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Dragon-kings death in nonlinear wave interactions

Moises S. Santos [a,b,*], José D. Szezech Jr [b,c], Antonio M. Batista [b,c,d], Kelly C. Iarosz [d], Iberê L. Caldas [d], Ricardo L. Viana [a]

[a] *Department of Physics, Federal University of Paraná, 80060-000, Curitiba, PR, Brazil*
[b] *Graduate Program in Science - Physics, State University of Ponta Grossa, 84030-900, Ponta Grossa, PR, Brazil*
[c] *Department of Mathematics and Statistics, State University of Ponta Grossa, 84030-900, Ponta Grossa, PR, Brazil*
[d] *Institute of Physics, University of São Paulo, 05508-900, São Paulo, SP, Brazil*

HIGHLIGHTS

- The distribution tail has humps, that is an evidence of dragon-kings extreme events in three-wave model.
- Aiming to suppress the dragon-kings, we propose a fourth wave as a control method.
- Fourth-wave applied during a short time can prevent catastrophic dragon-kings events in the three-wave interactions.

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ABSTRACT

Extreme events are by definition rare and exhibit unusual values of relevant observables. In literature, it is possible to find many studies about the predictability and suppression of extreme events. In this work, we show the existence of dragon-kings extreme events in nonlinear three-wave interactions. Dragon-king extreme events, identified by phase transitions, tipping points and catastrophes, affects fluctuating systems. We show that these events can be avoided by adding a perturbing small amplitude wave to the system.

1. Introduction

Extreme events are by definition rare and exhibit unusual values of relevant observables. These events have been observed in weather [1], optics [2], plasma physics [3], records with long-range memory [4], and stock markets [5]. In optical rogue waves, extreme events emerge from a turbulent state [6]. Riccardo et al. [7] reported that extreme events are compatible with general halo current trends in magnetically confined plasma physics in the tokamak Joint European Torus (JET).

Extreme events have been estimated through the extrapolation of power law frequency-size distributions [8]. Many extreme events, known as dragon-kings, do not belong to a power law distribution [9]. Sornette [9] introduced the concept of dragon-kings corresponding to meaningful outliers. Dragon-kings events are characterised by frequency distribution with extreme valued outliers about the power law tails [10]. Devastating effects can happen due to these events, then it is important to find a way to suppress or anticipate them. The dragon-kings were reported by Johansen and Sornette [11] in the distribution of financial drawdowns. Sornette and Ouillon [12] discussed the mechanisms, statistical tests, and empirical evidences of dragon-kings. Dragon-king extreme events were found in neuronal networks by Mishra et al. [13]. They presented evidence of these events in coupled bursting neurons.

* Corresponding author at: Graduate Program in Science - Physics, State University of Ponta Grossa, 84030-900, Ponta Grossa, PR, Brazil.
E-mail address: moises.wz@gmail.com (M.S. Santos).

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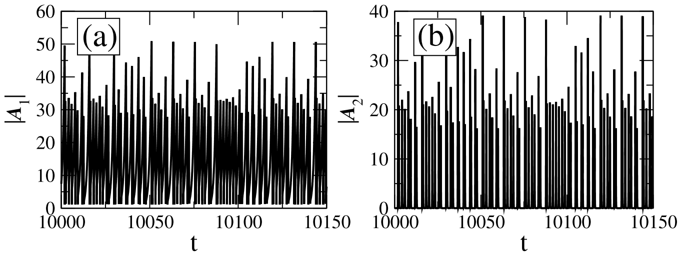

Fig. 1. Temporal evolution of wave amplitudes (a) $|A_1|$ and (b) $|A_2|$ of the nonlinear three-wave interactions model ($r = 0$). The figure exhibits chaotic behaviour for $\delta_3 = 2.2$ and $\nu = -14.5$.

Recently, many articles about the suppression of extreme events have been published. Direct corrective reset to the system was used as a suppression method [14]. Cavalcante et al. [15] studied coupled chaotic oscillators and showed that extreme events can be suppressed by means of tiny perturbations. Small perturbations were used to reduce the appearance of extreme events in damped one-dimensional nonlinear Schrödinger equation [16]. Krüger et al. [17] demonstrated that the existence of extreme events can be controlled by means of the properties of the phase space in Hamiltonian systems.

