Dynamic range in small-world networks of spiking neurons

Ricardo Luiz Viana
Department of Physics
Federal University of Paraná
Curitiba – Paraná - Brazil

Many thanks to:

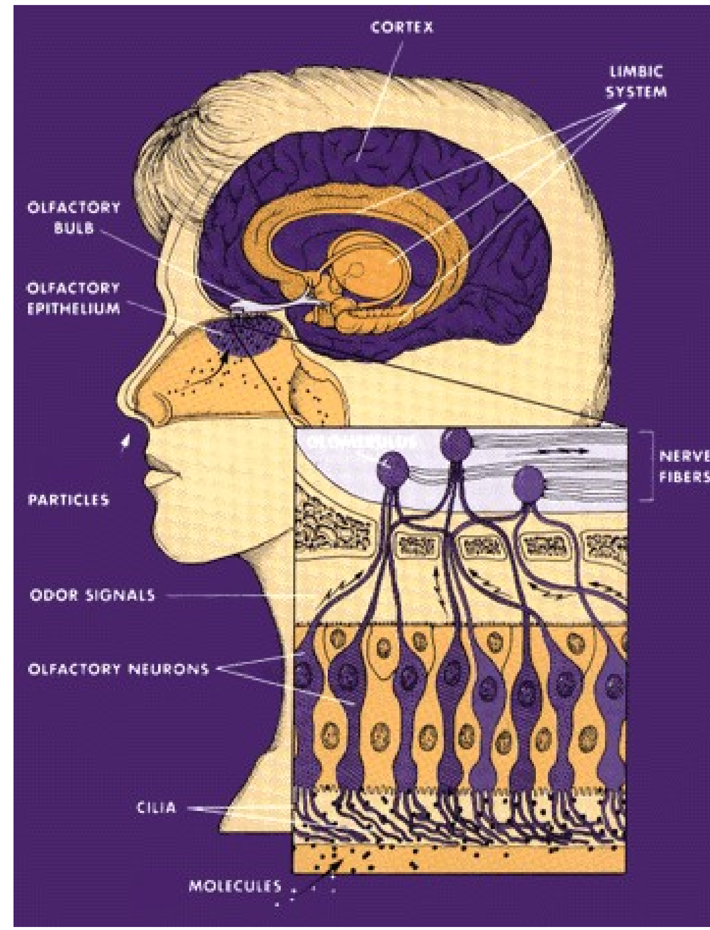
- Carlos A. S. Batista (UFPR- Campus Pontal do Sul)
- Sérgio R. Lopes (UFPR – Curitiba)
- Antonio M. Batista (UEPG-Ponta Grossa)

Introduction

- Stimulus (r) - response (F) relationship
- Weber-Fechner: F ~ log r
- Stevens: $F \sim r^m$ $(m > 0)$
- Anatomical and physiological constraints impose limits on the minimum (F_{min}) and maximum (F_{max}) responses
- Dynamical range is the log-difference between stimuli levels corresponding to the maximum and minimum responses

Example: olfatory system

- Stevens law: m = 0.55 (smell of coffee), 0.60 (heptane), 0.25 (rubber)
- Microscopic level (receptor neurons): dynamic range of 10 dB
- Macroscopic level (dendro-dendrictic neuronal network in the glomeruli): 30 dB
- Enhancement of the dynamic range as a network effect



This work

- What are the possible network effects on the stimuli-response relationship?
- Network properties = number of neurons, connection architecture, coupling strength...
- Effects to be studied = form of stimuli-response relationship (exponent, maximum and minimum levels, dynamic range...)
- Computational model: small-world network of Hodgkin-Huxley neurons with chemical synapses

Modeling of small-world networks

- $V_i(t)$: membrane potential of the i-th neuron at time t, where i = 1, 2, … N is the network index
- V measured in mV, time in ms
- The time evolution of $V_i(t)$ is given by the Hodgkin-Huxley equations
- The connection of the i-th neuron with other neurons is modeled by a synaptic current $I_{i,syn}(t)$
- The external signal acting on the i-th neuron is modeled by an external current $I_{i,ext}(t)$

