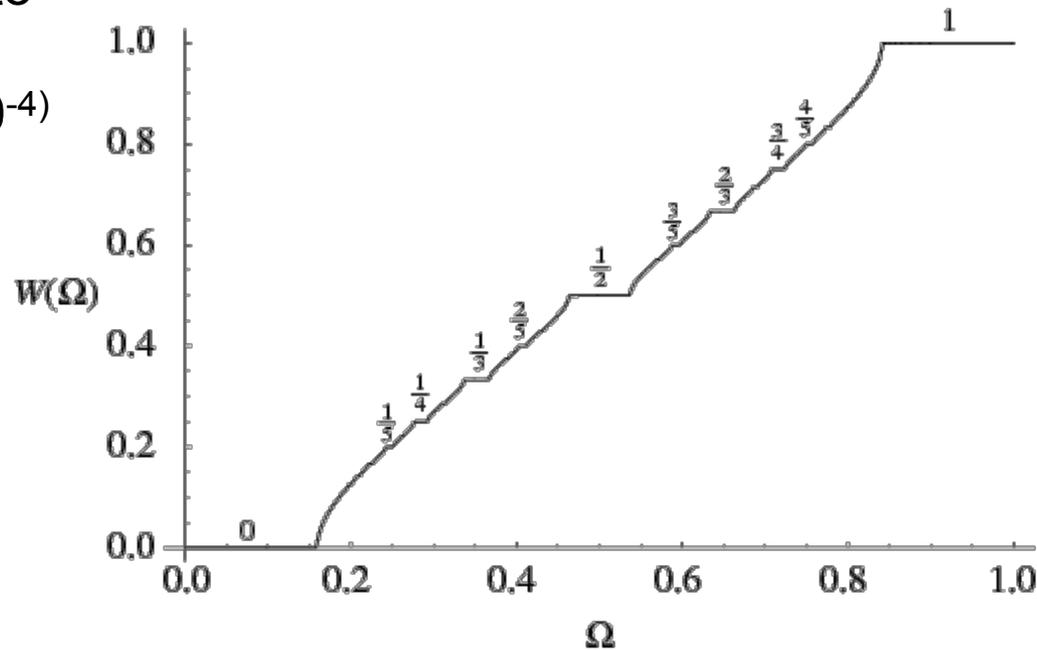
Escada do Diabo (Fractal)



Número de rotação \times frequência (amplitude fixa)

Número de Rotação de Algumas Órbitas Periódicas

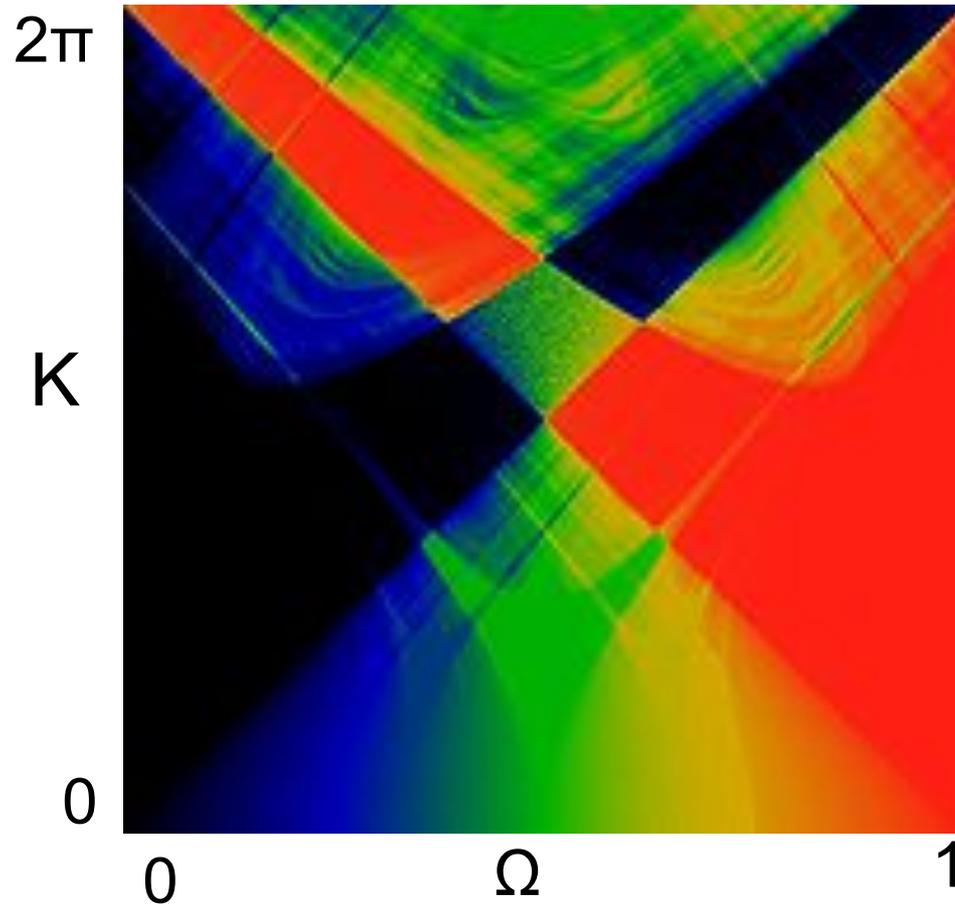
$W = m/n$, constante,
menos no conjunto
de Cantor
(dimensão $0,9 \times 10^{-4}$)



