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Damping control law for a chaotic impact oscillator

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Abstract

We apply a feedback control technique to suppress chaotic behavior in dissipative mechanical systems by using a small-amplitude damping signal. The control signal is obtained by varying the damping coefficient according to the velocity direction. As an application of this method, we present numerical simulations of an impact oscillator and the required damping law used to achieve the control. Our numerical results show the method effectiveness even for high levels of noisy perturbation.

1. Introduction

Chaotic behavior, sensitively depending on initial conditions, has been identified in many dynamical systems [1]. One important type of dynamical system that exhibits chaotic behavior is formed by impact oscillators, also called vibro-impact systems. These systems arise whenever their components collide with each other or with rigid obstacles. These impact oscillators have been the subject of growing interest in dynamical systems literature [2,3]. In addition, it is interesting to point out that these oscillators do not satisfy the usual smoothness assumptions. Thus, classical mathematical methods are applicable only to a limited extent and require extensions both for analytical and numerical methods.

For several years, this behavior was thought to be quite undesirable and, consequently, it was strongly avoided, mainly in mechanical systems designed for technological applications. It turns out, however, that chaotic behavior, if properly handled, can be of practical interest in real-world applications.

In 1990, an original scheme of chaos control was put forward by Ott et al. [4]. The control procedure is known nowadays as the OGY method and has had a great impact on nonlinear science. The OGY method consists on stabilizing a desired unstable periodic orbit embedded in a chaotic attractor by using only a tiny perturbation on an available control parameter. This is in marked contrast with usual control methods, such those used for periodic motion, for which tiny perturbations causes only small-size effects.

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Another interesting chaos control strategy was proposed by Pyragas in 1992 [5]. The Pyragas method also considers the dynamical properties of a chaotic attractor to stabilize unstable periodic orbits. In this case, the method implementation requires a delayed feedback signal.

After OGY and Pyragas methods, a wide variety of control chaos strategies was developed and verified experimental and numerically. Recently, a new kind of feedback control method was proposed [6,7], and consists in suppressing chaos by using a small-amplitude control signal, applied to alter the energy of a chaotic system. For that, it was used a sigmoid function in the forcing term of the equations of motion to represent the control signal. This analytical function was useful to show the conceptual applicability of the proposed method. However, in a real system it would be worthful to know a signal prescription to implement the control considered in [6,7].

In order to overcome possible difficulties to apply the method of controlling chaos by using a small-amplitude signal, we use a piecewise-linear absolute value function to describe the control signal instead of using the sigmoid function proposed in Refs. [6,7]. With this choice, the control signal considered can be readily obtained by varying the damping coefficient [16–18]. As an example, we apply this control signal to suppress chaos in an impact oscillator [8–15].

This paper is structured as follows: in Section 2 we present the mathematical model for the impact oscillator considering the control function. Section 3 explores some aspects of the model dynamics from numerical simulations, emphasizing the performance of the control method. Our conclusions are presented in Section 4.

2. Theoretical model

Fig. 1 depicts the model of the impact oscillator. This system is composed by a periodically forced and damped linear one-dimensional oscillator whose displacement is limited by an amplitude constraint, $x = x_c$, which is a rigid wall with which the oscillator collides inelastically.

Between two successive impacts, the smooth motion, without control input, is given by

$$\ddot{x} + c\dot{x} + x = F\cos\omega t, \tag{1}$$

where c is the damping coefficient, F the forcing amplitude, and ω the forcing frequency. Both the oscillator mass and the elastic constant have been normalized to unity for simplicity.

Considering the damping control law $u = k|\dot{x}|$, the resulting equation of motion is described by

$$\ddot{x} + f_d(\dot{x}) + x = F\cos\omega t, \tag{2}$$

where

$$f_d(\dot{x}) = c\dot{x} - k|\dot{x}| = \begin{cases} (c-k)\dot{x} & \text{if } \dot{x} \geqslant 0, \\ (c+k)\dot{x} & \text{if } \dot{x} < 0 \end{cases}$$

and k is a constant coefficient.