Hodgkin-Huxley equations

$$C_m \frac{dV_i}{dt} = -I_{i,Na} - I_{i,K} - I_{i,L} + I_{i,ext} + I_{i,syn},$$

$$\alpha_n(V_i) = 0.01 \frac{V_i + 50}{1 - \exp[-0.1(V_i + 50)]},$$

$$\beta_n(V_i) = 0.125 \exp\left[\frac{V_i + 60}{80}\right],$$

$$\alpha_m(V_i) = 0.1 \frac{V_i + 35}{1 - \exp[-0.1(V_i + 35)]},$$

$$\beta_m(V_i) = 4 \exp\left[\frac{V_i + 60}{18}\right],$$

$$\alpha_n(V_i) = 0.07 \exp\left[\frac{V_i + 60}{20}\right],$$

$$\beta_h(V_i) = \frac{1}{1 + \exp[-0.1(V_i + 30)]},$$

$$I_{i,K} = \bar{g}_K n^4 (V_i - E_K),$$

$$I_{i,Na} = \bar{g}_{Na} m^3 h (V_i - E_{Na}),$$

$$I_{i,K} = \bar{g}_L n^4 (V_i - E_L),$$

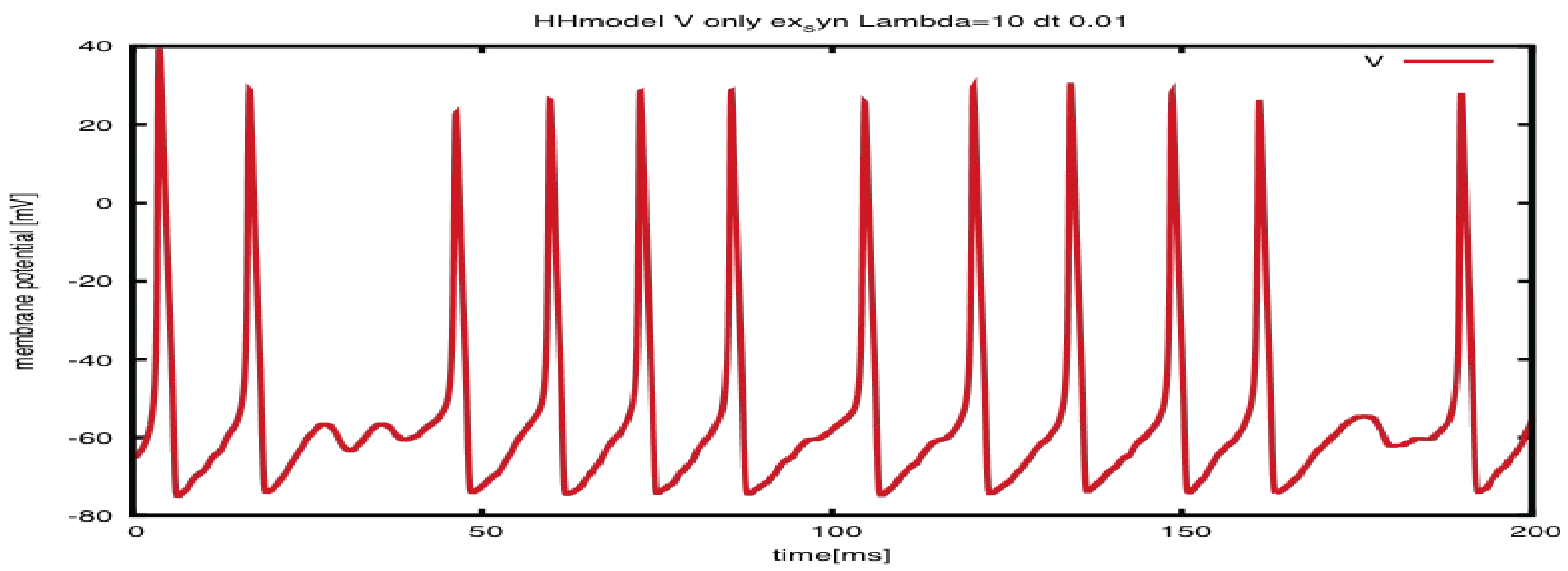
$$\frac{dn}{dt} = -(\alpha_n + \beta_n) n + \alpha_n,$$

$$\frac{dm}{dt} = -(\alpha_m + \beta_m) m + \alpha_m,$$

$$\frac{dh}{dt} = -(\alpha_h + \beta_h) h + \alpha_h,$$

Parameter values

Membrane specific capacitance	$C = 1.0\mu F/cm^2$	
Maximum specific conductances(mS/cm^2)		
$\bar{g}_{Na} = 120$	$\bar{g}_K = 36$	$\bar{g}_L = 0.3$
Nernst (reversal) potentials(mV)		
$E_{Na} = 55$	$E_K = -72$	$E_L = -49$



Modeling of chemical synapses

- g_c: synaptic specific conductance
- a_{ij}: adjacency matrix
- r_j: number of bond receptors of the j-th pre-synaptic neuron

$$I_{syn} = \bar{g}_c \sum_{j=1}^{N} a_{ij} r_j(t)(V_{syn} - V_j),$$

Characteristic times(ms)	
$\tau_r = 0.5$	$\tau_d = 8$
Reversal potentials(mV)	
$V_{syn} = 20$	$V_0 = -20$

