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Minimizing Chaos During the Reconnection Process

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Abstract—Chaos around the separatrices of resonant chains during the reconnection process in Tokamaks with non-monotonic profiles is analysed. To characterize the extension of chaos in the system, we estimate the convergence of the chaotic lines by computing the winding number profiles in the neighborhood of the separatrices. Some simulations were performed and we have detected, for these non-twist mappings, a reduction of chaos during the reconnection process. A theoretical interpretation based on the overlapping of resonance pictures is proposed in order to explain the decrease of chaos. These results may contribute to interpret the recently observed improvement in reversal magnetic shear in Tokamaks. © 1997 Elsevier Science Ltd

1. INTRODUCTION

Reconnection in a phase space is defined as a process where the number and the index of the fixed points remain the same, but the trajectories assume a new arrangement [1–3]. In this paper we focus attention on the relation between the reconnection process and chaos. As we shall see, the stochastic layer diminishes during the reconnection process. We present a theoretical interpretation in order to explain the decrease of chaos during the reconnection based on the overlapping of resonance pictures (ORP). The paper of Chirikov [4] presents a method to obtain an analytical estimate of the transition to chaos established in the ORP. The creation of a stochastic layer can be understood in the Chirikov picture; according to that picture, chaos in nonlinear Hamiltonian systems is originated from overlapping of adjacent resonant island chains in the phase space. Whenever the amplitude of the resonant islands increases, they will become larger and overlap each other. The ORP will be used in order to understand chaos during the reconnection.

Two papers have already treated the relationship between reconnection process and chaos. One of these [5] deals with a continuous Hamiltonian that represents a reconnection process, but in this work the reconnection is generated by increasing the amplitude of the islands. As one increases the island amplitude the stochastic layer naturally increases [4], hiding the most important phenomenon. The reconnection picture we use in this paper, contrary to Ref. [5], is that caused by the approximation of the island chains without increasing the amplitude. The other paper [6] deals with a non-twist map and uses the residue criterion to compute the critical value for the destruction of the KAM curves. It is concluded that the transition to chaos in this class of systems is different from the twist maps. In this work we emphasize the behavior of the stochastic layer, keeping the island amplitude constant.

The physical reconnection model used in this work is extracted from the magnetic confined plasma context. MHD equilibrium plasmas with central hole current density profiles confined in Tokamaks present reconnection of the magnetic field lines when perturbed by resonant fields. Although these perturbations can be due to natural plasma oscillations or other external perturbations, for numerical applications we consider only those created by external resonant helical windings [7, 8]. The reconnection of magnetic islands is important in the analysis of the stability of plasma confinement and in the disruption of equilibrium plasmas [9]. Particularly, the magnetic field configuration chosen for our numerical application corresponds to the reversed magnetic shear in Tokamaks, which has been obtained by a new improved confinement regime [10, 11].

The paper is therefore organized as follows: in Section 2 we define the physical model and discuss some features related to non-monotonic profiles in Tokamaks; in Section 3 we analyze the behavior of the stochastic layer during the reconnection process; in Section 4 we present some results of numerical simulations; and in Section 5 we conclude the paper.

2. THE PHYSICAL MODEL

We consider MHD equilibrium plasmas confined in Tokamaks. If the configuration of the magnetic field, which confines the plasma, presents any spatial symmetry, the magnetic field line trajectories lie on magnetic surfaces of constant Hamiltonian [12].

We consider non-monotonic winding number ρ radial profile plasmas, or reversed magnetic shear, which correspond to the initial and final stages of the Tokamak discharge [9]. Recently, this magnetic configuration has also been used as an alternative to improve plasma confinement [10, 11]. Also, a large aspect-ratio Tokamak ($R \gg a$, where R is the major radius of the Tokamak and a is the plasma column radius) is considered, with m pairs of external helical winding conductors wound along the vessel. This arrangement is illustrated in Fig. 1.

In this case, the description of the Hamiltonian of the system is made by the sum of an integrable part, due to the cylindrical approximation of the equilibrium plasma in the torus wound by the helices, and a non-integrable part, which corresponds to the toroidal correction [9]:

$$H = H_{\text{equilibrium}}(I_p) + H_{\text{helical perturbation}}(I_h) + H_{\text{toroidal correction}} \quad (1)$$

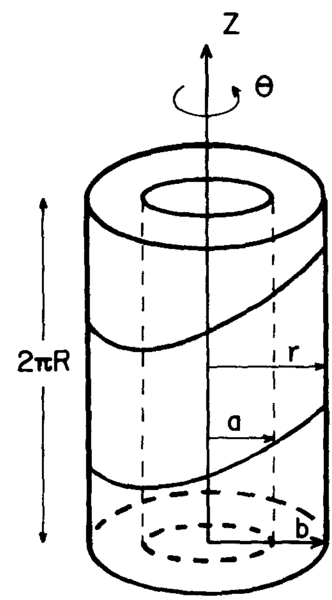

Fig. 1. Scheme of a Tokamak in the cylindrical approximation.