Número de rotação x frequência ($K = 1$)
(W não depende da condição inicial)

increasing "rational numbers" have

Cálculo do Número de Rotação ω



Preto: $\omega = 0$
Verde: $\omega = 0,5$
Vermelho: $\omega = 1$

II- Cenário de Ruelle-Takens

Ruelle-Takens Theorem

D. Ruelle and F. Takens, *Comm. Math. Phys.* **20**, 167 (1971)

An immensely influential paper in the history of the study of chaos was the work of Ruelle and Takens [1], later extended with Newhouse [2] on the robustness of quasiperiodic dynamics with 3 or more incommensurate frequencies. This addressed the question of whether complexity in dynamics was likely to occur through the accumulation of Hopf-like bifurcations adding additional frequencies (and presumably spatial “modes”), or instead through the onset of low dimensional chaos. The statement of the theorem was quite mathematical, which led initially to a misunderstanding of the strength of the result in the physics community, but nevertheless it remains a powerful result.

[1] D. Ruelle and F. Takens, *Comm. Math. Phys.* **20**, 167 (1971)

[2] S.E. Newhouse, D. Ruelle and F. Takens, *Comm. Math. Phys.* **64**, 35 (1978)

Rota em um Mapa de Poincaré Bidimensional (Fluxo Tridimensional)

C. Grebogi, E. Ott and J.A. Yorke Phys. Rev. Lett. **51**, 339 (1983); Physica **D15**, 354 (1985)

$$\begin{aligned}\theta_{n+1} &= \theta_n + \omega_1 + \varepsilon P_1(\theta_n, \phi_n) \pmod{1} \\ \phi_{n+1} &= \phi_n + \omega_2 + \varepsilon P_2(\theta_n, \phi_n) \pmod{1}\end{aligned}$$

where P_1, P_2 are nonlinear functions periodic in both θ and ϕ with period 2π . The explicit forms used were sums of sinusoidal functions $A_{rs} \sin[2\pi(r\theta + s\phi + B_{rs})]$ with A_{rs} and B_{rs} chosen randomly for (r, s) taking the values $(0, 1), (1, 0), (1, 1), (1, -1)$. Equation (21.2) takes the form of two, nonlinearly coupled circle maps.

(P), as well as chaos. The types of motion were identified through the values of the two Lyapunov exponents (together with the third value 0 for the corresponding flow):

Map exponents	Flow Exponents	Dynamics
0, 0	0, 0, 0	3-frequency QP
0, -	0, 0, -	2-frequency QP
-, -	0, -, -	1-frequency (P)
+, ?	+, 0, ?	Chaotic

They found the percentage of the values of leading to each type of motion

Attractor	$\varepsilon/\varepsilon_c = 0.375$	$\varepsilon/\varepsilon_c = 0.75$	$\varepsilon/\varepsilon_c = 1.125$
3-frequency QP	82%	44%	0%
2-frequency QP	16%	38%	32%
Periodic	2%	11%	31%
Chaotic	0%	7%	36%

where ε_c is the value of ε for which the map becomes noninvertible. (When the map is noninvertible it can be shown that there is no 3-frequency periodic motion, c.f. the 1d circle map at $K > 1$.) Thus in this example at least, 3-frequency quasiperiodic

Evidências Experimentais

Experimentally 3-frequency quasiperiodic motion has been documented by Gollub and Benson [4], and 4-frequency and 5-frequency quasiperiodic motion by Walden et al.[5], both in fluid convection.