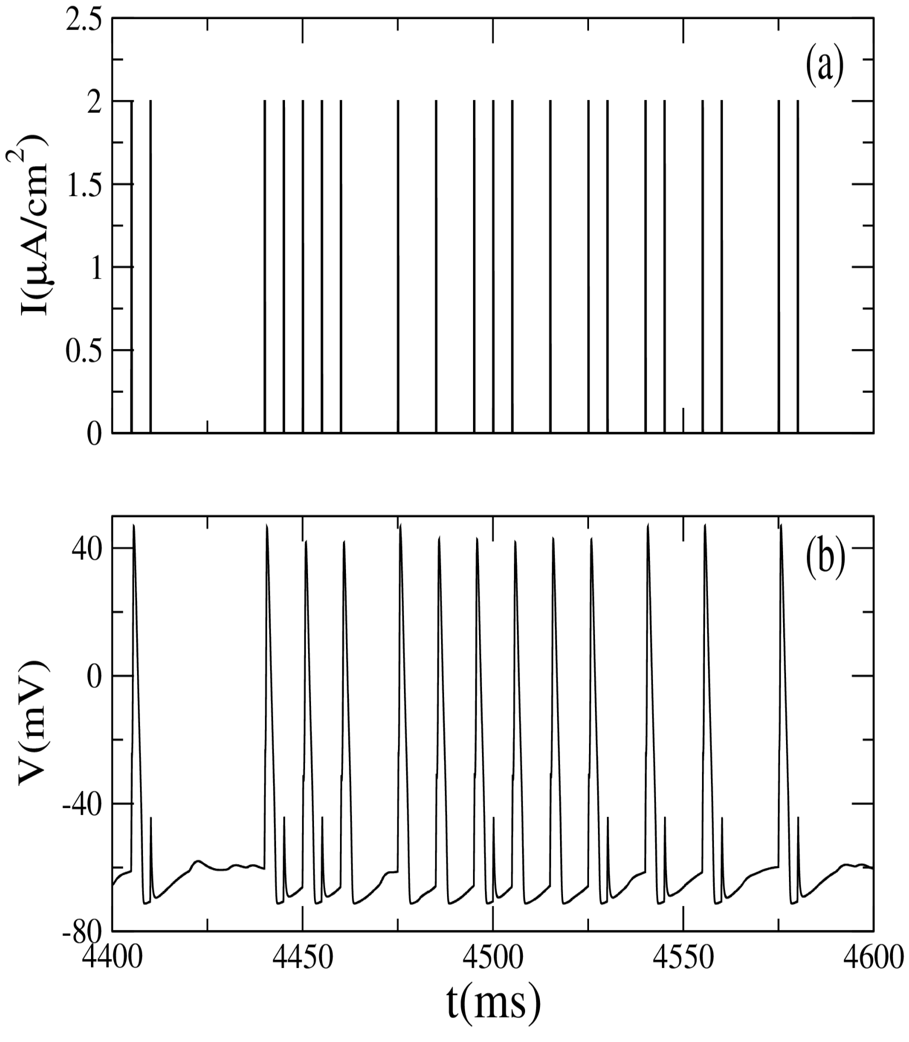
$$\frac{dr_j}{dt} = \left(\frac{1}{\tau_r} - \frac{1}{\tau_d}\right) \frac{1 - r_j}{1 + \exp(-V_j + V_0)} - \frac{r_j}{\tau_d},$$

