

Isochronous bifurcations in a two-parameters twist map

Mecânica Clássica

2024

Introdução

Mapa twist com dois parâmetros (amplitudes de dois modos)

Ilhas Isócronas excitadas por dois modos acoplados

Bifurcações das ilhas, de um modo dominante ao outro

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Isochronous bifurcations in a two-parameter twist map

Michele Mugnaine ^{*1}, Bruno B. Leal¹, Alfredo M. Ozorio de Almeida², Ricardo L. Viana³, and Iberê L. Caldas¹

¹Institute of Physics, University of São Paulo, São Paulo, SP, 05508-090, Brazil

²Brazilian Center for Research in Physics, Rio de Janeiro, RJ, 22290-180, Brazil

³Department of Physics, Federal University of Paraná, Curitiba, PR, 81531-980, Brazil

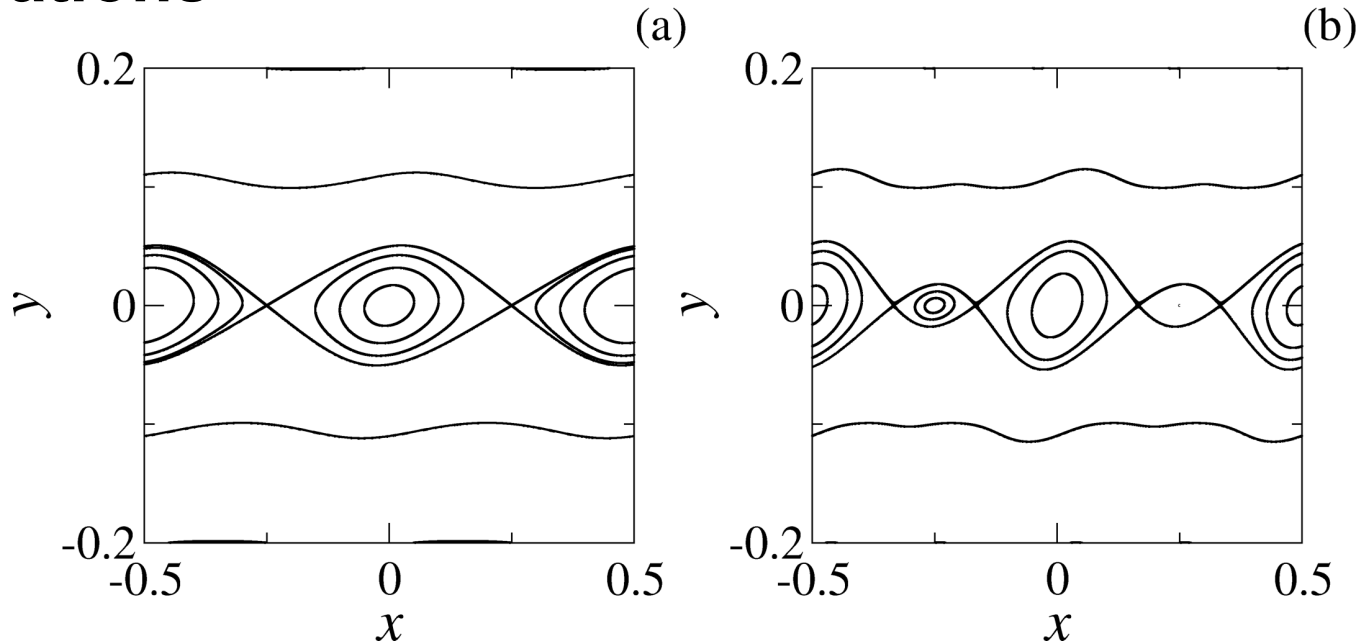
Abstract

Isochronous islands in phase space emerge in twist Hamiltonian systems as a response to multiple resonant perturbations. According to the Poincaré-Birkhoff theorem, the number of islands depends on the system characteristics and the perturbation. We analyze, for the two-parameter standard map, also called two-harmonic standard map, how the island chains are modified as the perturbation amplitude increases. We identified three routes for the transition from one chain, associated with one harmonic, to the chain associated with the other harmonic, based on a combination of pitchfork and saddle-node bifurcations. These routes can present intermediate island chains configurations. Otherwise, the destruction of the islands always occurs through the pitchfork bifurcation.

Keywords: Hamiltonian system, resonance, symplectic map, isochronous islands

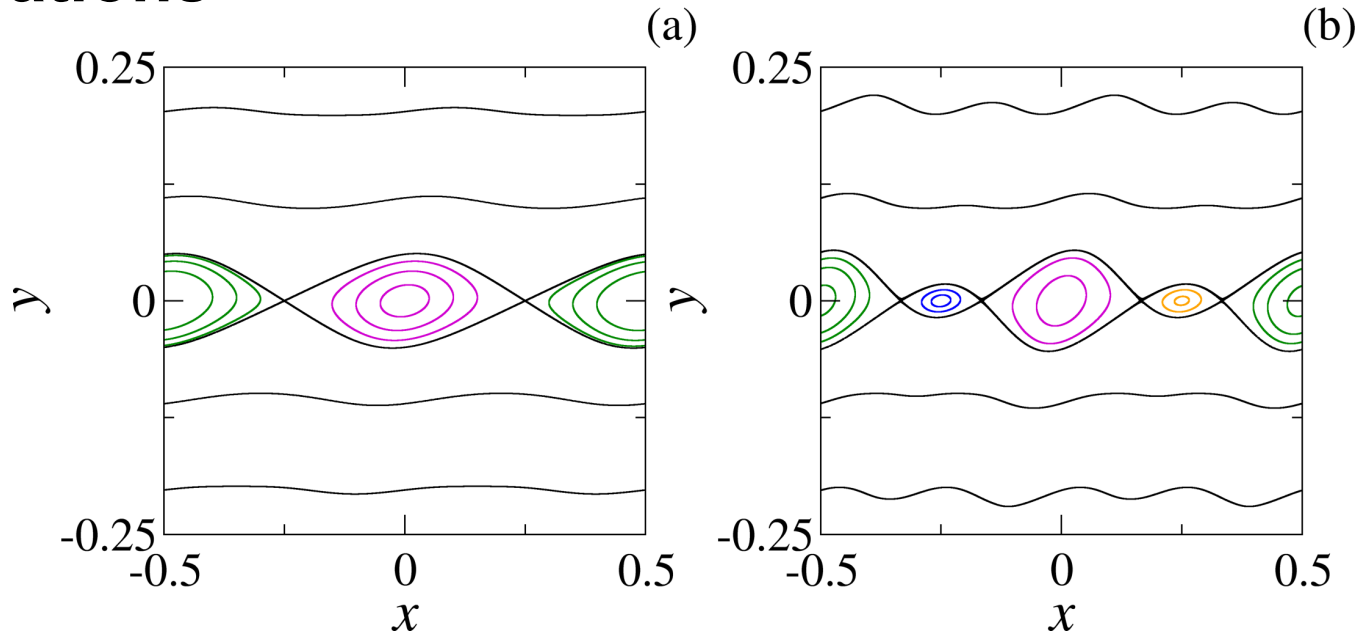
Isochronous Bifurcations

- **Isochronous islands** in phase space emerge in twist Hamiltonian systems as a response to **multiple resonant perturbations**