[4] J.P. Gollub and S.V. Benson, *J. Fluid. Mech.* **100**, 449 (1980)

[5] R.W. Walden, P. Kolodner, A. Passner, C.M. Surko, *Phys. Rev. Lett.* **53**, 242 (1984)

III- Janelas Periódicas: Um Cenário Homoclínico

Rene O. Medrano-T., Iberê L. Caldas

IF-USP

Motivação :

Espaço dos parâmetros das equações não lineares com soluções caóticas

Propriedades a serem explicadas :

a - Janelas periódicas (para mapas e equações diferenciais)

b - Distribuição dessas janelas

c - Sequência de janelas com períodos progressivamente crescentes

Sistema de Roessler

- $\dot{x} = -y - z$
- $\dot{y} = x + a y$
- $\dot{z} = b + (x - c) z$

Variáveis: $x, y, z \rightarrow$ espaço de fase tridimensional

Parâmetros de controle: a, b, c

Atrator Caótico de Roessler

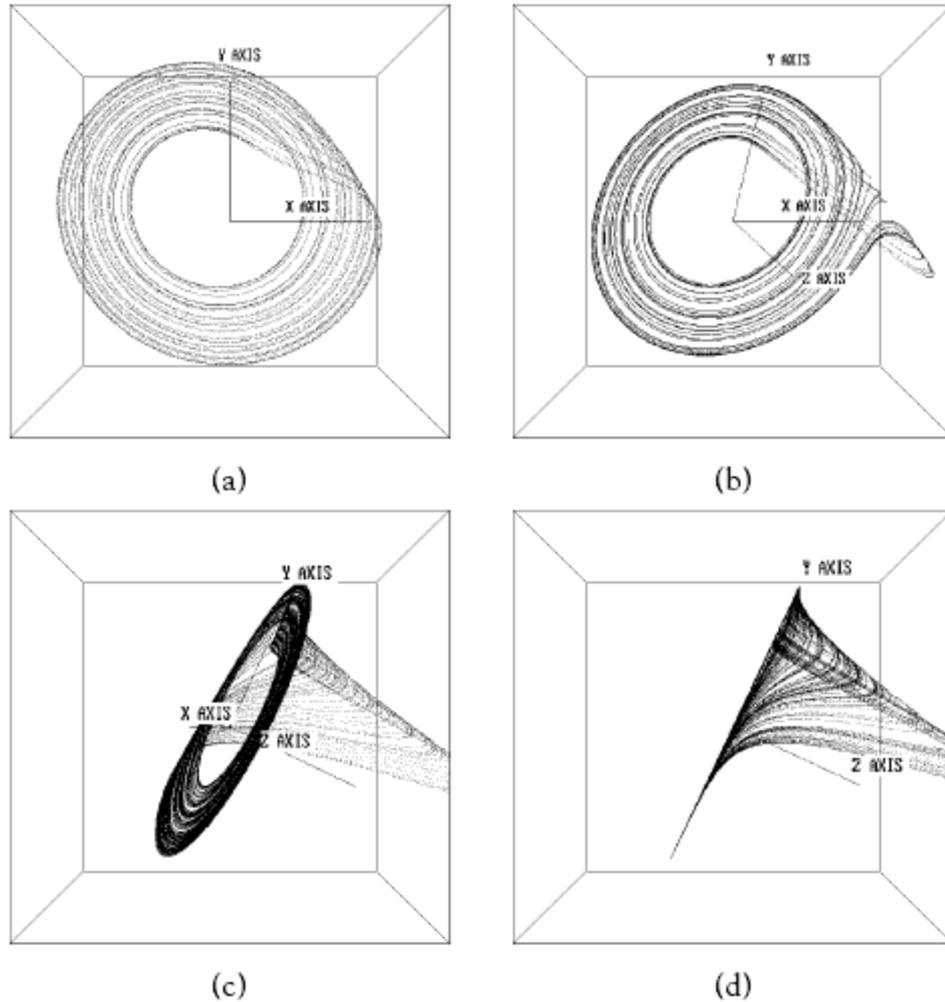


Figure 9.6 The Rössler attractor.