External signal

- train of stereotyped electric impulses with external current

  $I_{i,ext}(t) = I_0 \Sigma_k \delta(t-t_k)$

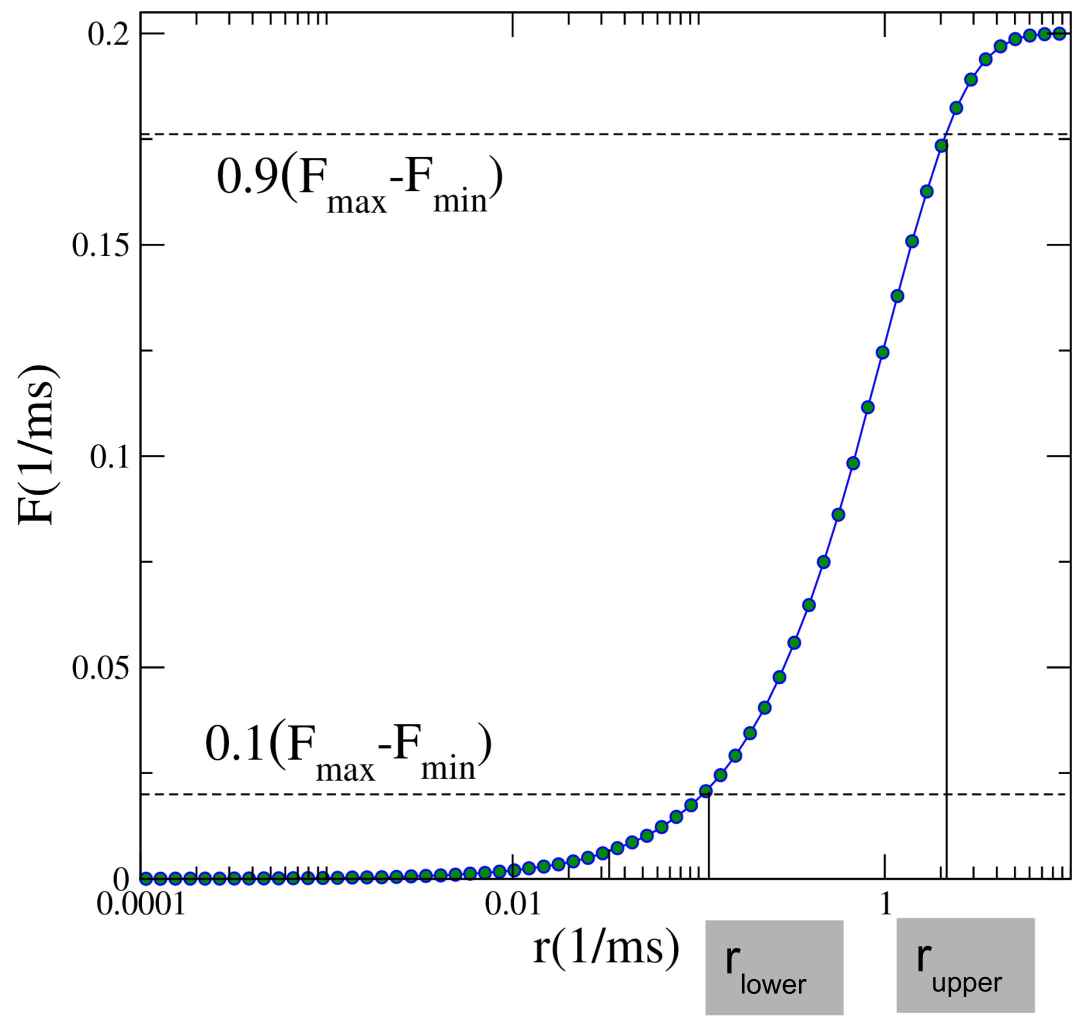
- interspike intervals follow a Poisson process with input rate r (stimulus)



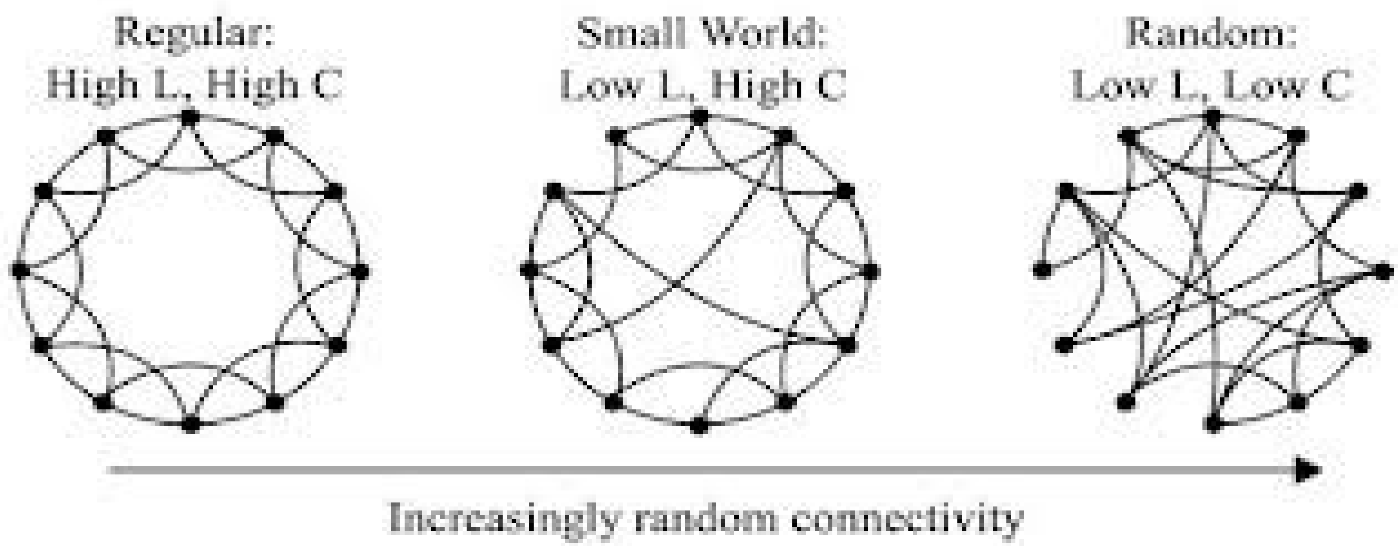
Stimulus-response relationship for a single neuron