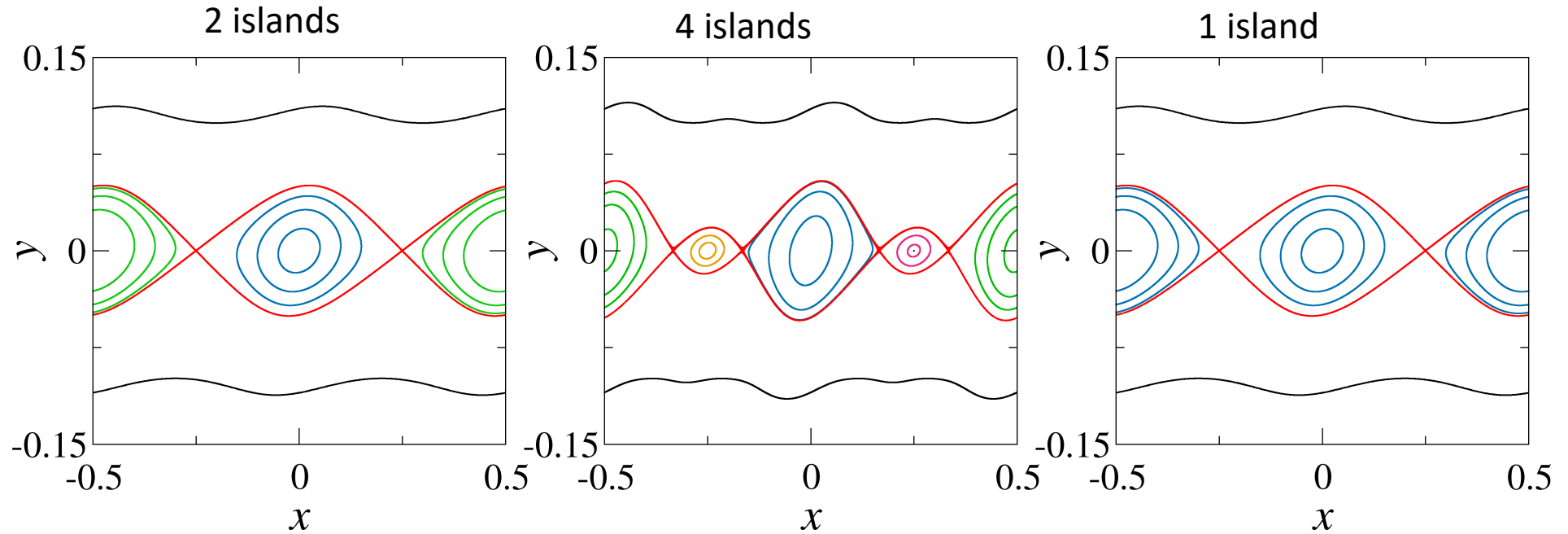
Isochronous Bifurcations

- **Isochronous islands** in phase space emerge in twist Hamiltonian systems as a response to **multiple resonant perturbations**



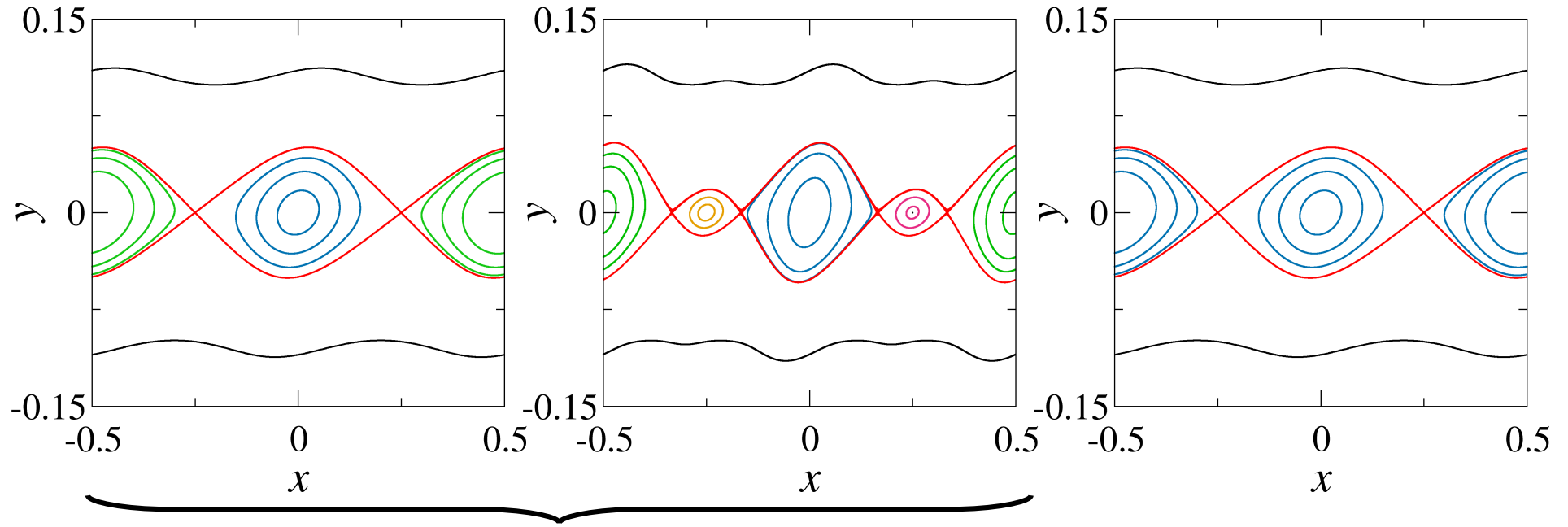
Isochronous islands

- Islands with same period



Isochronous islands

- Islands with same period



Isochronous islands

Two-harmonic standard map

$$y_{n+1} = y_n - \frac{K_1}{2\pi m_1} \sin(2\pi m_1 x_n) - \frac{K_2}{2\pi m_2} \sin(2\pi m_2 x_n)$$

$$x_{n+1} = x_n + y_{n+1}$$

Two-harmonic standard map

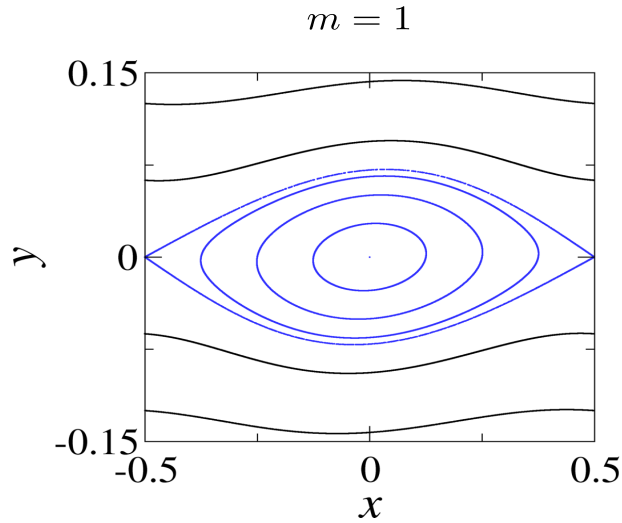
$$y_{n+1} = y_n - \frac{K_1}{2\pi m_1} \sin(2\pi m_1 x_n) - \frac{K_2}{2\pi m_2} \sin(2\pi m_2 x_n)$$