An impact occurs whenever $x = x_c$. After each impact, we reset the velocity of the oscillator using the Newton impact law. In other words, we model the collisions with a rigid wall (amplitude constraint) by the law of inelastic impact: the velocity after the impact is taken to be $-R$ times the velocity before the impact, where R is a constant restitution coefficient ($0 < R < 1$).

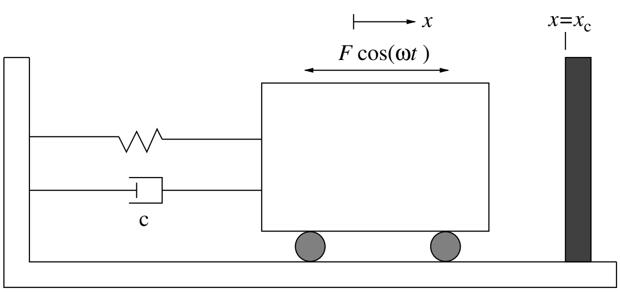

Fig. 1. Model of an impact oscillator.

3. Numerical results

Numerical simulations were performed by using a fourth order Runge–Kutta method. We adopted a fixed step for displacements far away from the rigid wall (amplitude constraint) and an adaptive step if we are close enough to the wall. The adaptive step was obtained using the Newton–Raphson's method. Here, the control parameter values are fixed at $x_c = 0$, $R = 0.8$, $F = 2.0$, and $\omega = 2.8$.

For the control switched off, in Fig. 2(a) we show a bifurcation diagram of the velocity, $\dot{x}$, in terms of the damping coefficient c. Hence, as can be seen, there is a wide range of the parameter for which the system presents chaotic dynamics, with one, two and three bands, occasionally interrupted by narrow periodic windows. In Fig. 2(b) for $c = 0.5$, we plot the phase portrait of a chaotic attractor, that we choose to implement the control method.

In order to investigate the influence of the control input (with a control function $u = k|\dot{x}|$) on a chaotic attractor, in Fig. 3 we present a bifurcation diagram in terms of parameter k, showing the transformation of the chaotic attractor for small k (Fig. 2(b)) into a period-1 orbit as k is increased from zero, through a reverse period-doubling bifurcation cascade. As examples of periodic behavior identified in this diagram, in Fig. 4(a) we plot the phase portrait of a period-1 orbit (for $k = 0.1$) and in Fig. 4(b) a period-2 orbit (for $k = 0.04$). In the background of these figures, it was depicted the chaotic attractor shown in Fig. 2(b).

We would like to emphasize that the control input u, for the case shown here, is always a small-amplitude signal. An illustrative example is in Fig. 5 where its evolution is shown, for the controlled period-1 orbit shown in Fig. 4(a), as compared with value of the system acceleration $\ddot{x}$ with and without control. The control input remains smaller than 5% of the maximum amplitude of $\ddot{x}$ (i.e., it is only necessary to apply a small quantity of energy to the control implementation).

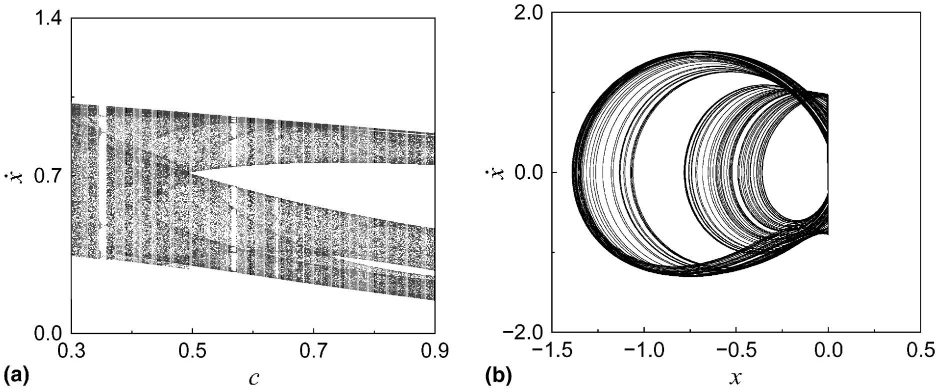

Fig. 2. (a) Bifurcation diagram of the velocity, $\dot{x}$, just before the impact as a function of the damping parameter c; (b) the phase portrait of the chaotic behavior for $c = 0.5$.

