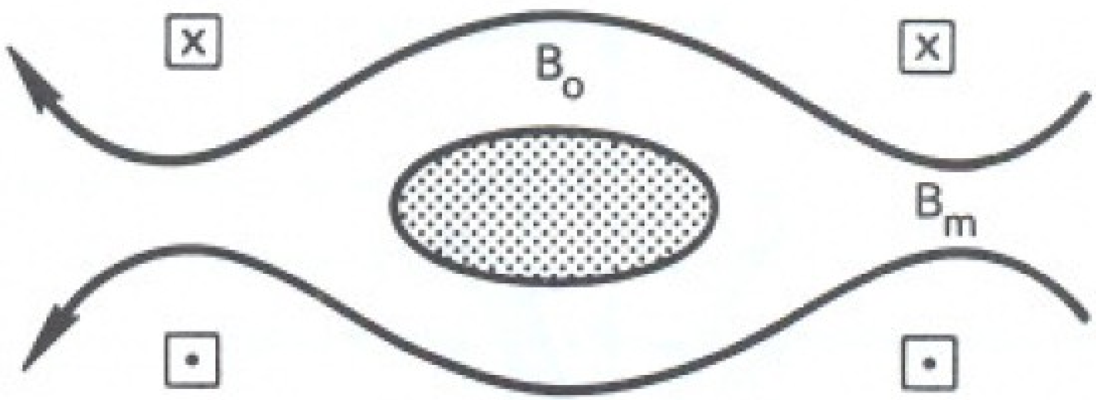
Espelho Magnético



A plasma trapped between magnetic mirrors. **FIGURE 2-8**

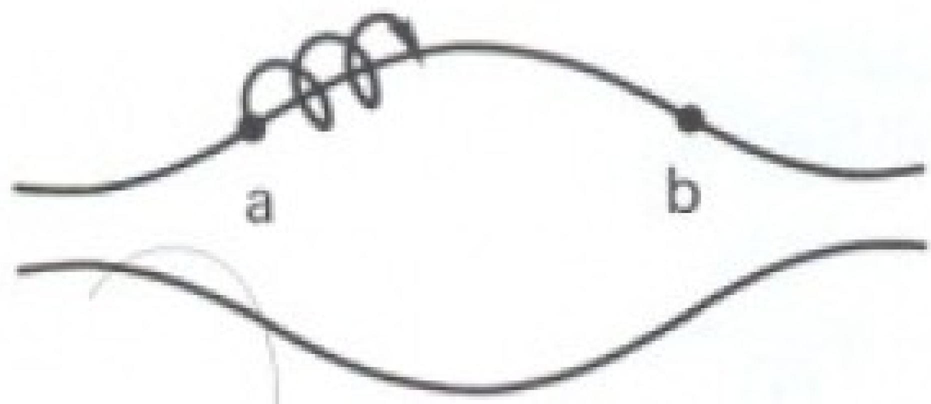


FIGURE 2-15 A particle bouncing between turning points a and b in a magnetic field.

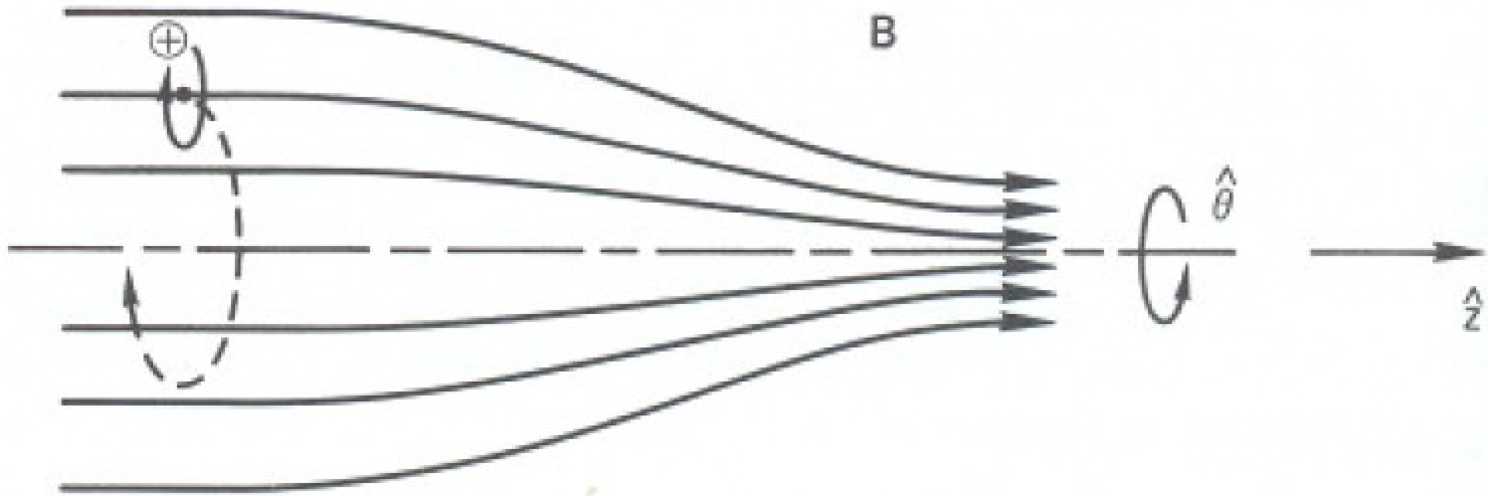


FIGURE 2-7 Drift of a particle in a magnetic mirror field.