We analyse a nonlinear three-wave interactions model. The nonlinear three-wave interaction is the lowest order effect in systems described by waves superposition [18]. It plays an important role in nonlinear optics, plasma physics and hydrodynamics [19]. Chian and Abalde [20] developed a nonlinear theory of three-wave interactions of Langmuir waves with whistler waves in the solar wind. Batista et al. [21] considered three-wave interaction to investigate drift-wave turbulence in tokamak edge plasma. In our simulations, we identify Lorentzian pulses. These pulses have been observed in magnetised plasmas and interactions of drift-Alfvén waves [22]. They can appear in flow trajectories due to topological alterations and in chaotic orbits [23,24].

We calculate the frequency-size distributions of the wave amplitude in the three-wave interactions model. Three-wave coupling was used to analyse drift wave turbulence and transport in magnetised plasma [25]. Experimental evidence of three-wave coupling was observed on plasma turbulence by Hidalgo et al. [26]. Galuzio et al. [16] showed the occurrence of extreme events in wave turbulence. In our results, depending on the parameter, the distribution follows a power law, except for large sizes. This behaviour indicates the existence of dragon-kings extreme events. In this work, we focus on the suppress of the dragon-kings through a fourth wave. Batista et al. [27] used a fourth resonant wave of small amplitude to control chaotic behaviour of three interacting modes. Coexistence of attractors was observed in a dissipative nonlinear parametric four-wave interactions [28]. We show that a fourth wave is able to kill the dragon-kings in nonlinear three-wave interactions.

The paper is organised as follows: In Section 2, we describe the nonlinear three-wave interactions model. Section 3 shows the dragon-kings extreme events and presents our method to suppress these events. In the last Section, we draw our conclusions.

2. Nonlinear three-wave interactions model

The nonlinear three-wave interactions are described by three first-order differential equations [29–31]. We include a fourth wave interacting with the first and second waves [28]. The equations of the four-wave interactions are given by

$$\dot{A}_1 = \nu_1 A_1 + A_2 A_3 - r A_2^* A_4, \quad (1)$$

$$\dot{A}_2 = \nu_2 A_2 - A_1 A_3^* - r A_1^* A_4, \quad (2)$$

$$\dot{A}_3 = \nu_3 A_3 - A_1 A_2^* + i\delta_3 A_3, \quad (3)$$

$$\dot{A}_4 = \nu_4 A_4 - i\delta_4 A_4 + r A_1 A_2, \quad (4)$$

where A_i is the wave amplitude ($i = 1, 2, 3, 4$), A_i^* denotes the conjugate complex, ν_1 is the energy injection coefficient, ν_i ($i = 2, 3, 4$) is the dissipation parameter, and $\delta_{3,4}$ are a small mismatch. We consider $\nu_1 = 1$, $\nu_2 = \nu_3 = \nu$, $\delta_4 = 0$, $\nu_4 = -0.83$, transient time equal to 10^4, and random initial condition in the interval [0, 1] for A_i, and A_i^*. The r parameter controls the intensity of the fourth wave, at the limit when $r = 0$, the four-wave interaction model becomes the three-wave interaction model.

The temporal evolution of waves $|A_1|$ and $|A_2|$ for the three-wave model is exhibited in Figs. 1(a) and 1(b), respectively. The time evolution of wave $|A_3|$ is similar to $|A_2|$, so it was not shown in Fig. 1. We consider $\delta_3 = 2.2$, $\nu = -14.5$, and $r = 0$. For the considered parameters, the system exhibits chaotic behaviour, namely the dynamical system is sensitive to initial conditions. The chaotic attractor is plotted in Fig. 2(a). The system has a large number of periodic orbits that can be found by varying the parameter δ_3. Fig. 2(b) shows a period-4 stable periodic orbit for $\delta_3 = 1.1$ and $\nu = -14.5$.

The short time interval of the temporal evolution (pulse) of $|A_2|$ (Fig. 1(b)) can be fitted by a Lorentzian function, as shown in Fig. 3(a) (red line) for the same parameter values of Fig. 1. Recently, Lorentzian pulses have been associated with

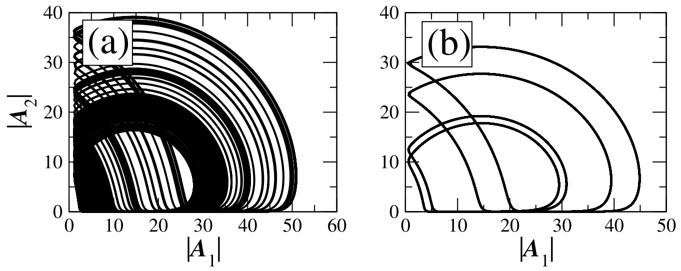

Fig. 2. $|A_2|$ versus $|A_1|$ for the nonlinear three-wave interactions model ($r = 0$). (a) Chaotic attractor for $\delta_3 = 2.2$ and $\nu = -14.5$. (b) Period-4 stable orbits for $\delta_3 = 1.1$ and $\nu = -14.5$ (periodic evolution). We consider a transient equal to 10^4 and $r = 0$.