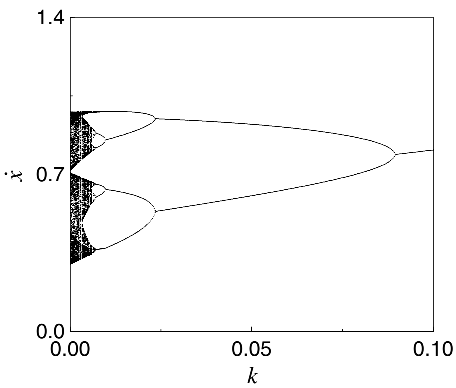

Fig. 3. Bifurcation diagram of the velocity, $\dot{x}$, just before the impact as a function of the control parameter k.

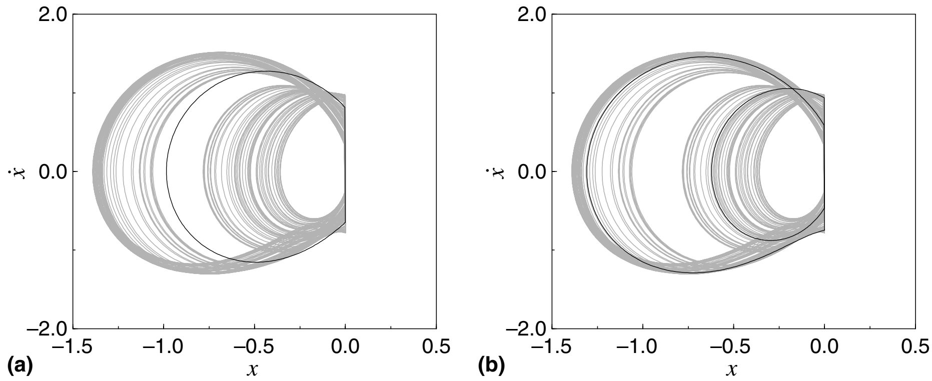

Fig. 4. (a) Period-1 orbit for $k = 0.10$; (b) period-2 orbit for $k = 0.04$. In gray is plotted the chaotic attractor of Fig. 2b.

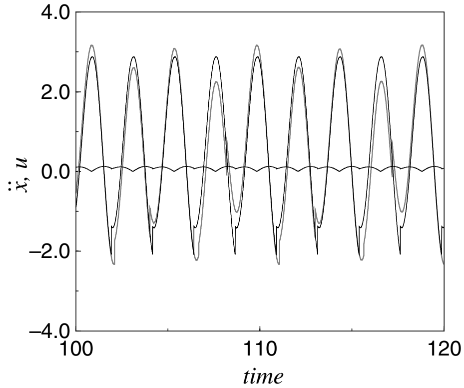

Fig. 5. Time histories of the control input u, for $k = 0.1$, in black heavy line. Evolution of acceleration, $\ddot{x}$, for control on (black light line) and for control off (gray line).

In Fig. 6, for the obtained period-1 orbit, we show the variation of the damper due to control. In dashed and solid lines are plotted the damping curve for the system with chaotic and periodic behaviors, respectively. Notice that the addition of control input corresponds to the variations of the damping coefficient. In such way, the damping coefficient decreases for positive velocities and increases for negative velocities.