- Firing rate F = number of spikes per unit time
- Power-law relationship $F \sim r^m,$
- valid within the dynamic range

$$\Delta = 10 \log_{10} \frac{r_{upper}}{r_{lower}},$$



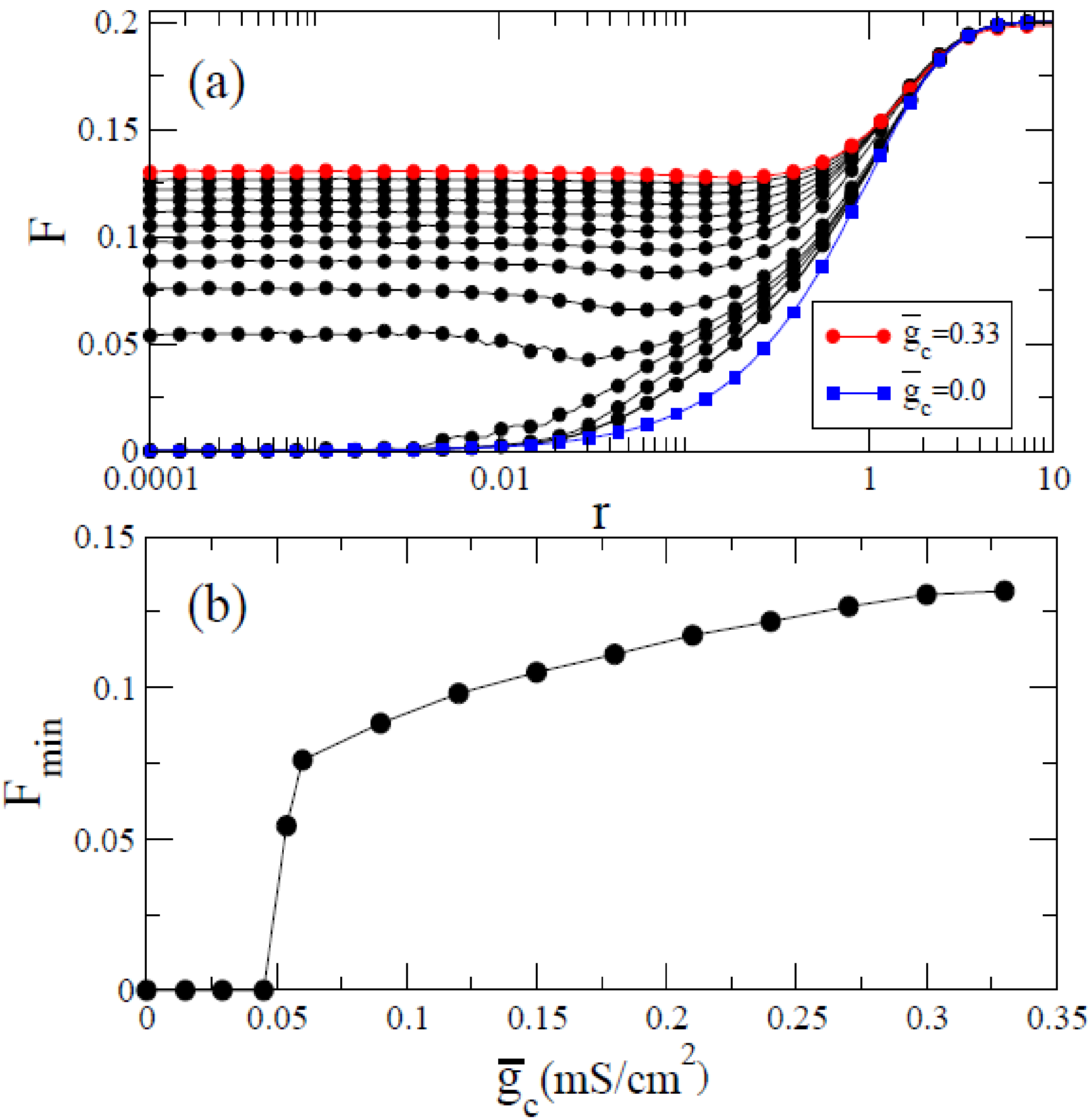
Small-world network of HH neurons



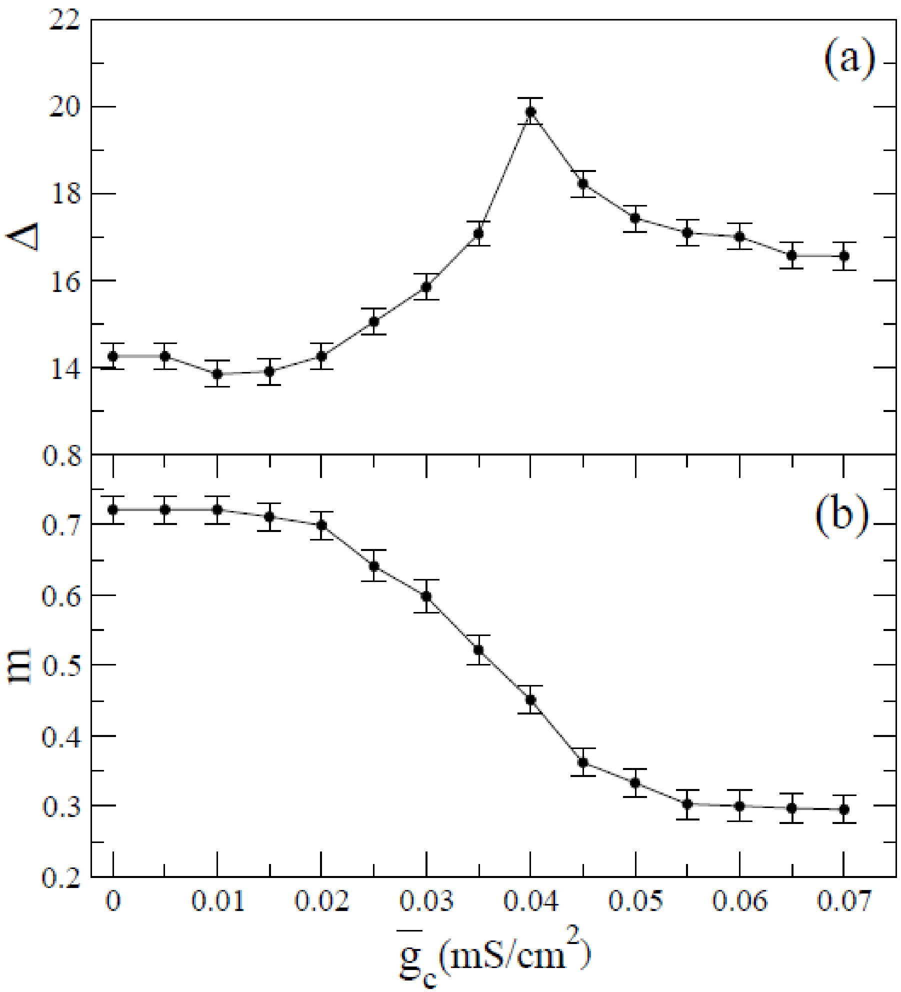
- Adjacency matrix obtained from a one-dimensional chain (local connections only)
- Watts-Strogatz procedure: some local connections are randomly replugged into nonlocal shortcuts with probability p

Stimulus-response relationship for a SW network

- Network of N = 2000 neurons
- fixed probability of shortcuts p = 0.001
- Varying the connection strength the value of F_{min} increases, whereas F_{max} remains constant
- A phase transition?