$$B_\theta = 0 \text{ and } \partial/\partial\theta = 0$$

We can obtain B_r from $\nabla \cdot \mathbf{B} = 0$:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \tag{2-31}$$

If $\partial B_z/\partial z$ is given at $r = 0$ and does not vary much with r, we have approximately

$$\begin{aligned} rB_r &= -\int_0^r r\frac{\partial B_z}{\partial z}\,dr \simeq -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \\ B_r &= -\frac{1}{2}r\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \end{aligned} \tag{2-32}$$

The variation of $|B|$ with r causes a grad-B drift of guiding centers about the axis of symmetry, but there is no radial grad-B drift, because $\partial B/\partial\theta = 0$. The components of the Lorentz force are

$$\begin{aligned} F_r &= q(\underset{\textcircled{1}}{v_\theta B_z} - v_z \cancel{B_\theta}) \\ F_\theta &= q(\underset{\textcircled{2}}{-v_r B_z} + \underset{\textcircled{3}}{v_z B_r}) \\ F_z &= q(v_r \cancel{B_\theta} - \underset{\textcircled{4}}{v_\theta B_r}) \end{aligned} \tag{2-33}$$

$$F_z = \tfrac{1}{2} q v_\theta r (\partial B_z / \partial z) \qquad [2\text{-}34]$$

We must now average over one gyration. For simplicity, consider a particle whose guiding center lies on the axis. Then v_θ is a constant during a gyration; depending on the sign of q, v_θ is $\mp v_\perp$. Since $r = r_L$, the average force is

$$\bar{F}_z = \mp \frac{1}{2} q v_\perp r_L \frac{\partial B_z}{\partial z} = \mp \frac{1}{2} q \frac{v_\perp^2}{\omega_c} \frac{\partial B_z}{\partial z} = -\frac{1}{2} \frac{m v_\perp^2}{B} \frac{\partial B_z}{\partial z} \qquad [2\text{-}35]$$

We define the *magnetic moment* of the gyrating particle to be

$$\boxed{\mu \equiv \tfrac{1}{2} m v_\perp^2 / B} \qquad [2\text{-}36]$$

$$\bar{F}_z = -\mu(\partial B_z/\partial z) \qquad [2\text{-}37]$$

This is a specific example of the force on a diamagnetic particle, which in general can be written

$$\mathbf{F}_\| = -\mu\ \partial B/\partial \mathbf{s} = -\mu\ \nabla_\| B \qquad [2\text{-}38]$$

where $d\mathbf{s}$ is a line element along $\mathbf{B}$. Note that the definition [2-36] is the same as the usual definition for the magnetic moment of a current loop with area A and current I: $\mu = IA$. In the case of a singly charged ion, I is generated by a charge e coming around $\omega_c/2\pi$ times a second: $I = e\omega_c/2\pi$. The area A is $\pi r_L^2 = \pi v_\perp^2/\omega_c^2$. Thus

$$\mu = \frac{\pi v_\perp^2}{\omega_c^2}\frac{e\omega_c}{2\pi} = \frac{1}{2}\frac{v_\perp^2 e}{\omega_c} = \frac{1}{2}\frac{m v_\perp^2}{B}$$

$$m\frac{dv_\parallel}{dt} = -\mu\frac{\partial B}{\partial s} \qquad [2\text{-}39]$$

Multiplying by $v_\parallel$ on the left and its equivalent ds/dt on the right, we have

$$mv_\parallel\frac{dv_\parallel}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{\partial B}{\partial s}\frac{ds}{dt} = -\mu\frac{dB}{dt} \qquad [2\text{-}40]$$

Here dB/dt is the variation of B as seen by the particle; B itself is constant. The particle's energy must be conserved, so we have

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right) = \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \mu B\right) = 0 \qquad [2\text{-}41]$$

With Eq. [2-40] this becomes

$$-\mu\frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

so that

$$d\mu/dt = 0 \qquad [2\text{-}42]$$

$$\tfrac{1}{2}mv_{\perp 0}^2/B_0 = \tfrac{1}{2}mv_{\perp}'^2/B' \tag{2-43}$$

Conservation of energy requires

$$v_{\perp}'^2 = v_{\perp 0}^2 + v_{\parallel 0}^2 \equiv v_0^2 \tag{2-44}$$

Combining Eqs. [2-43] and [2-44], we find

$$\frac{B_0}{B'} = \frac{v_{\perp 0}^2}{v_{\perp}'^2} = \frac{v_{\perp 0}^2}{v_0^2} \equiv \sin^2\theta \tag{2-45}$$

where θ is the pitch angle of the orbit in the weak-field region. Particles with smaller θ will mirror in regions of higher B. If θ is too small, B' exceeds B_m; and the particle does not mirror at all. Replacing B' by B_m in Eq. [2-45], we see that the smallest θ of a confined particle is given by