Mode 1

Mode 2

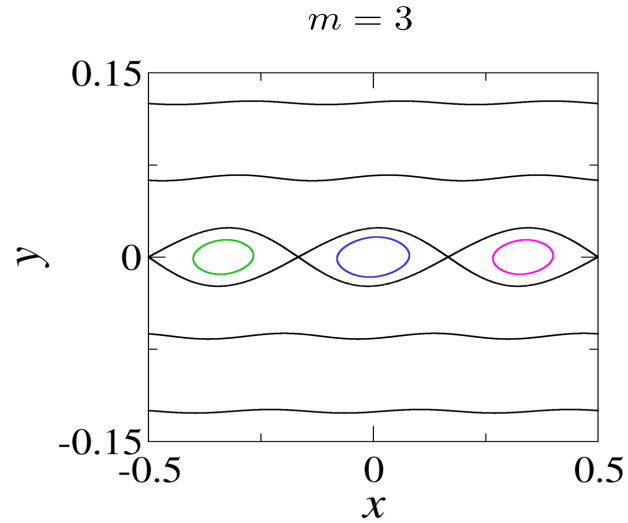
$$x_{n+1} = x_n + y_{n+1}$$

Two-harmonic standard map



Mode 1

$$y_{n+1} = y_n - \frac{K_1}{2\pi} \sin(2\pi x_n)$$
$$x_{n+1} = x_n + y_{n+1}$$



Mode 2

$$y_{n+1} = y_n - \frac{K_2}{6\pi} \sin(6\pi x_n)$$
$$x_{n+1} = x_n + y_{n+1}$$

Two-harmonic standard map

$$y_{n+1} = y_n - \frac{K_1}{2\pi m_1} \sin(2\pi m_1 x_n) - \frac{K_2}{2\pi m_2} \sin(2\pi m_2 x_n)$$

$$x_{n+1} = x_n + y_{n+1}$$

Two-harmonic standard map

$$y_{n+1} = y_n - \frac{K_1}{2\pi m_1} \sin(2\pi m_1 x_n) - \frac{K_2}{2\pi m_2} \sin(2\pi m_2 x_n)$$

$$x_{n+1} = x_n + y_{n+1}$$

$m_1 = 1$
$m_2 = 3$

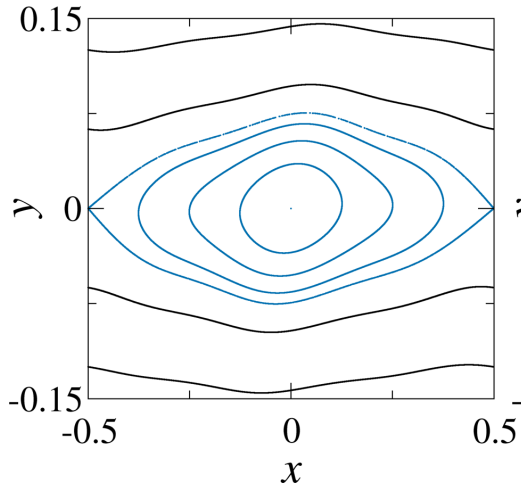
Two-harmonic standard map

$$y_{n+1} = y_n - \frac{K_1}{2\pi m_1} \sin(2\pi m_1 x_n) - \frac{K_2}{2\pi m_2} \sin(2\pi m_2 x_n)$$