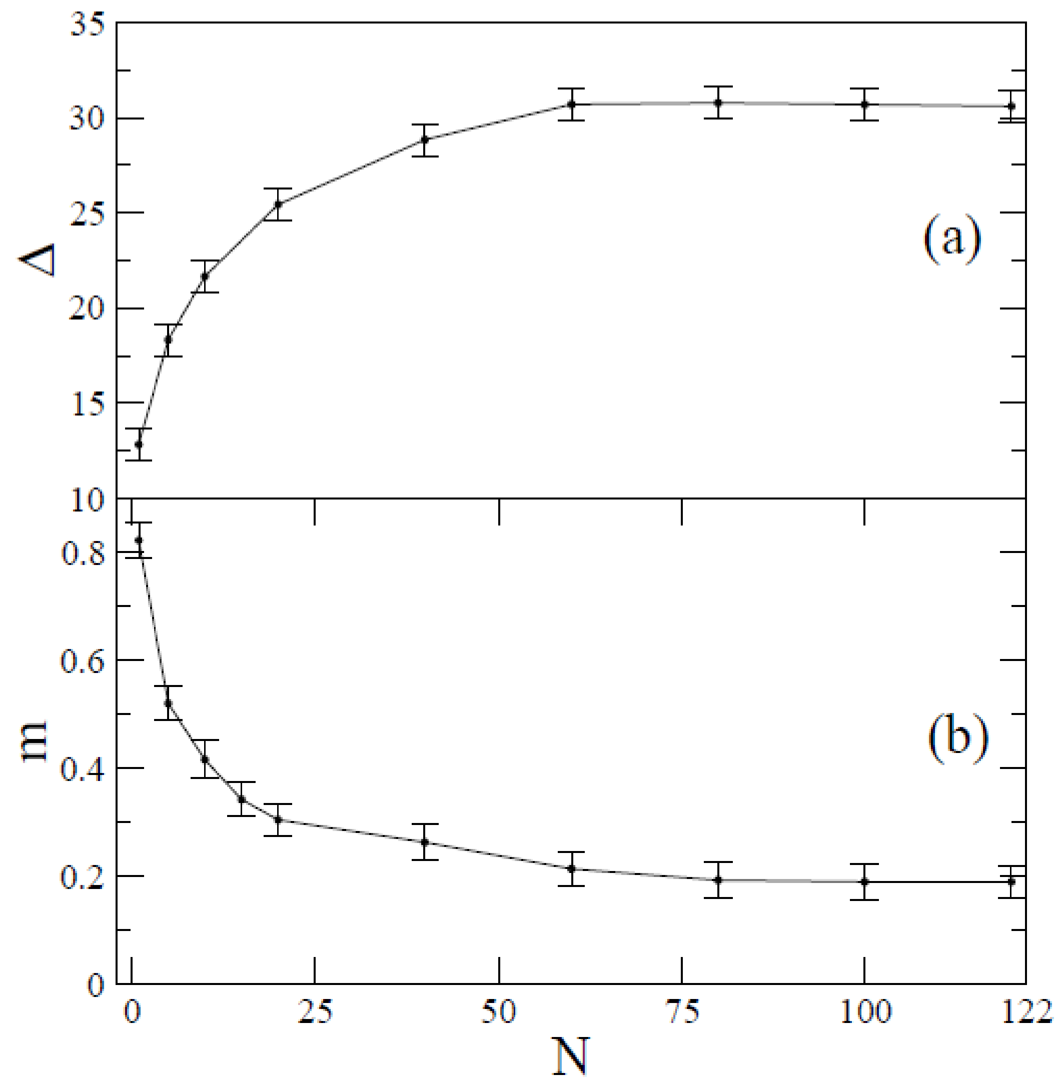
Increasing the coupling strength



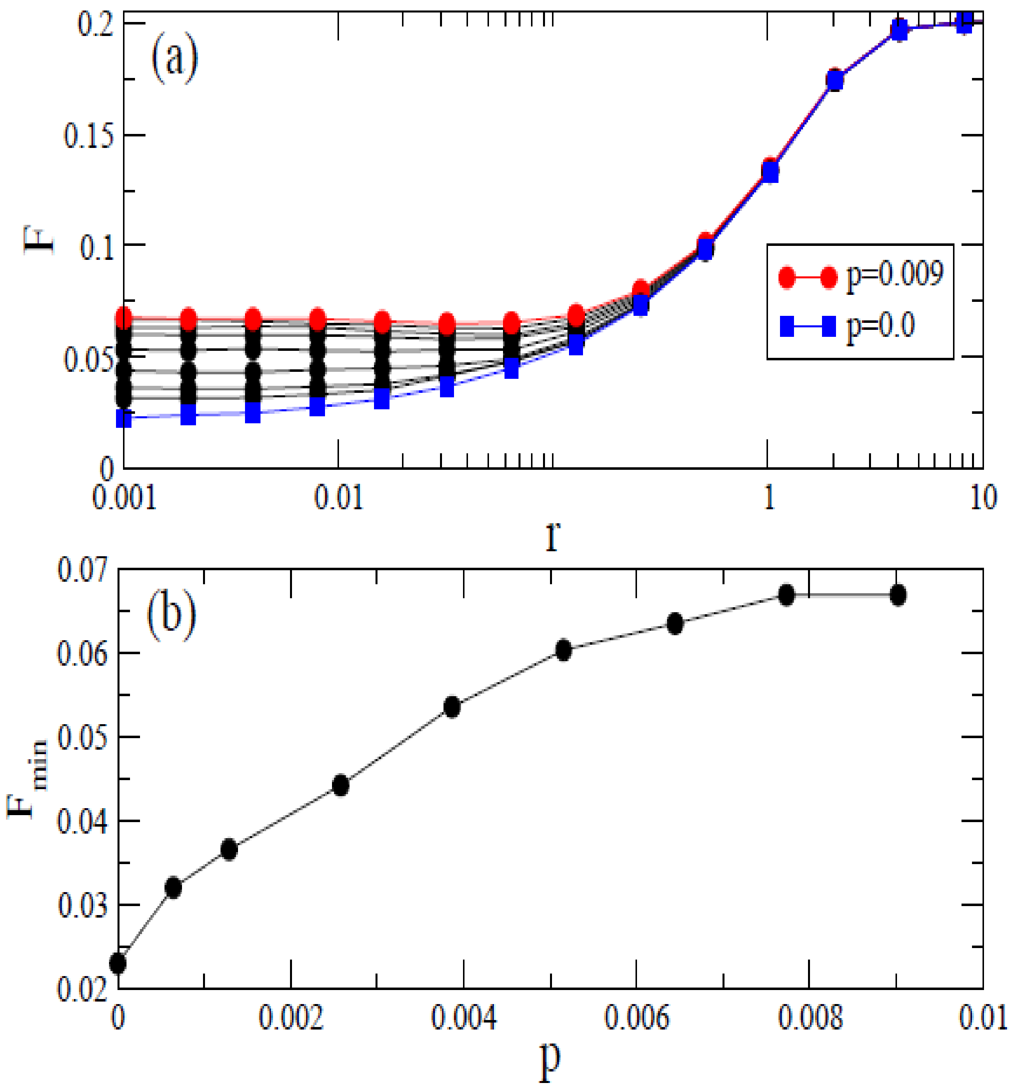
- SW network of N = 2000 neurons with p = 0.001
- Dynamic range initially increases to a maximum but decreases afterwards (dynamical effect?)
- Slope of power-law relationship monotonically decreases