$$\sin^2\theta_m = B_0/B_m \equiv 1/R_m \tag{2-46}$$

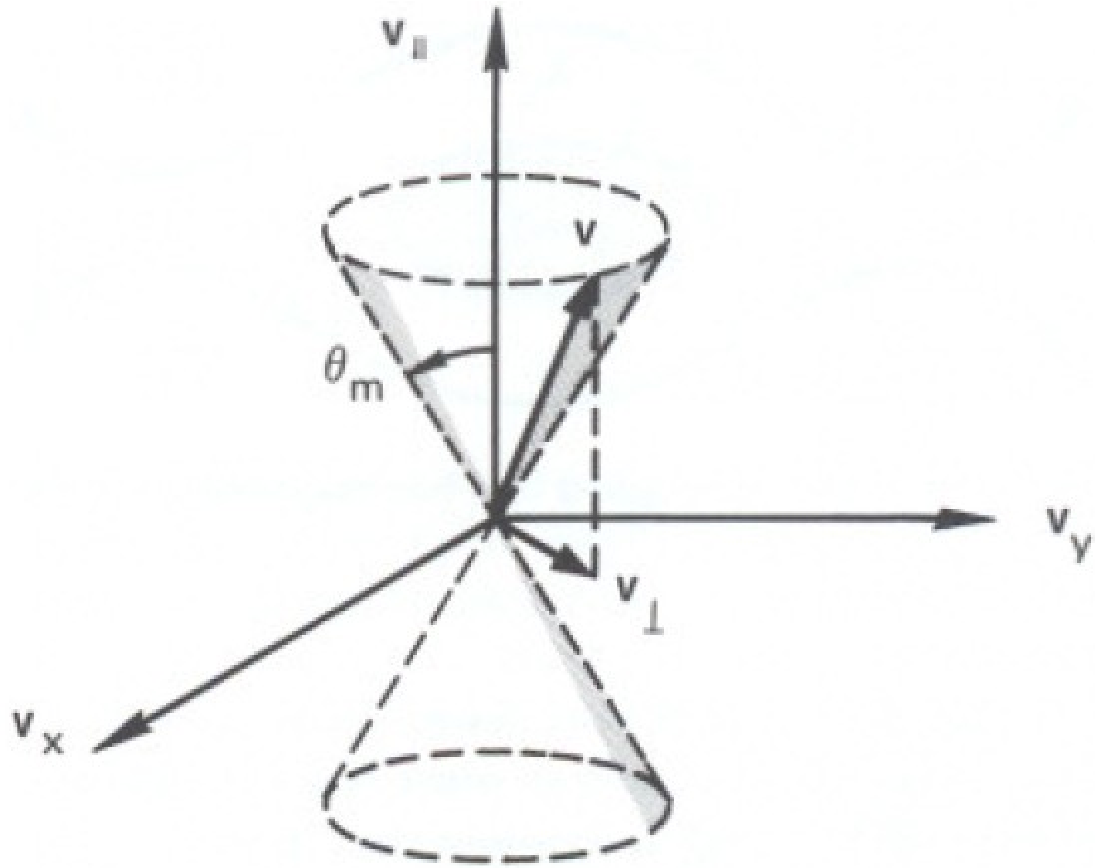
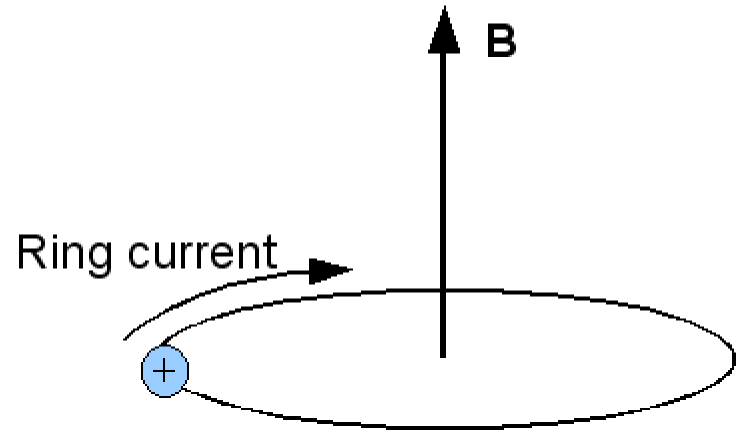


FIGURE 2-9 The loss cone.