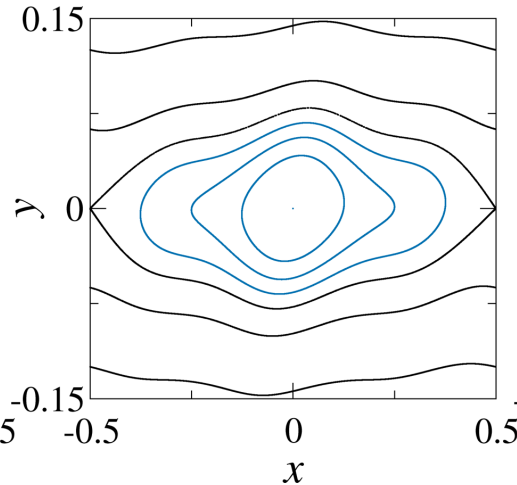
$$x_{n+1} = x_n + y_{n+1}$$

$$\begin{aligned} m_1 &= 1 \\ m_2 &= 3 \end{aligned}$$

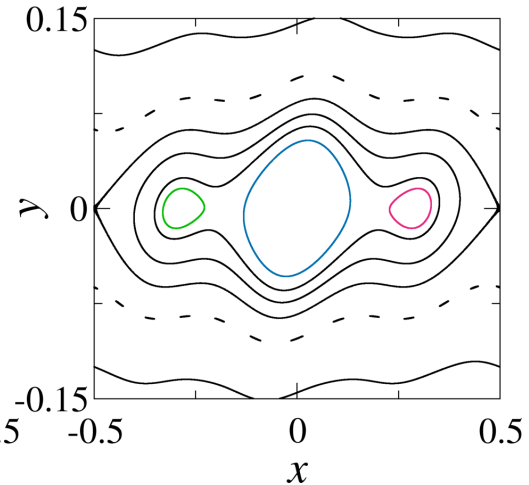
$K_1=K_2=0.05$



$K_1=0.05, K_2=0.10$



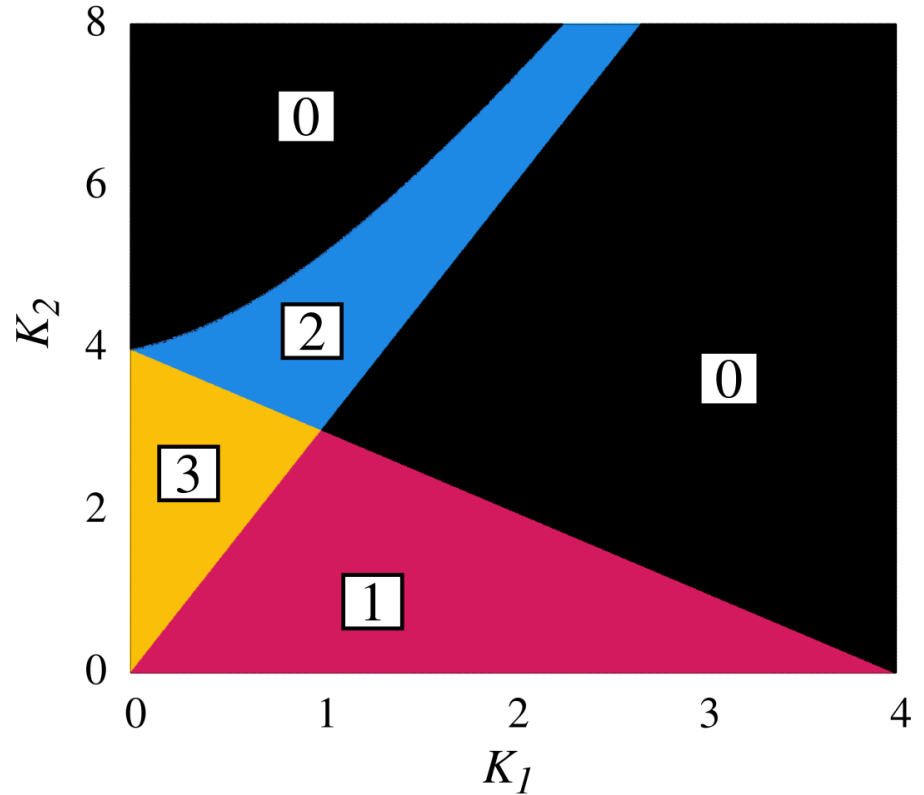
$K_1=0.05, K_2=0.20$



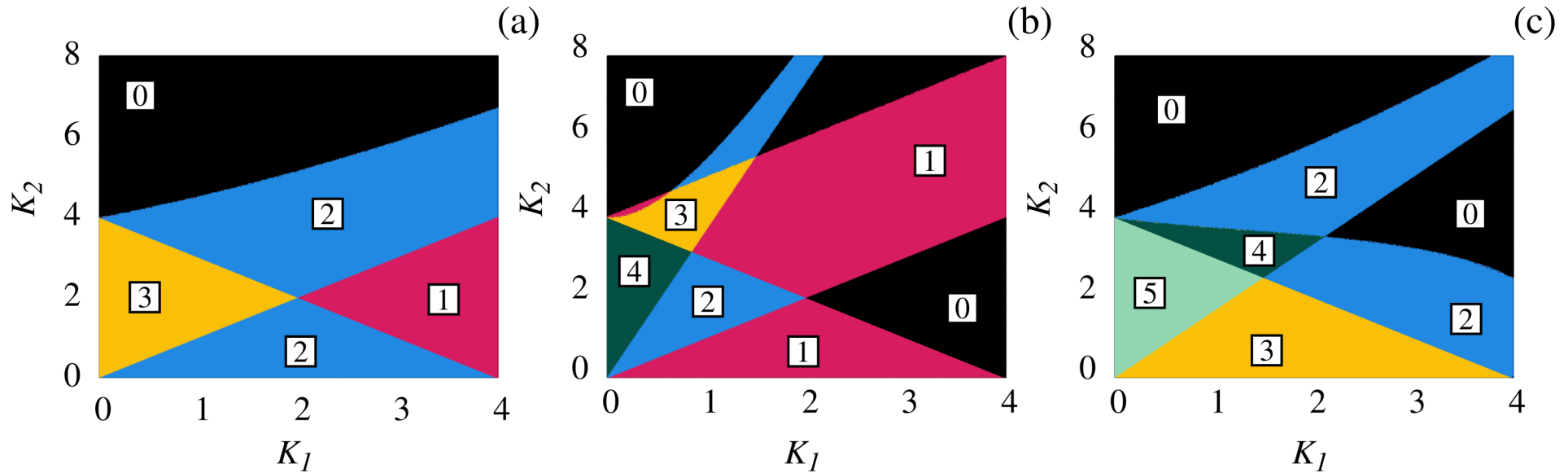
Two-harmonic standard map

- How many islands?