Parameters are set at $a = 0.1$, $b = 0.1$, and $c = 14$. Four different views are shown. The dynamics consists of a spiraling out from the inside along the xy -plane followed by a large excursion in the z -direction, followed by re-insertion to the vicinity of the xy -plane. Part (d) shows a side view. The Lyapunov dimension is 2.005—indeed it looks like a surface.

Chaos
Alligood et al.

Resultados obtidos para as equações de Roessler

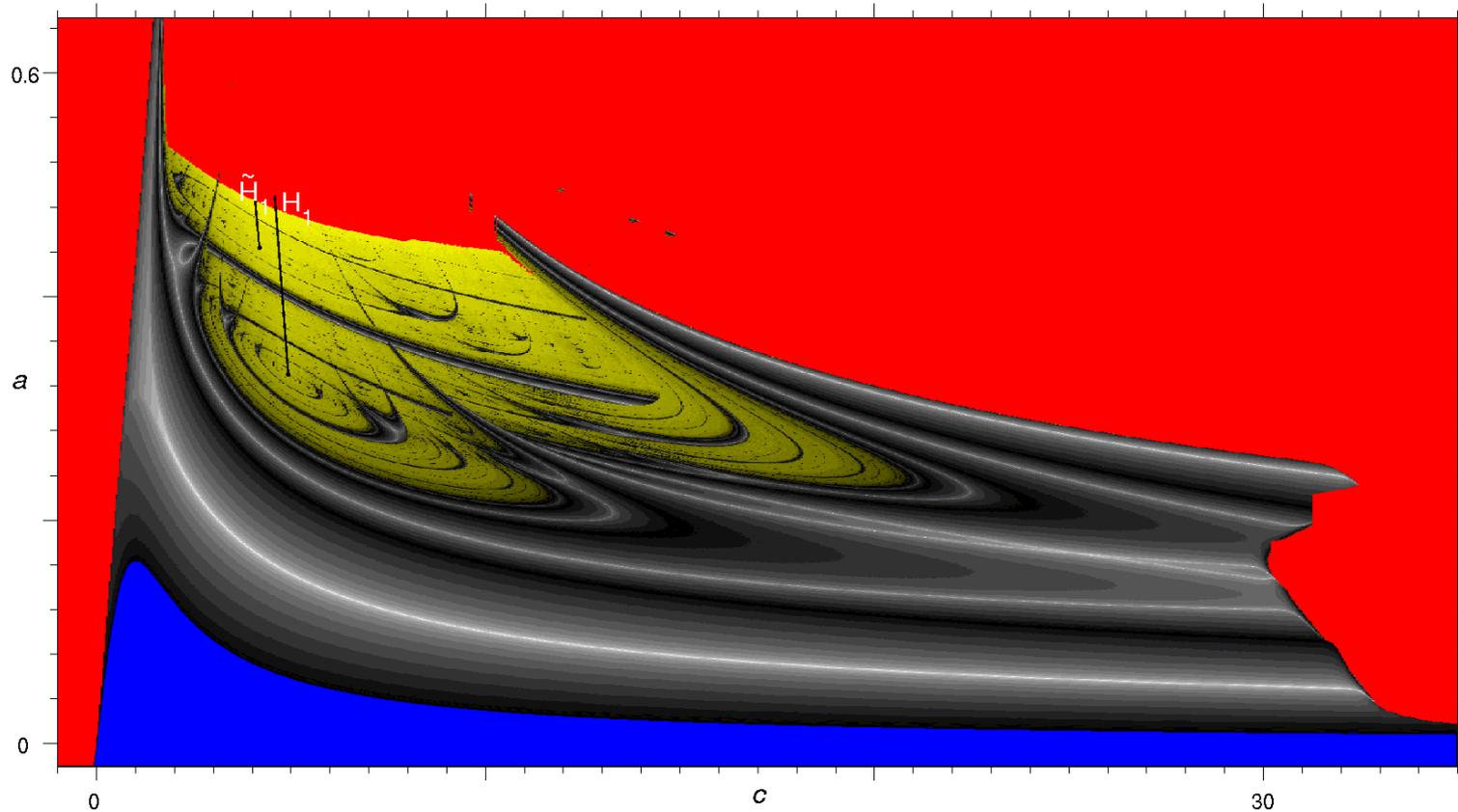
Previsto pelo teorema de Shilnikov \Rightarrow $\left\{ \begin{array}{l} \text{órbitas homoclínicas} \\ \text{distribuição, no espaço dos parâmetros,} \\ \text{de janelas periódicas em torno} \\ \text{das órbitas homoclínicas} \end{array} \right.$