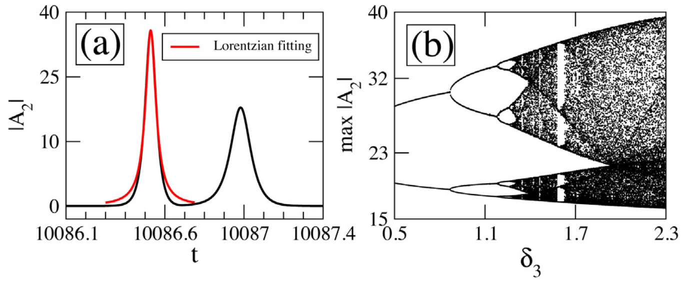

Fig. 3. (a) Temporal evolution of $|A_2|$ (black line) and its fit by a Lorentzian function (red line) for $\delta_3 = 2.2$, $\nu = -14.5$, and $r = 0$. (b) Bifurcation diagram for δ_3 showing periodic and chaotic regimes.

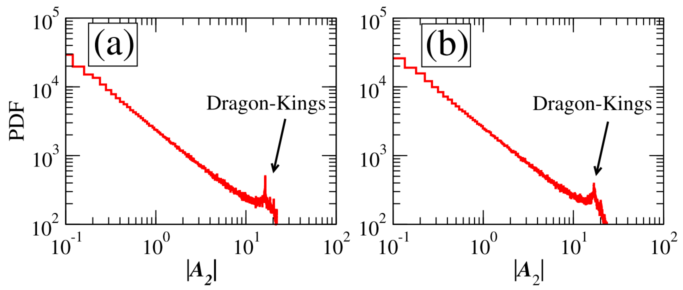

Fig. 4. Probability density function (PDF) for $r = 0$, (a) $\delta_3 = 2.2$ and $\nu = -14.5$, and (b) $\delta_3 = 2$ and $\nu = -15$.

chaotic behaviour [23,24]. Chaotic dynamics are characterised by exponential power spectrum. Maggs and Morales [24] reported that Lorentzian pulses are responsible for power spectrum with exponential form. They showed results from different plasma devices, where it is observed exponential fluctuation power spectra due to the occurrence of Lorentzian pulses in time signals of observed quantities [23]. In fact, the chaotic behaviour for these parameters is seen in Fig. 3(b) where is plotted the bifurcation diagram for the maximum value of $|A_2|$ as a function of δ_3.

3. Dragon-kings death

Dragon-kings are extreme events that belong to a special class of events. The extreme events are characterised by power law statistics. However, the dragon-kings are beyond the power laws. They exhibit humps in the tail of the distributions for the extended size of events [9].

We calculate the probability density function (PDF) of the temporal evolution of $|A_2|$. Fig. 4 shows the appearance of dragon-kings in the three-wave interactions model. In Figs. 4(a) and 4(b), we consider $\delta_3 = 2.2$ and $\nu = -14.5$, and $\delta_3 = 2$ and $\nu = -15$, respectively. The dragon-kings are identified by a hump in the PDF tail. The events with amplitude above 10 are outliers and generate the dragon-kings.

With regard to the four-wave model, the fourth-wave interacts with the first and second waves. Lopes and Chian [31] used a small sinusoidal wave to control chaos in the nonlinear three-wave coupling. They demonstrated that a desired

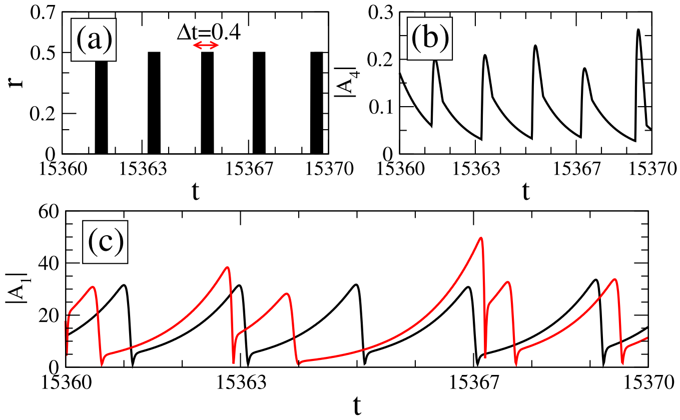

Fig. 5. The three-wave interactions model with a fourth wave interacting with the first and second waves, where r is the parameter that controls the intensity of the fourth wave. (a) r and (b) $|A_4|$ as a function of t. (c) Temporal evolution of $|A_1|$ for $\delta_3 = 2.2$, $\nu = -14.5$, $r = 0$ (red line), $r = 0.5$ (black line), and $\Delta t = 0.4$.