$$m_1 = 1$$
$$m_2 = 3$$



Two-harmonic standard map



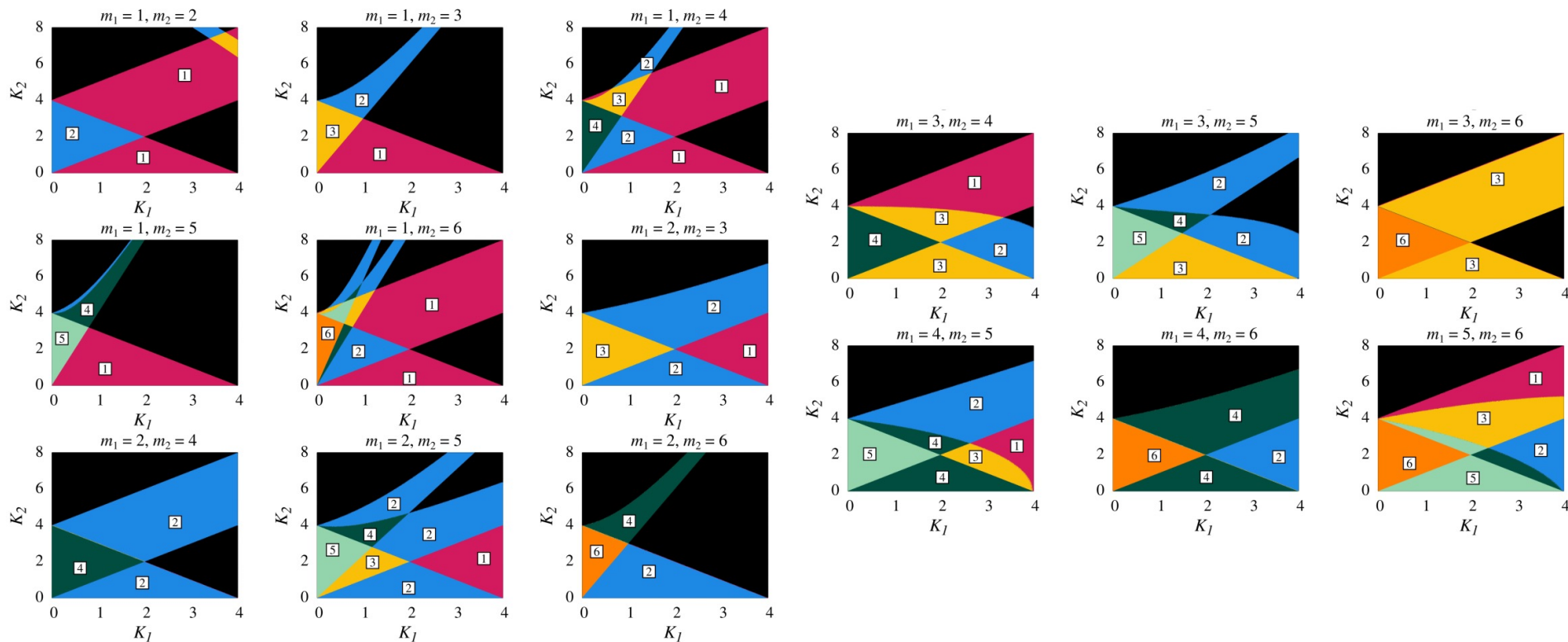
$m_1=2,$
 $m_2=3$

$m_1=1,$
 $m_2=4$

$m_1=3,$
 $m_2=5$

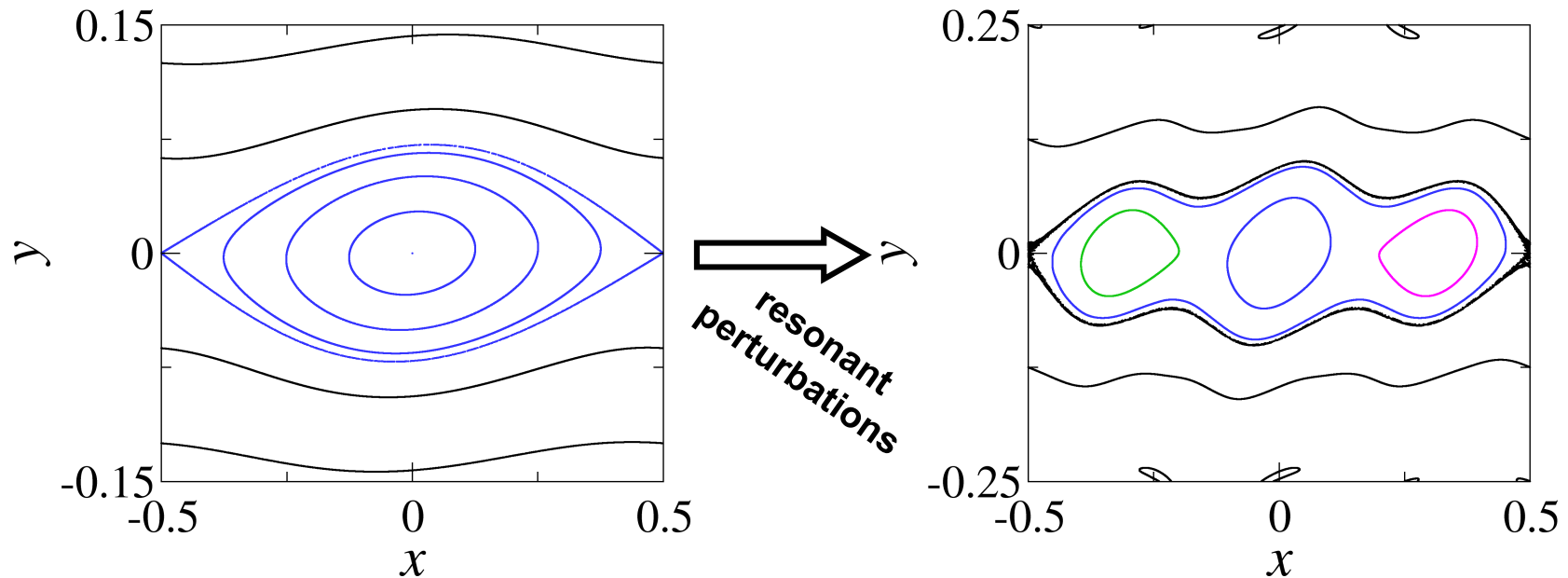
Two-harmonic standard map

- How many islands?



Isochronous Bifurcations

- We analyze the transition from one **island chain** associated with **one harmonic** to the island chain associated with the **other harmonic**.

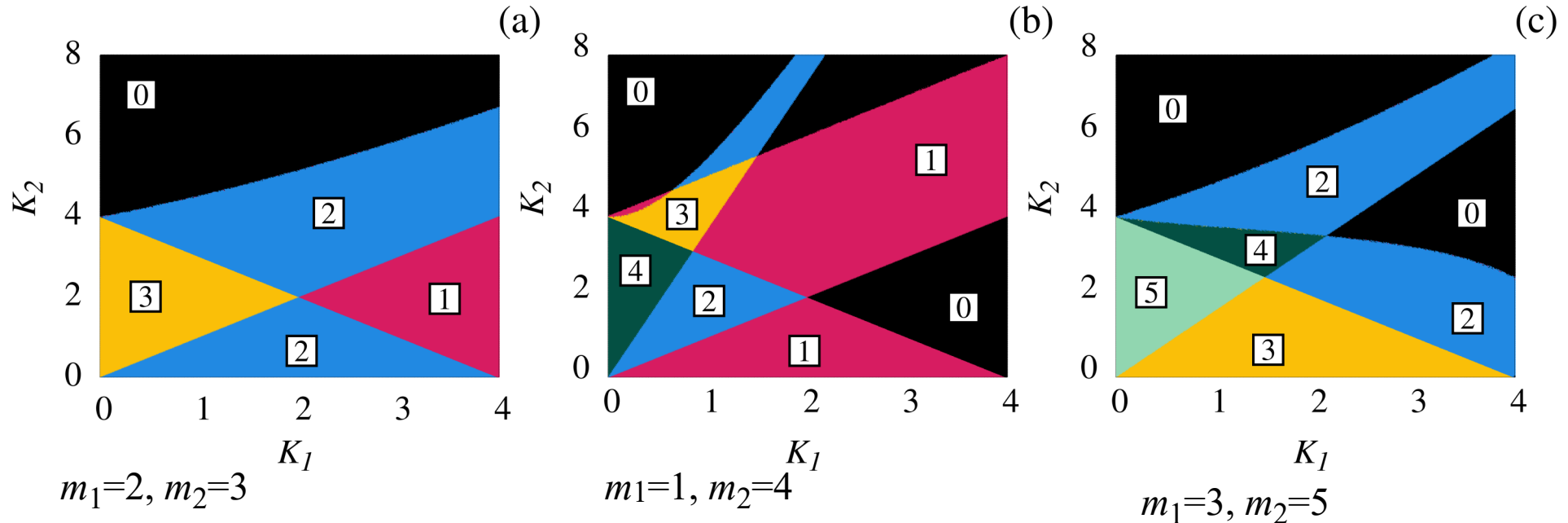


Two-harmonic standard map

- How is the **transition** from one **island chain** associated with **one harmonic** to the island chain associated with the **other harmonic**?

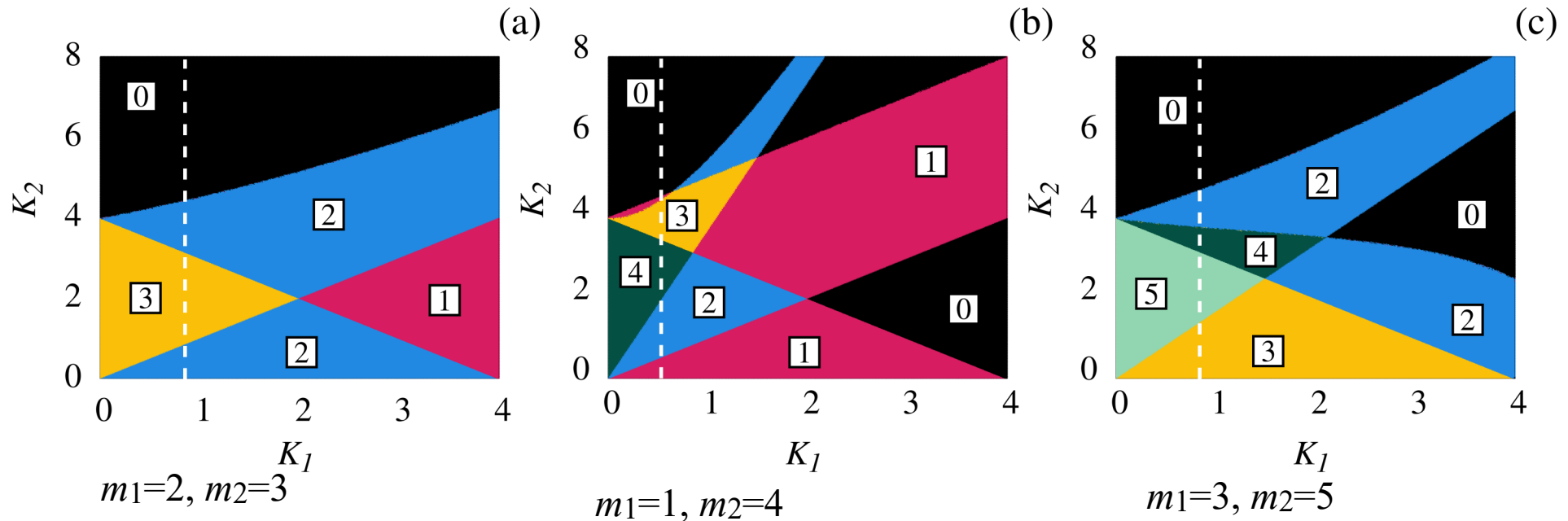
Two-harmonic standard map

- How is the **transition** from one **island chain** associated with one **harmonic** to the island chain associated with the **other harmonic**?