Conservação do Momento Magnético

$$\frac{\partial \mu}{\partial t} = 0 \qquad \mu = \frac{m v_\perp^2}{2B}$$

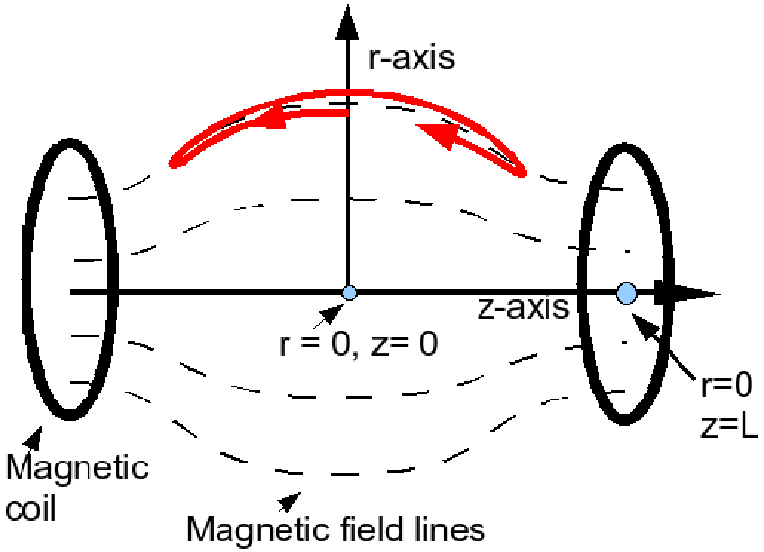


Espelho Magnético

- Theta pinch tem perda pelas extremidades
- Mas a reflexão confina partículas

$$\mathbf{F} = -\mu \nabla B$$

- Partículas vindas do centro são refletidas nas extremidades



Movimento da partícula no espelho magnético

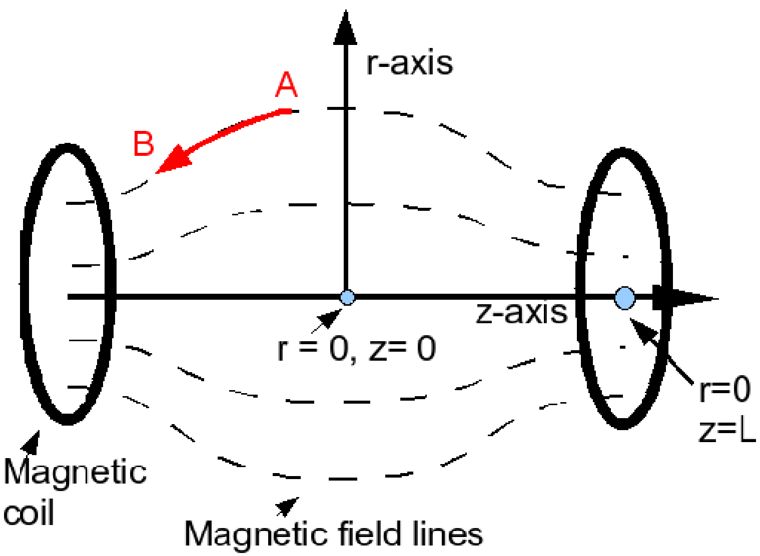
Espelho Magnético

- Energia cinética perpendicular é obtida da conservação do momento magnético

$$\mu = \frac{mv_{\perp A}^2}{2B_A} = \frac{mv_{\perp B}^2}{2B_B}$$

$$\frac{1}{2}mv_{\perp B}^2 = \frac{B_B}{B_A}\frac{1}{2}mv_{\perp A}^2$$

- A conservaçaõ de energia acarreta a diminuição da velocidade na direção z



Particle moving from A to B

$$\frac{1}{2}mv_{\parallel A}^2 + \frac{1}{2}mv_{\perp A}^2 = \frac{1}{2}mv_{\parallel B}^2 + \frac{1}{2}mv_{\perp B}^2$$

Condição de Reflexão

- Particle vai de A para B e é refletida em B

$$\frac{1}{2}mv_{\parallel A}^2 + \frac{1}{2}mv_{\perp A}^2 = \frac{1}{2}mv_{\parallel B}^2 + \frac{1}{2}mv_{\perp B}^2$$

Zero porque a partícula é refletida

$$\frac{B_B}{B_A}\frac{1}{2}mv_{\perp A}^2$$

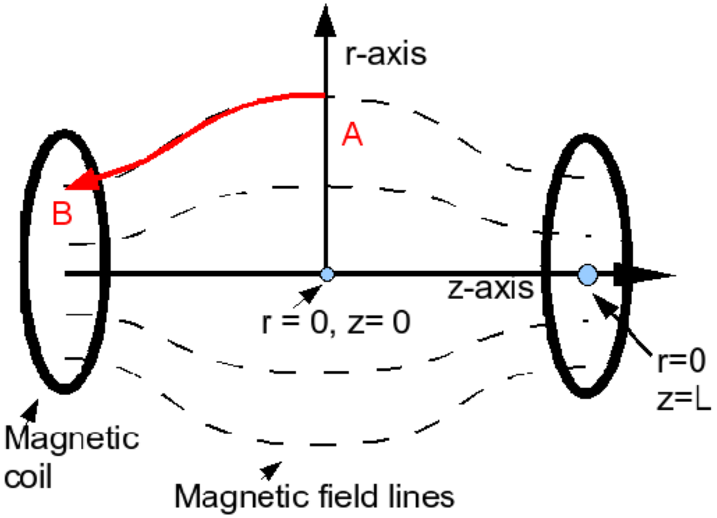
$$v_{\parallel A}^2 = v_{\perp A}^2\left[\frac{B_B}{B_A} - 1\right]$$