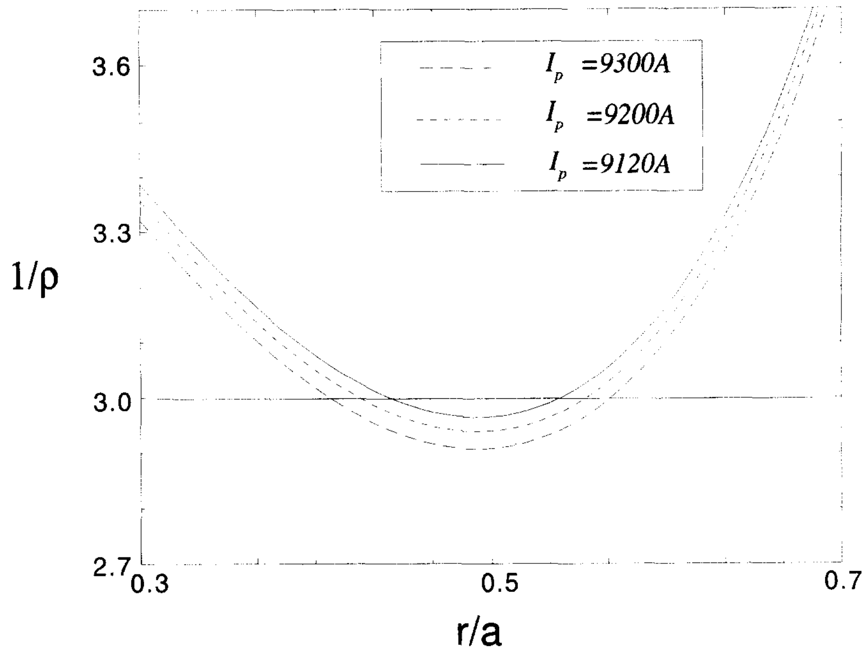

Fig. 2. Non-monotonic radial profile showing the inverse of winding number $1/\rho$, safety factor, for the total plasma current $I_p = 9300$ A, $I_p = 9200$ A and $I_p = 9120$ A. The line corresponding to the resonance 1/3 is indicated in the figure.

where the control parameters are I_p, the total plasma current, and I_h, the helical current.

The magnetic surfaces of the system are described by Poincaré maps. Considering first the Poincaré maps of the system under cylindrical approximation, the unperturbed equilibrium KAM surfaces constitute a family of nested cylindrical surfaces.

The effect of the integrable helical perturbation is the creation of two main twin island chains with m islands at the two resonant surfaces with $\rho = n/m$ (n, m integers), presented by the non-monotonic safety factor profile plasma, that is equal to the inverse of the winding number, as we can see in Fig. 2. In this work, we use $m = 3$. The toroidal correction breaks the integrability of the system and gives rise to a stochastic layer mainly concentrated in the neighborhood of the separatrices.

The reconnection process can be achieved in one of two ways: by the variation of the parameter I_h that increases the amplitude of the islands and approximates the chains, or by the variation of I_p that only approximates the islands. In this work we will decrease I_p, thus approximating the chains and keeping the amplitude of the islands constant. Since an increase in the amplitude of the islands produces a known effect of increasing the stochastic layer [4], in order to observe the chaotic behavior due only to the reconnection, we will keep I_h constant during the reconnection process.

3. THE STOCHASTIC LAYER DURING RECONNECTION

In this chapter we will expose a theoretical interpretation based on the ORP in order to explain the decrease of chaos during the reconnection process. We begin with an analysis of the resonances and the route to chaos according to ORP.

After some canonical transformations, the Hamiltonian (1) that describes the reconnection of magnetic lines can be cast in the form $H = H_0(I) + \epsilon H_1(I,\psi,t)$ [9]. In the toroidal section of the Tokamak the variable action is directly related to the radius of the section, the angle variable to the polar angle, and the canonical time to the toroidal angle of the Tokamak.

This Hamiltonian has a natural frequency $\omega_0(I) = \partial_I H_0$ that explicitly depends on the action I. A resonance is fulfilled in the Hamiltonian when

$$q\omega_0(I_{p/q}) = p\omega \tag{2}$$

where we have used the notation $I_{p/q}$ to denote the value of the action at the resonance p/q, and ω for the perturbing frequency of H_1.

According to the Poincaré–Bendixon theorem [13], for each rational value p/q of the frequency in the phase space a resonant island chain appears with winding number $\rho = \omega_0(I_{p/q})$. In this way a nonlinear Hamiltonian phase space is populated by a denumerable set of resonant island chains. The ratio between the number of resonant chains in an interval and the corresponding interval of action defines the density of chains.

If one varies a parameter that increases the amplitude of the islands, they will become larger and eventually overlap with each other, developing chaos. However, there are two main factors that can enlarge the overlapping of resonances in a Hamiltonian system: an increase in the amplitude of the islands of the resonant chains and an increase in the density of chains.