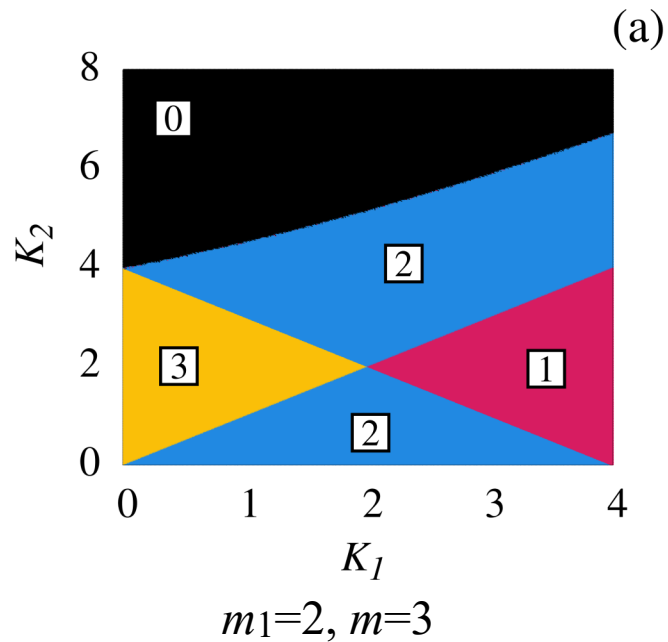


Two-harmonic standard map

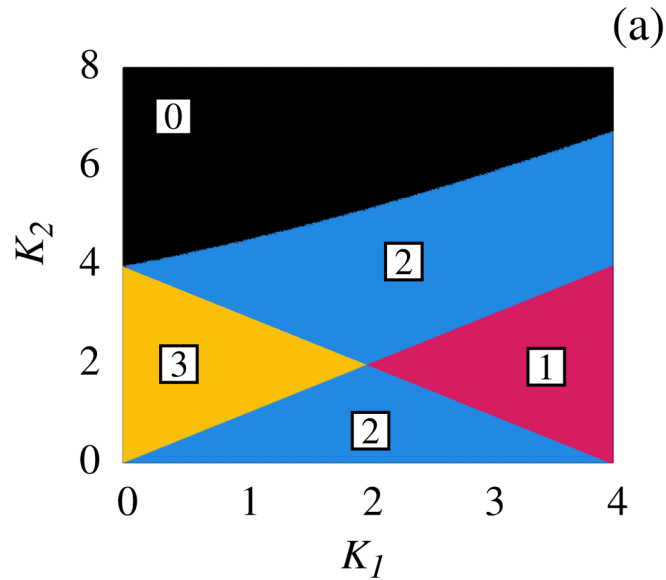
- How is the **transition** from one **island chain** associated with one **harmonic** to the island chain associated with the **other harmonic**?