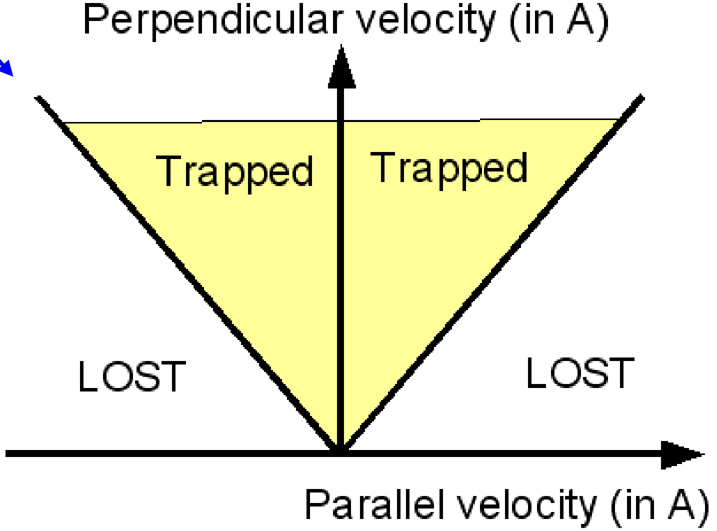
$$v_{\parallel A} = \pm v_{\perp A}\sqrt{\frac{B_B}{B_A} - 1}$$

Perda de Partículas no Espelho

- Apenas uma parte das partículas são confinadas (colisões levam a uma perda rápida de partículas)

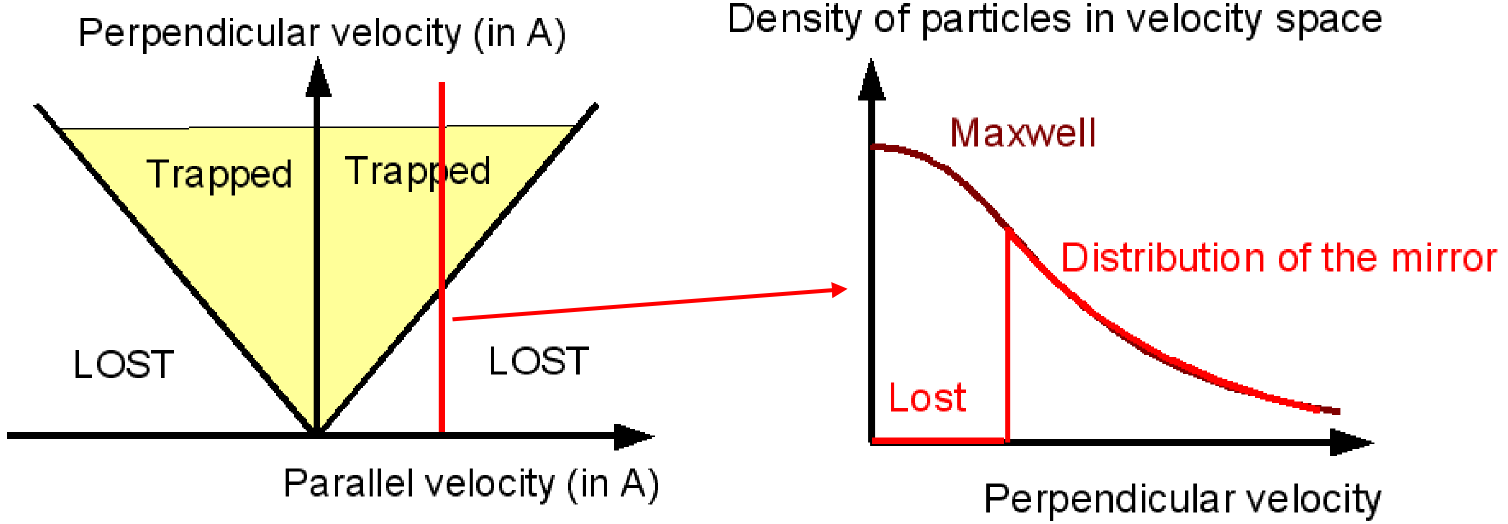
$$v_{\parallel A} = \pm v_{\perp A}\sqrt{\frac{B_B}{B_A} - 1}$$





Instabilidade

- A perda rápida de partículas altera a distribuição de velocidades, que deixa de ser Mawelliana e fica fora do equilíbrio termodinâmico
- A alteração da distribuição gera instabilidades