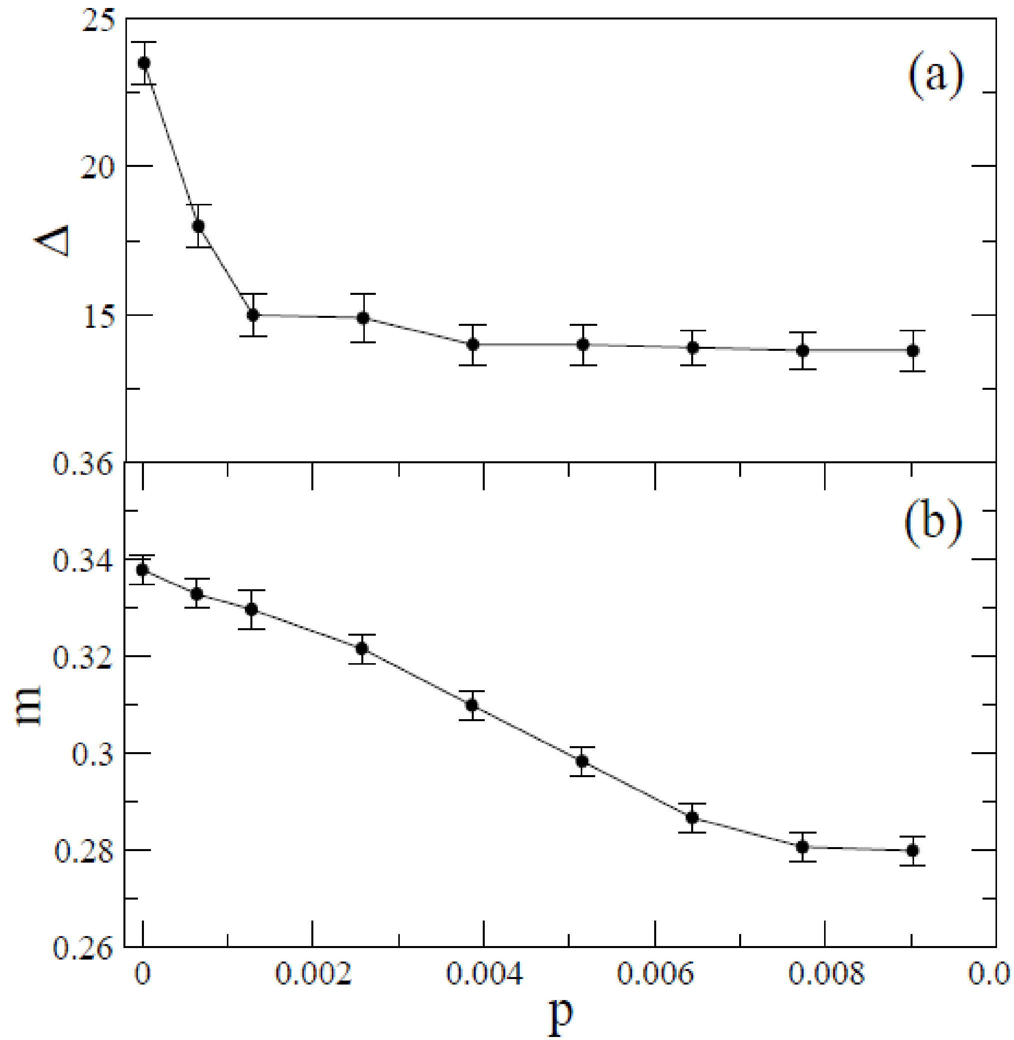
Increasing the number of neurons

- Fixed g_c and p
- Dynamic range <u>increases</u> (by a factor of two): agrees with olphatory system
- Exponent of power-law (F *vs.* r) relation decreases with N
- This is related to the increase of F_{min}



Increasing the shortcut probability





Conclusions

- The stimulus-response relationship is a power-law (within specified limits) in both microscopic (single neurons) and macroscopic levels (neuronal networks)
- The main effect of the network structure (with respect to the behavior of single neurons) is the enhancement of the dynamic range
- The decrease of the power-law exponent follows from the increase of the minimum response level associated with the constancy of the maximum response level (the curve is “pushed upwards”)

Thank you very much!

- Publications (download at fisica.ufpr.br/viana)
- C. A. S. Batista, R. L. Viana, S. R. Lopes, and A. M. Batista, Physica A **410** (2014) 628-640
- R. L. Viana, F. S. Borges, K. C. Iarosz, A. M. Batista, S. R. Lopes, and I. L. Caldas, Comm. Nonlin. Sci. Numer. Simulat. **19** (2014) 164-172
- K. C. Iarosz, A. M. Batista, R. L. Viana, S. R. Lopes, I. L. Caldas, and T. J. P. Penna, Physica A **391** (2012) 819-827