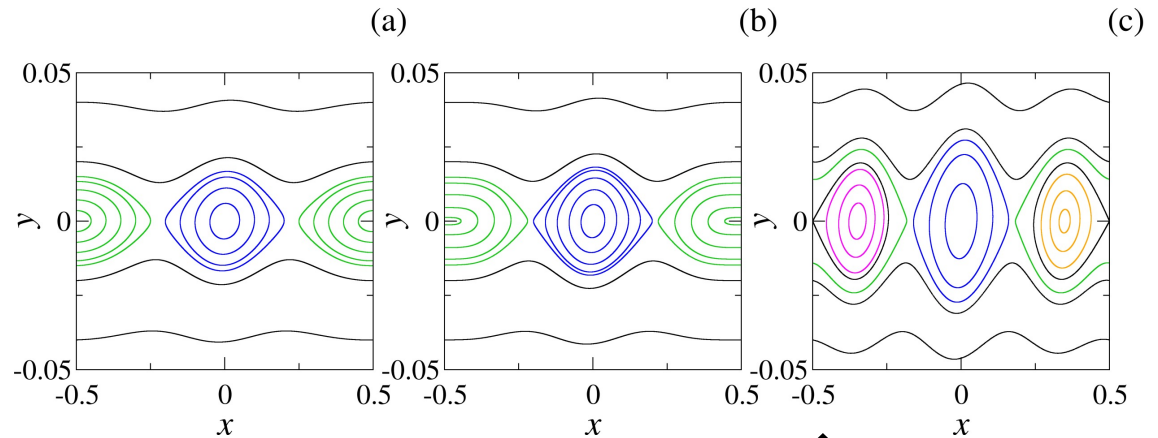
Two-harmonic standard map



Two-harmonic standard map

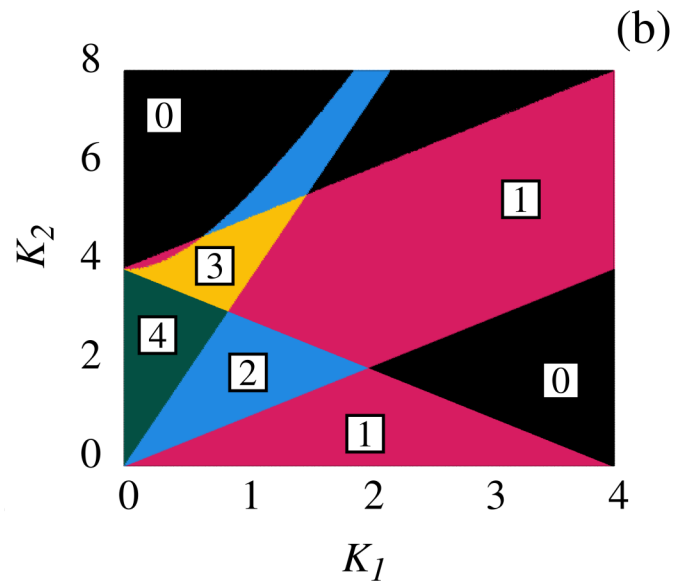


$$m_1=2, m_2=3$$



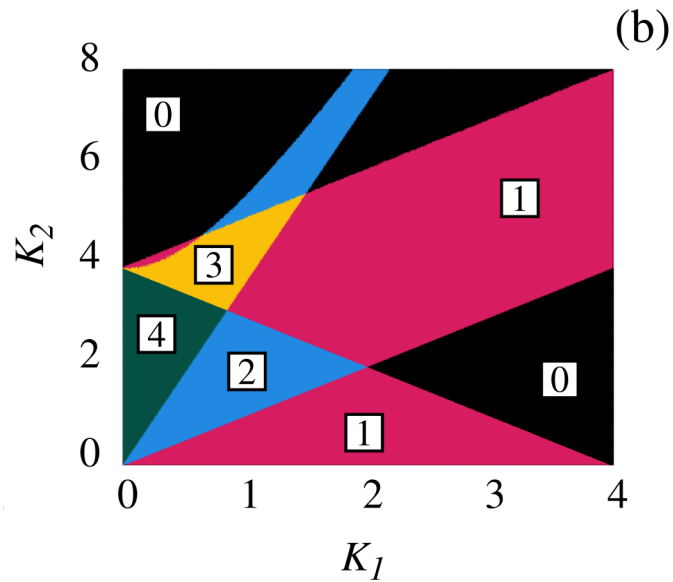
Pitchfork bifurcation

Two-harmonic standard map

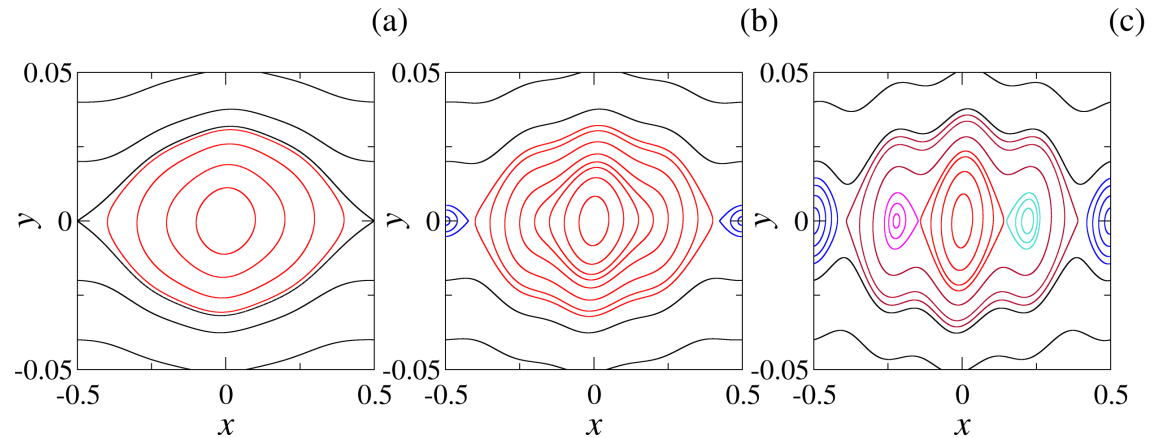


$$m_1=1, m_2=4$$

Two-harmonic standard map



$$m_1=1, m_2=4$$

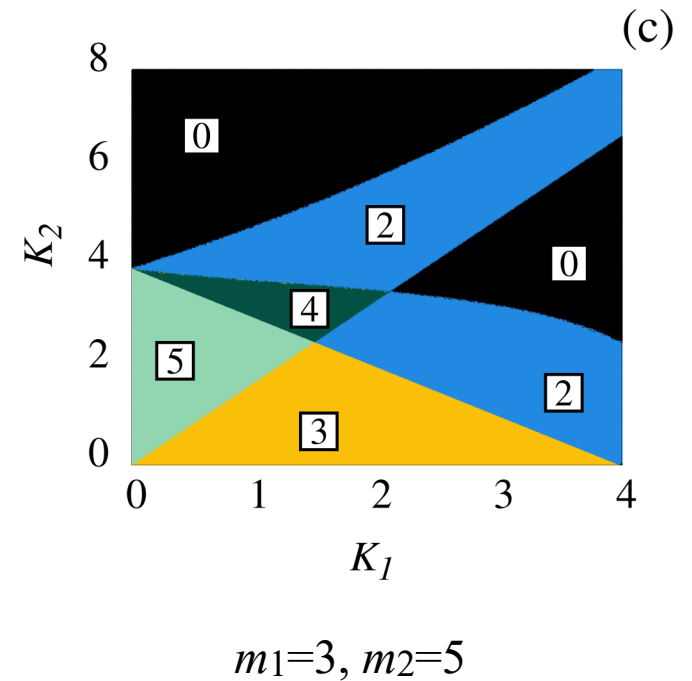


Pitchfork bifurcation

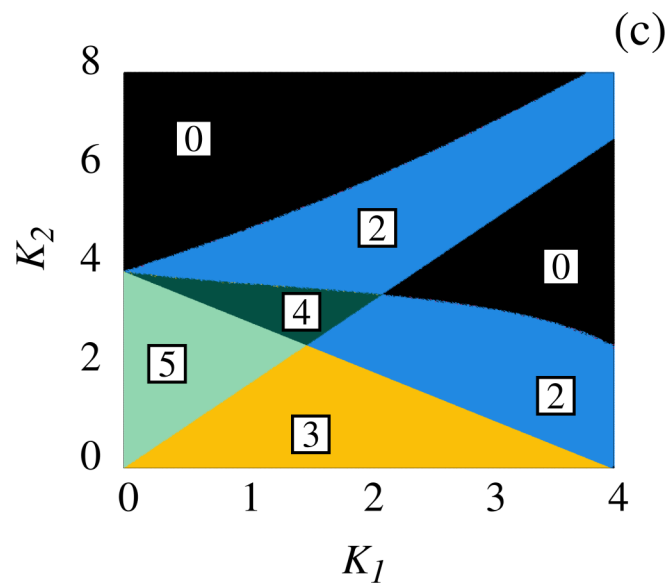


Saddle-node bifurcation

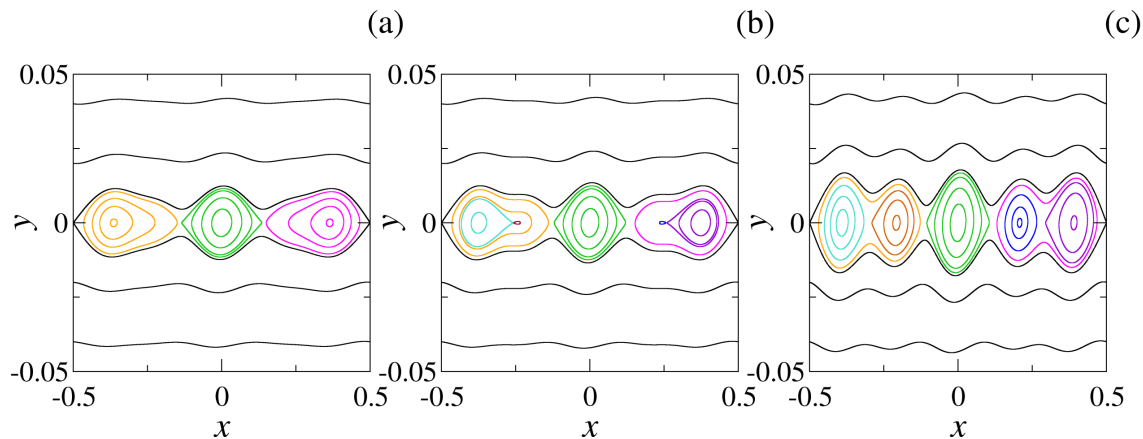
Two-harmonic standard map



Two-harmonic standard map



$$m_1=3, m_2=5$$

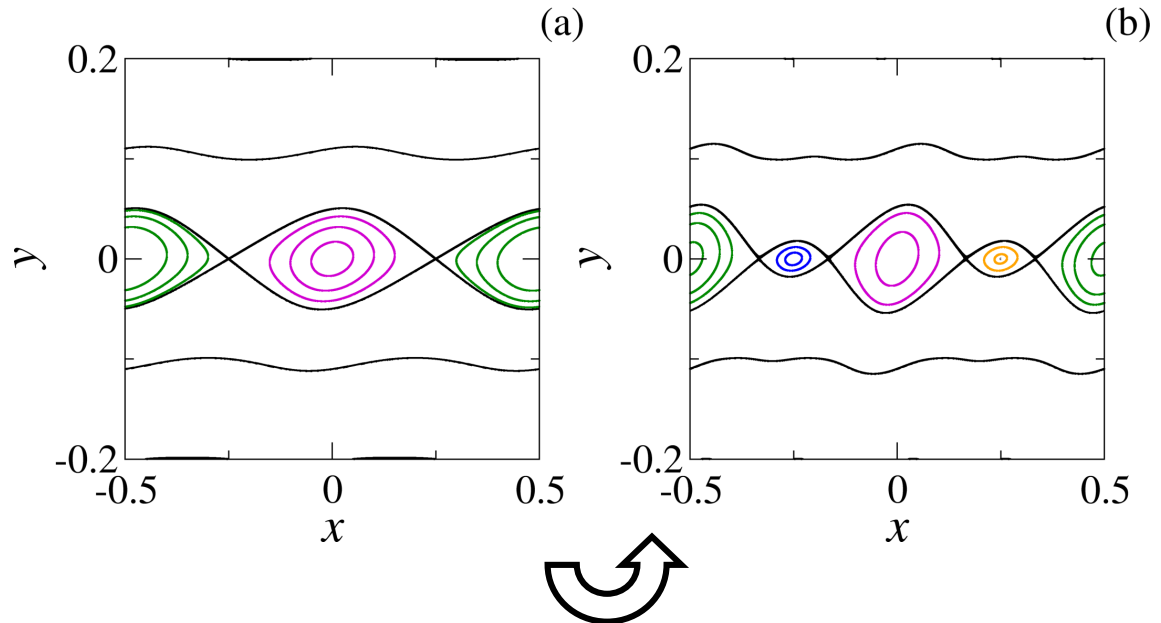


**Saddle-node
bifurcation**

Two-harmonic standard map

- How is the **transition** from one **island chain** associated with one **harmonic** to the island chain associated with the **other harmonic**?

Route 1: $m_1 \xrightarrow{P} m_2$		
m_1	m_2	Transitions
1	2	$1 \rightarrow 2$
2	3	$2 \rightarrow 3$
2	4	$2 \rightarrow 4$
3	4	$3 \rightarrow 4$
3	6	$3 \rightarrow 6$
4	5	$4 \rightarrow 5$