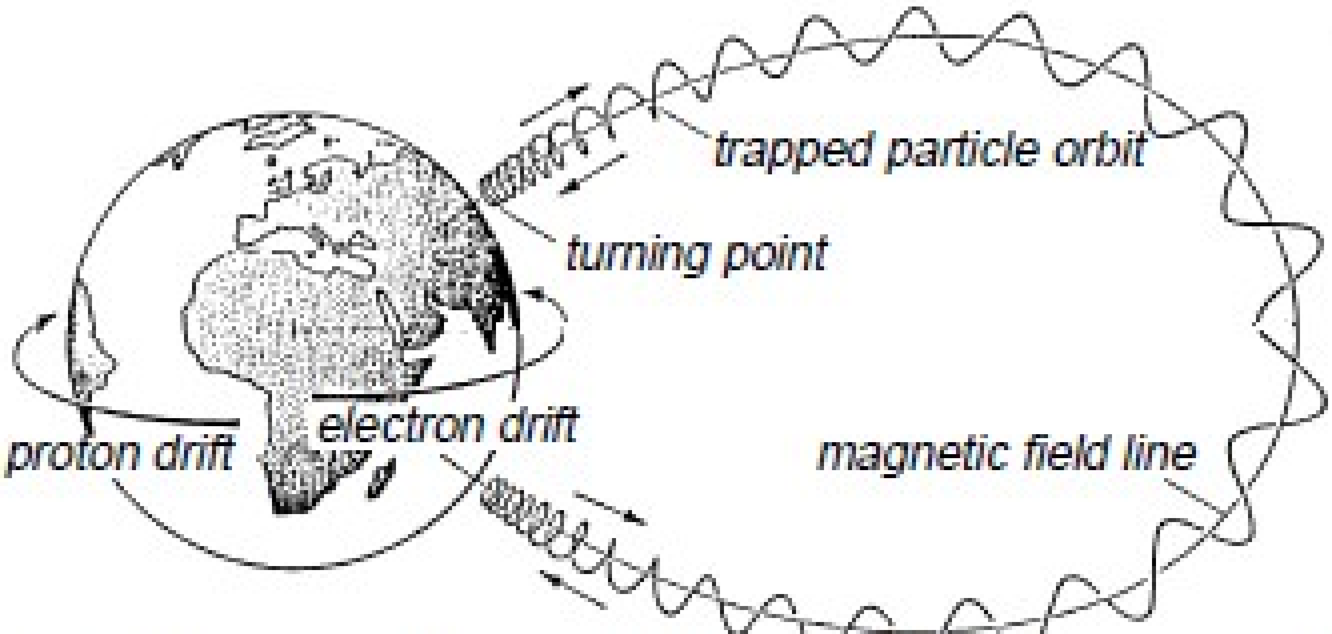
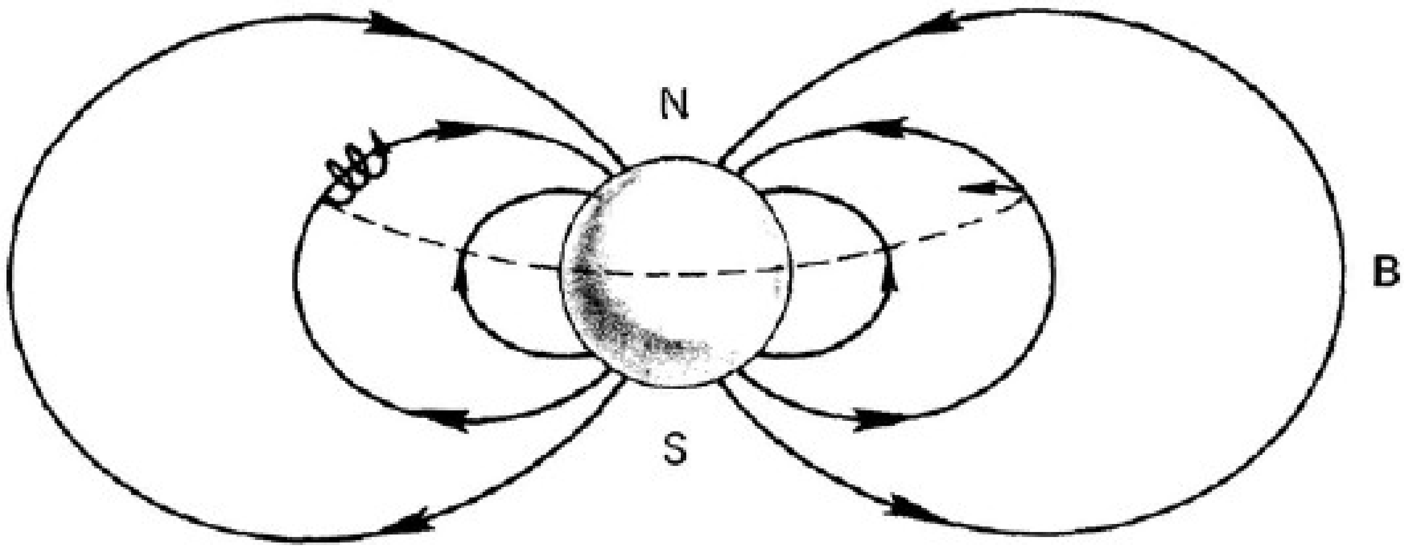
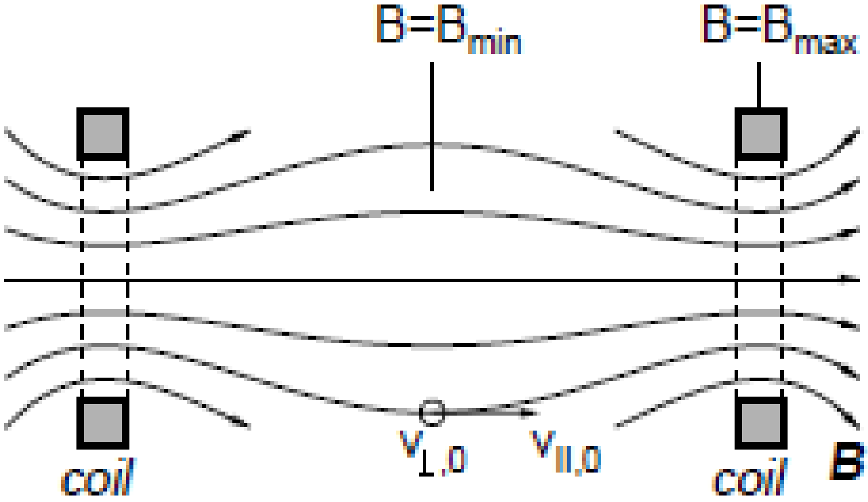