Aplicação do teorema de Shilnikov:

Descoberta da distribuição de órbitas homoclínicas no espaço dos parâmetros \Rightarrow distribuição das janelas periódicas

Sistema de Roessler

Atratores no Espaço dos Parâmetros

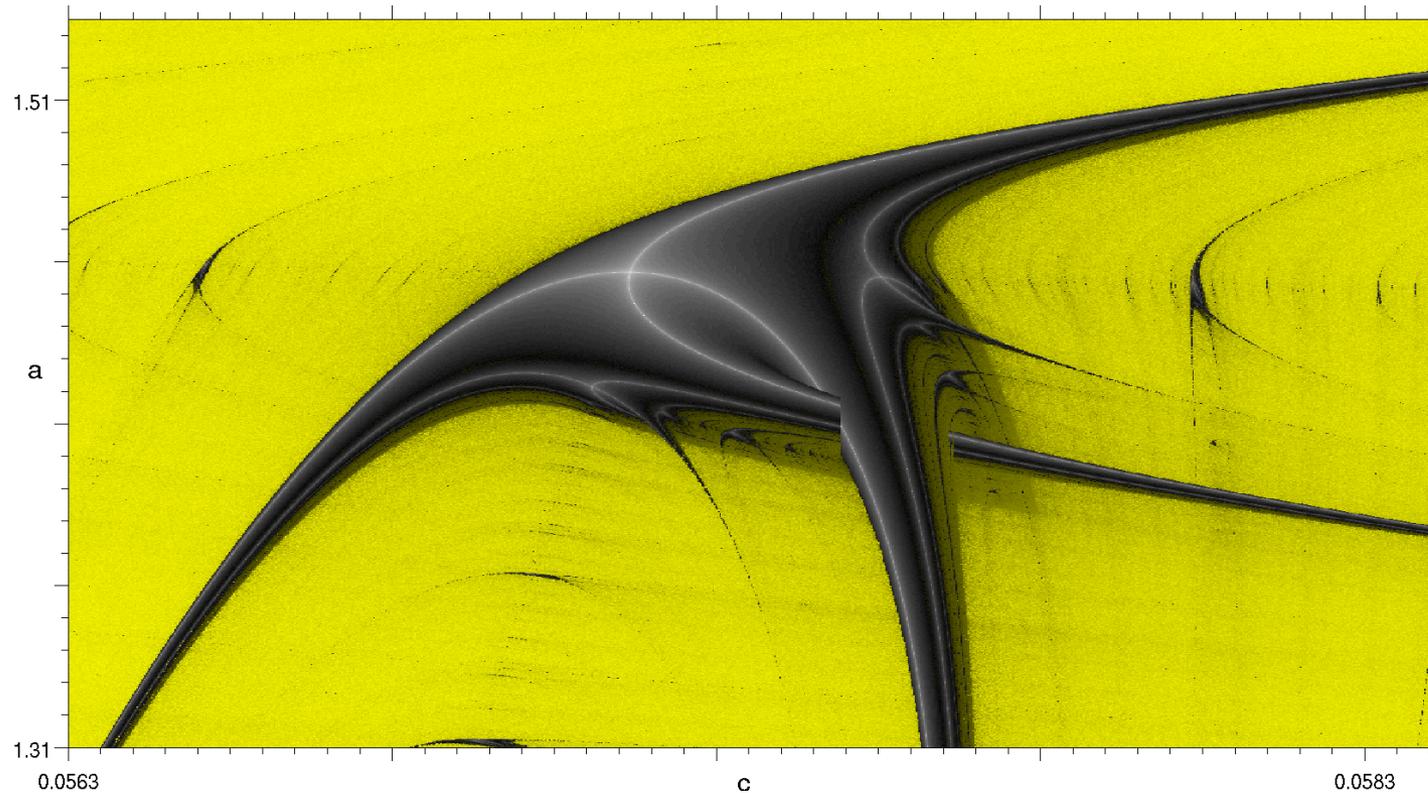


Azul: ponto fixo
Vermelho: infinito
Amarelo: atrator caótico
Preto a branco: atrator periódico