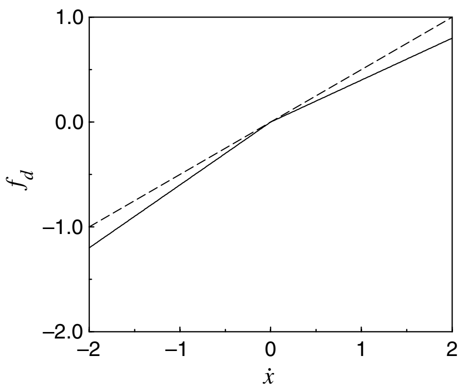

Fig. 6. Damping force $f_d = c\dot{x} - k|\dot{x}|$; for $k = 0$ uncontrollable damper (dashed line) and for $k = 0.1$ controllable damper (solid line).

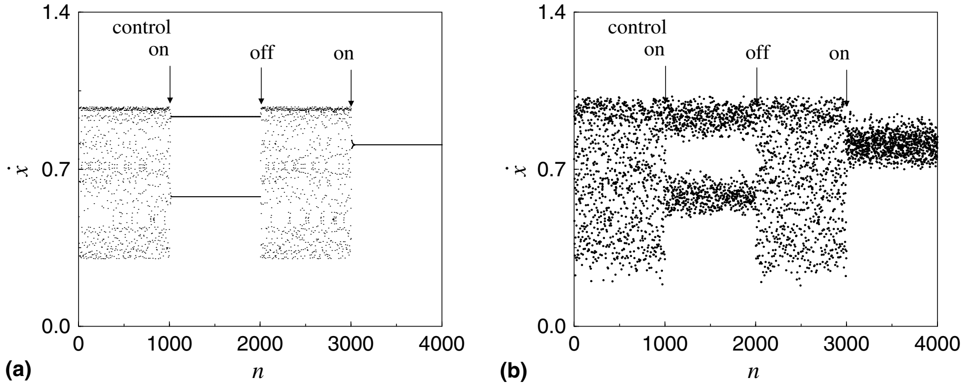

Fig. 7. Dynamical variable $\dot{x}$ (just before impact) as a function of the impact number n; (a) the evolution without noise; (b) with noise for $\sigma = 0.1$.

Finally, we investigate the effects of an extrinsic noise on the performance of the applied control method. For that, we use a random perturbation of the control parameter F (forcing amplitude). In other words, it is considered a parametric noise on the parameter F in following form:

$$F \to F(1 + \sigma p_n), \tag{3}$$

where p_n represents a uniform random variable on the interval $(-1, +1)$ with unit variance and zero mean, and σ is referred to as the noise level of the parametric fluctuations. In addition, the σp_n term is applied just after the moment of the impact.

Let us begin showing in Fig. 7(a) an example of the control implementation without noise. This figure shows the evolution of velocity, collected just before the impacts, as a function of impact number n. The control is only switched on at time $n = 1000$ (for $k = 0.04$) and, as shown in Fig. 4(b), a period-2 orbit is obtained. At $n = 2000$ the control is switched off, and, at $n = 3000$ is switched on again, for $k = 0.1$, obtaining the period-1 orbit, shown in Fig. 4(a).

In Fig. 7(b) we show a similar process, but now considering the noise perturbation. Indeed, even for the high noise used perturbation ($\sigma = 0.1$, i.e., 10% of the parameter variation) the control method is efficient to suppress chaotic behavior, what shows the robustness of the method proposed.

Furthermore, in Fig. 7(a) and (b), we also note that the suppression of chaos is obtained almost immediately after the control is turned on. This fact may be important in some technological applications requiring fast response to external control.

4. Conclusion

In this paper, we discuss a recently proposed procedure to suppress chaotic regimes of an impact oscillator. The applied small-amplitude signal we use to achieve this suppression is mathematically described by a piecewise-linear absolute value function instead of a smooth sigmoid function, which is a more difficult to be implemented in a laboratory setting. Thus, this control signal could be obtained in practical situations (experimentally) by varying the damping coefficient. We show numerically that the proposed control method is effective to suppress chaos even for a high level of noise perturbation, what shows the robustness of the control.

Acknowledgments

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