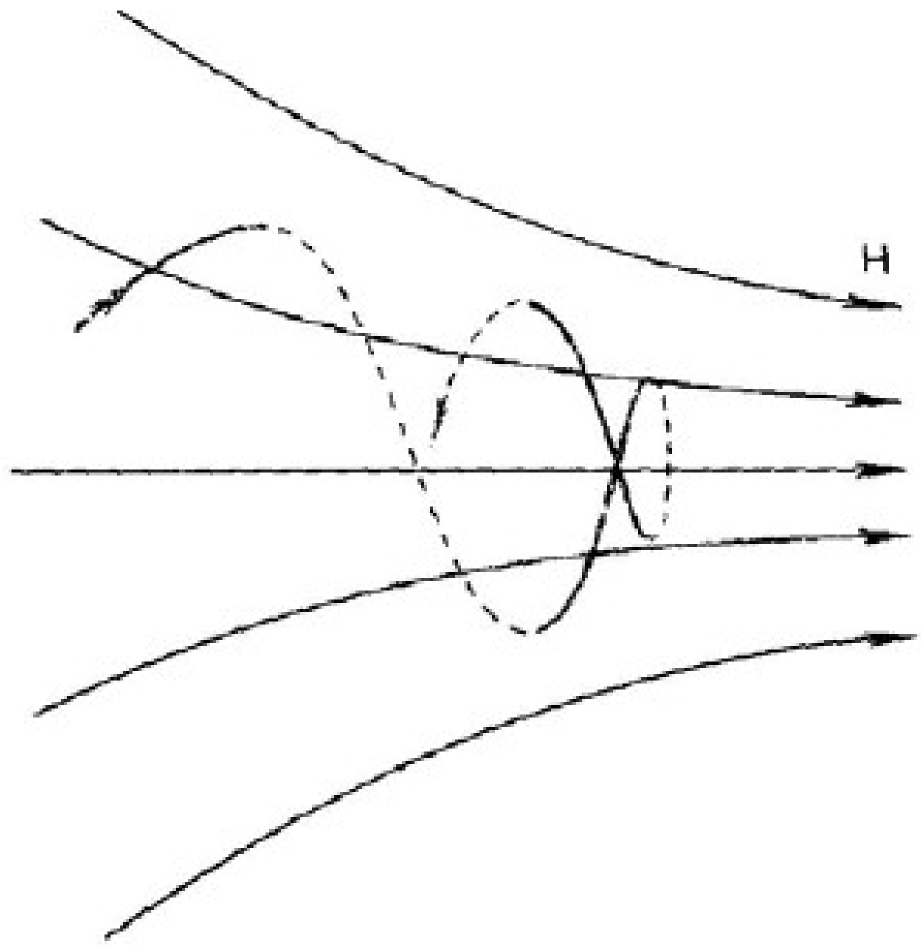
Figure 6: Electron and proton drifts in the Earth magnetic field.