4	6	$4 \rightarrow 6$
5	6	$5 \rightarrow 6$

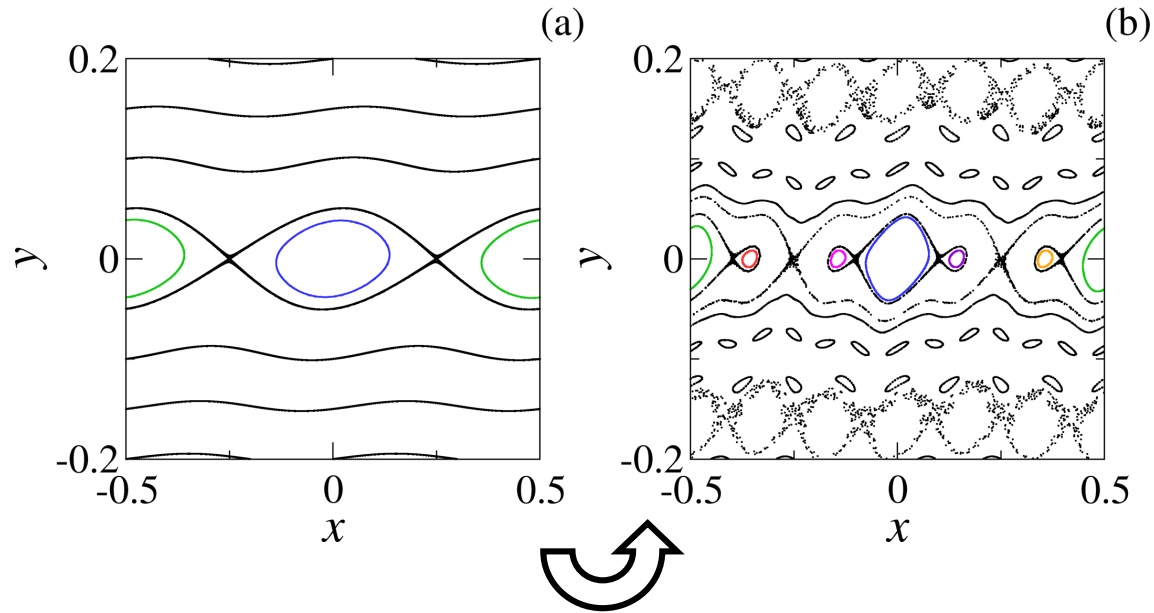


Pitchfork bifurcation

Two-harmonic standard map

- How is the **transition** from one **island chain** associated with one **harmonic** to the island chain associated with the **other harmonic**?

Route 2: $m_1 \xrightarrow{\text{SN}} m_2$		
m_1	m_2	Transitions
1	3	$1 \rightarrow 3$
1	5	$1 \rightarrow 5$
2	6	$2 \rightarrow 6$
3	5	$3 \rightarrow 5$

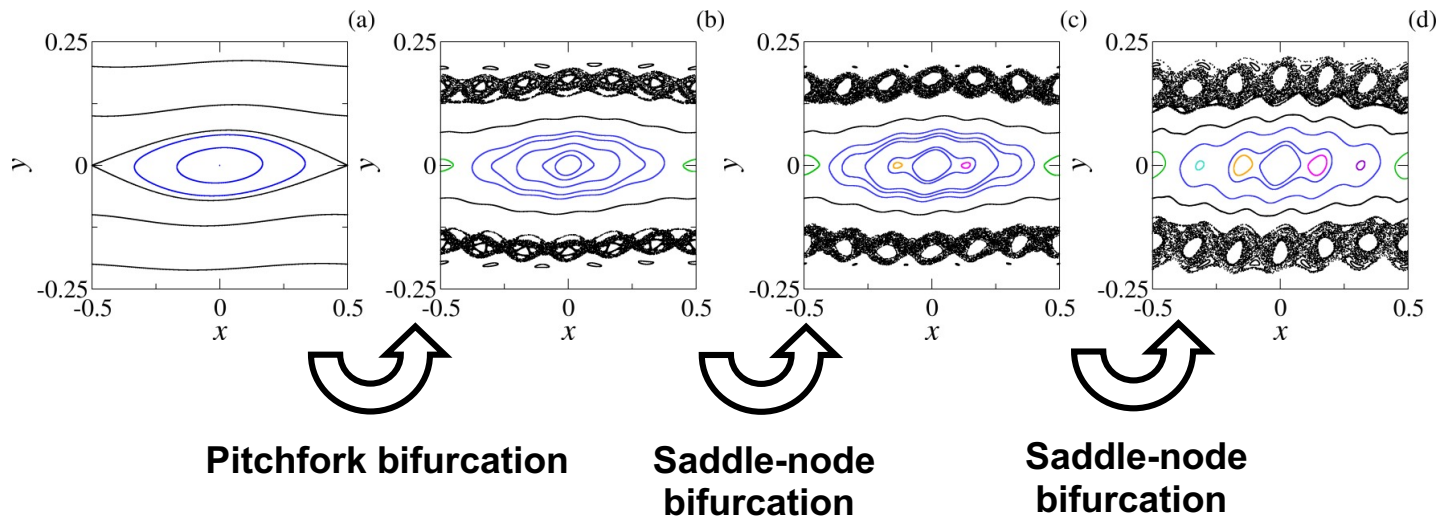


Saddle-node
bifurcation

Two-harmonic standard map

- How is the **transition** from one **island chain** associated with one **harmonic** to the island chain associated with the **other harmonic**?

Route 3: $m_1 \xrightarrow{P+SN} m_2$		
m_1	m_2	Transitions
1	4	$1 \rightarrow 2 \rightarrow 4$
1	6	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6$
2	5	$2 \rightarrow 3 \rightarrow 5$



Conclusões

Mapa introduzido com dois modos acoplados

Ilhas isócronas são observadas

Bifurcações entre uma configuração de ilhas de um modo para a configuração do outro modo