Expoente de Lypunov do atrator periódico
(graduação crescente de branco para preto)
 $b = 0,3$

Janela Periódica no Espaço dos Parâmetros

($b = 0.3$)

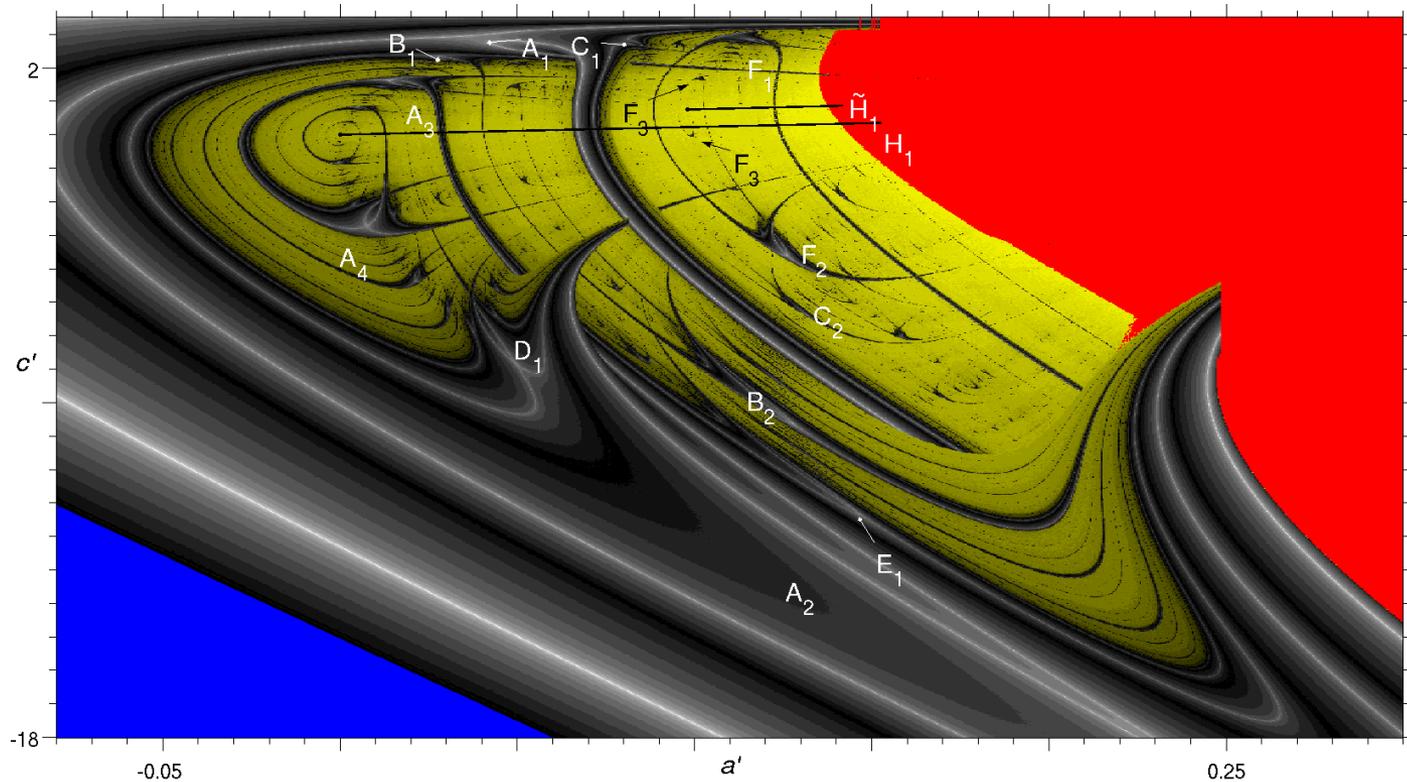


Amarelo: atrator caótico

Gradação crescente de branco para preto: expoente de Lyapunov do atrator periódico