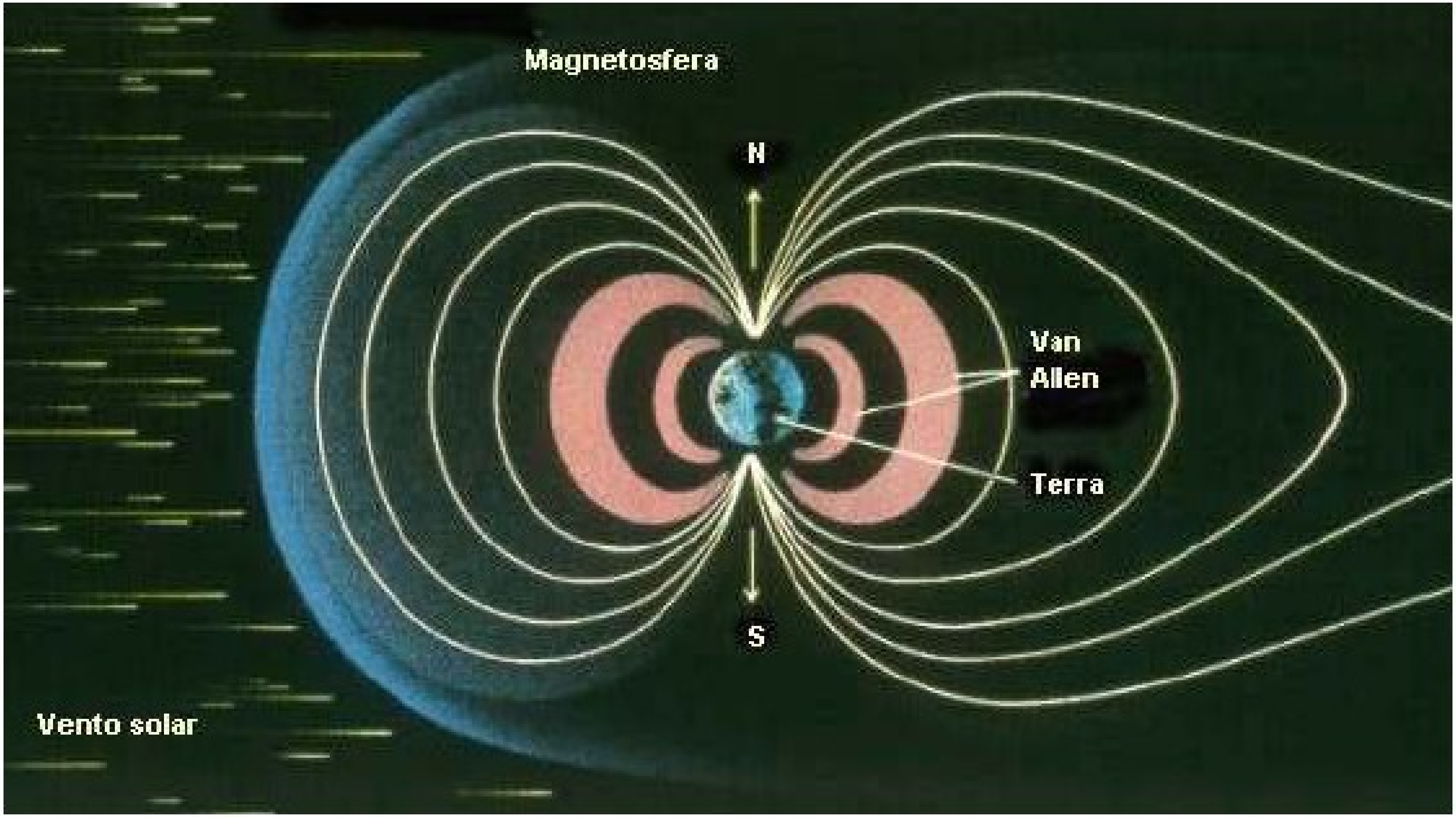
Movimento de uma única Partícula no Campo Magnético Terrestre.

Figure 8: Magnetic field lines of a simple axisymmetric magnetic mirror.

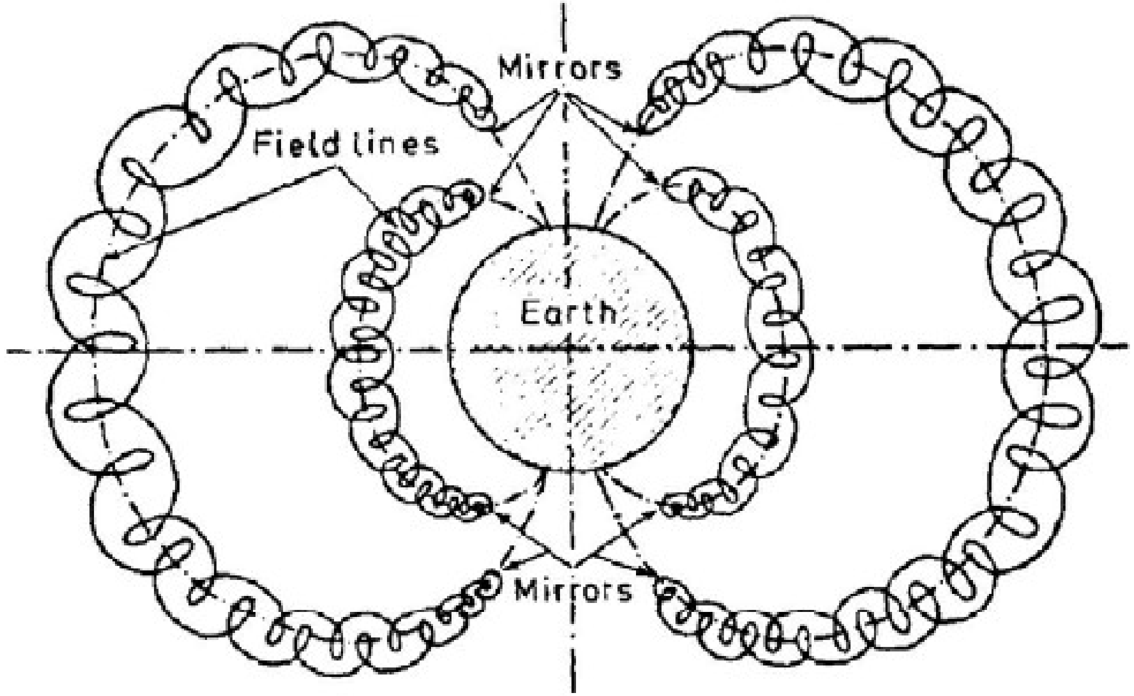




Reflexão da Partícula no Espelho Magnético.



Cinturão de Van Allen.



Trajetórias de Partículas Confinadas no Cinturão de Van Allen.