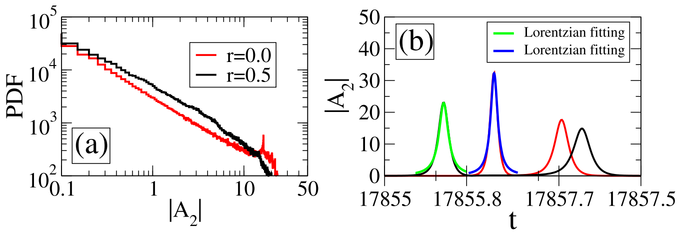

Fig. 6. (a) Probability density function (PDF) for $\delta_3 = 2.2$, $\nu = -14.5$, $r = 0$ (red line), $r = 0.5$ (black line), and $\Delta t = 0.4$. We verify that the dragon-kings events are suppressed for $r = 0.5$ (black line). (b) Lorentzian fitting for $r = 0$ (blue line) and $r = 0.5$ (green line).

periodic orbit can be obtained applying a fourth wave with small amplitude and coupled to the first and second waves. Aiming to suppress the dragon-kings, we apply a fourth wave ($|A_4|$) during a time interval $\Delta t = 0.4$ and $r = 0.5$ when $d|A_1|/dt > 0$, as shown in Fig. 5(a). Fig. 5(b) exhibits the temporal evolution of $|A_4|$. The energy of the first wave is decreased due to the application of the fourth wave (Fig. 5(c)). In Fig. 5(c), we see the behaviour of $|A_1|$ for $\delta_3 = 2.2$ and $\nu = -14.5$ when $r = 0$ (red line) and $r = 0.5$ (black line).

Fig. 6(a) displays the PDF for $r = 0$ (red line), $r = 0.5$ (black line), and $\Delta t = 0.4$. In our simulations, the fourth wave is able to produce a small alteration in the PDF. The PDF maintains the same slope, but the hump in the tail disappears, namely dragon-kings death. In Fig. 6(b), we see Lorentzian pulses not only in the nonlinear three-wave interactions (blue line), but also in the wave interactions (green line) where the dragon-kings are suppressed. Then, a fourth-wave kills the dragon-kings and does not destroy the Lorentzian pulses observed without the perturbation.

4. Conclusions

In conclusion, we study extreme events in a nonlinear three-wave interactions model. This model exhibits a rich variety of dynamical behaviour, such as periodic and chaotic regime. In the chaotic region, we find Lorentzian pulses, as well as extreme events. The Lorentzian pulses appear in many physical systems and have been used to characterise chaotic dynamics. Extreme events are usually rare and the event size distributions follow a power law. Depending on the system parameters, our simulations show a power law dependence in the wave amplitude distribution, except for larger values of the amplitude. The distribution tail has humps, that is an evidence of dragon-kings extreme events.

We consider an external perturbation in the three-wave interactions by means of a fourth wave. Lopes and Chian [31] reported that desired periodic orbits can be achieved applying a wave with small amplitude on the chaotic dynamics of nonlinear three-wave coupling. In this work, we focus on the suppression of dragon-kings events, for which the system remains chaotic after the inclusion of a new wave. To do this, we include a fourth-wave which is applied during a short time $\Delta t = 0.4$ and increase the intensity of this wave through the parameter r. Increasing r, we verify dragon-kings suppression for $r = 0.5$. In the interval $0 < r \leq 0.5$, the transition to dragon-kings death is not abrupt. As future work, we plan to analyse the regions in the parameter space Δt versus r for which the fourth-wave can prevent catastrophic dragon-kings events in the three-wave interactions.

We believe that in other systems there is a night king, i.e. an external perturbation, that is able to kill only the dragon-kings. Cavalcante et al. [15] showed that dragon-kings events can be suppressed [32] in a pair of electronic circuits by means of tiny perturbations.

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