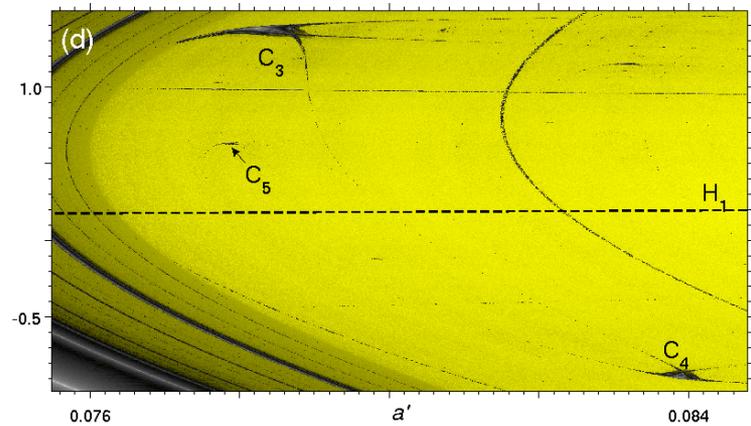
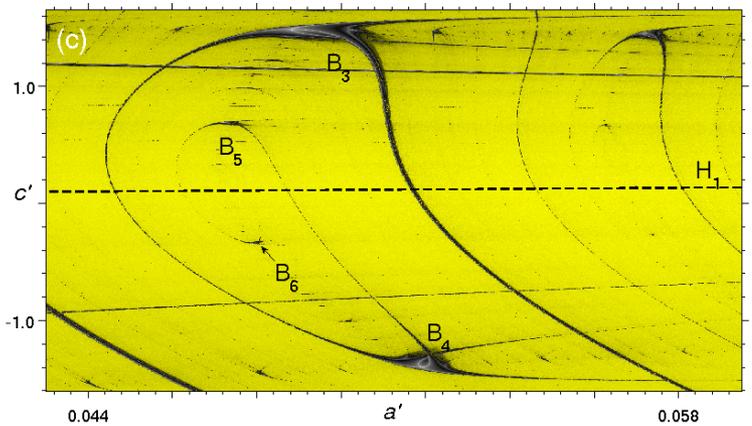
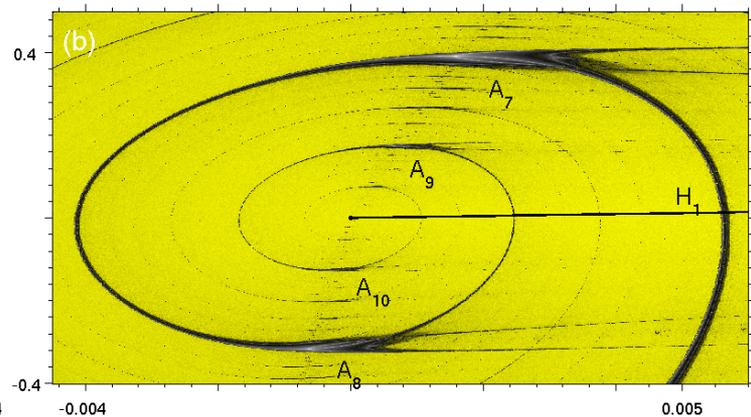
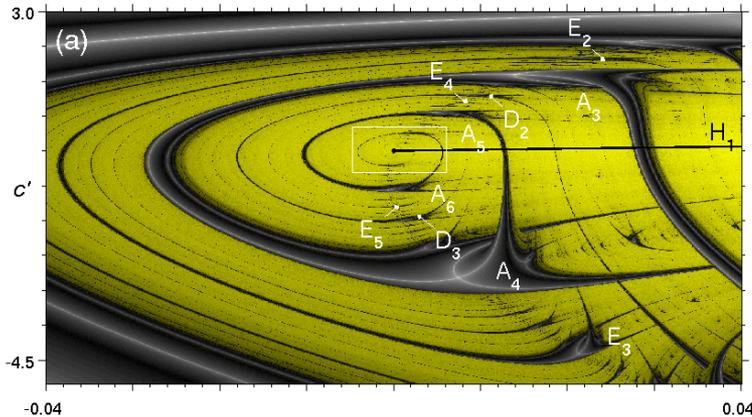
Sistema de Roessler

Órbitas Homoclínicas e Atratores no Espaço dos Parâmetros



Linhas H_1 e \tilde{H}_1 : parâmetros com órbitas homoclínicas

Sequências de Janelas Periódicas Órbitas Homoclínicas H1



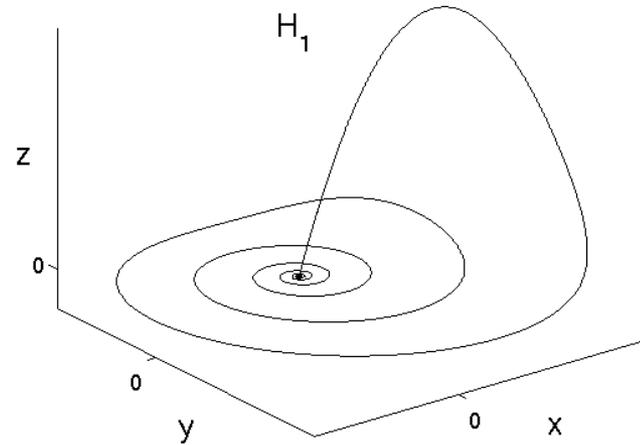
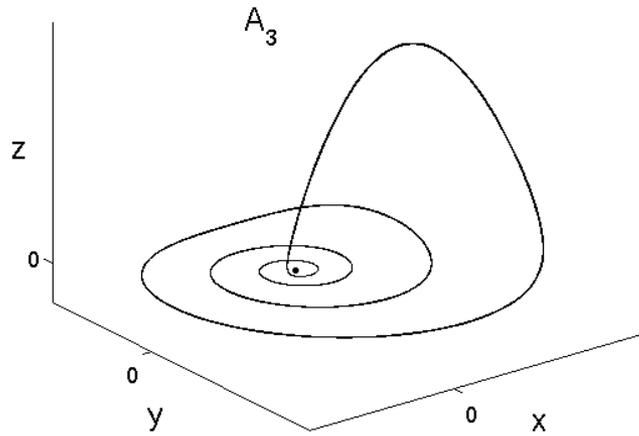
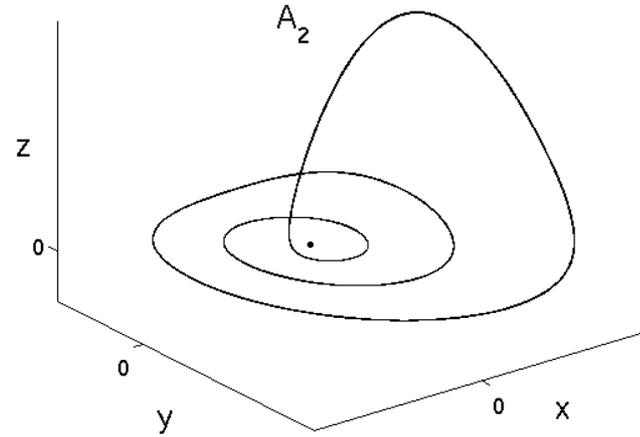
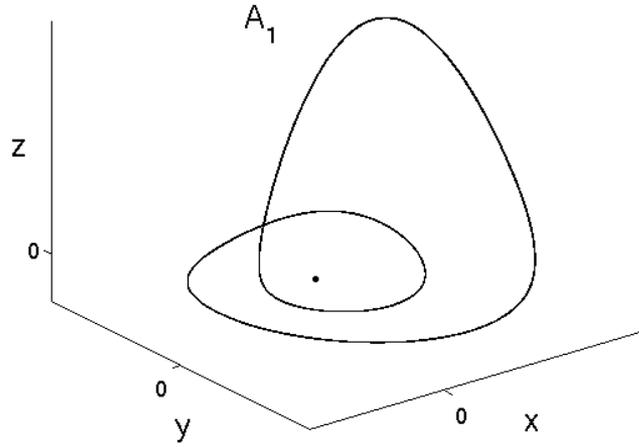
(a) Janelas periódicas A_3, A_4, A_5, A_6

(c) Janelas B_3, B_4, B_5, B_6

(b) Janelas A_7, A_8, A_9, A_{10}

(d) Janelas C_3, C_4, C_5

Órbita Homoclínica e Órbitas Periódicas Associadas



Órbita
homoclínica