In Fig. 2 we can observe a curve of the inverse of the winding number, the safety factor, versus the plasma radius, for the unperturbed equilibrium plasma. The slope of the curve is proportional to the density of the island chains. The higher the slope of the curve, the higher the amount of rational numbers p/q in an interval of radius, and consequently the density of chains. The two main resonant chains with winding number $\rho_0 = 1/3$ that suffer reconnection are also indicated in this figure. They are placed in the intersection between the line $1/\rho = 3$ and the winding curve.

As analyzed in Refs [9, 14, 15], a reconnection process always presents a non-monotonic behavior in the winding number relative to the action. So the function $\rho(I)$ has a point, in the middle of the two main chains, where $\dot{\rho} = \mathrm{d}\rho/\mathrm{d}I = 0$, as can be seen in Fig. 2. As the quantity $\dot{\rho}$ is related to the density of resonant island chains, we can say that close to the reconnection process, the phase space presents a local decrease in the number of resonant chains. In the context of Tokamak literature, $\dot{\rho}$ is named the magnetic shear, and the minimum of $\dot{\rho}$ is a region of small shear.

The stochastic layer in the separatrix of a resonant chain is generated by a local overlapping of resonant chains close to the separatrix [13]. We can expect that in the reconnection process the chaos will locally decrease due to a decrease of overlapping among the main resonant chains and their neighbor chains.

An analogous dependence between decreasing of chaos and $\dot{\rho}$ can be seen in the classical overlapping of resonance criteria presented in Appendix A. This analytical criterion remains for the case where $p = 1$ and q is large in the resonance condition (2). Under these circumstances the criterion says that the Hamiltonian phase space becomes chaotic when

$$1 < \sqrt{\frac{F\dot{\rho}}{\omega}}. \tag{3}$$

In fact, the Chirikov criterion of overlapping of resonances says that the increasing of chaos is proportional not only to the square of the amplitude of the chains, but also to the density of chains $\dot{\rho}$. This fact also suggests that during the reconnection process, as the density of chains decreases, the stochastic layer consequently diminishes.

4. NUMERICAL SIMULATIONS

In this section we present the results of some numerical simulations in order to illustrate the behavior of the stochastic layer during reconnection. The magnetic field line trajectories are characterized by a constant Hamiltonian (1). We obtain Poincaré plots by numerical integration of the expression

$$\frac{dr}{B_r} = \frac{r d\Theta}{B_\Theta} = \frac{dz}{B_z}.$$

For the following components:

$$B_z = B_z^0 + \frac{\partial \Phi}{\partial z} - \epsilon \frac{\partial \Phi}{\partial z} \cos \Theta$$

$$B_r = \frac{\partial \Phi}{\partial r} - \epsilon \frac{\partial \Phi}{\partial r} \cos \Theta$$

$$B_\Theta = B_\Theta^0 + \frac{\partial \Phi}{\partial \Theta} - \epsilon \frac{\partial \Phi}{\partial \Theta} \cos \Theta$$

where the components of B^0 correspond to the equilibrium plasma in cylindrical approximation, Φ corresponds to the potential of the helical current windings and $\epsilon = r/R$ precedes the toroidal correction terms [9].

In Fig. 3 we can see the Poincaré maps of the system for three values of the parameter I_p, the plasma current, corresponding to evolving stages of the reconnection process. The axes correspond to the normalized radius r/a and the angle Θ.

We can observe in the Poincaré plots shown in the last figure a weak decrease in the stochastic layer as one approximates to the separatrix. In order to estimate with greater accuracy the width of the stochastic layer, we will compute the winding number of the closed trajectories adjacent to the separatrix. The winding number is defined in Ref. [13] and characterizes the frequency of the movement of a closed trajectory. A definite winding number computed for a given initial condition indicates that there is a regular trajectory through this point. On the other hand, non-converging numerical results for a set of continuous initial conditions indicate the existence of a chaotic region in that interval.

In Fig. 4 we can see the inverse of the winding number ρ radial profile computed for initial points along the $\Theta = 2\pi/3$ line, which includes the hyperbolic point of the separatrix, showing the evolving stages of the reconnection process. We can observe in this figure a decrease of the width of the stochastic layer as the reconnection evolves.

The same plot for other initial points along the other hyperbolic points is shown in Fig. 5, here $\Theta = \pi/3$. This winding number profile presents another behavior different from that observed in Fig. 4. We can observe that the points in this profile present a decreasing dispersion as one approximates to the reconnection point, Fig. 5(c). This phenomenon is interpreted as a regularization in the phase space. What we are seeing in this figure is that the points are converging to a devil staircase that is more regular than a stochastic layer [13].

5. FINAL REMARKS

To summarize, we present the reconnection of magnetic island chains in a Tokamak, focusing attention on the behavior of the stochastic separatrix and in the decrease of chaos

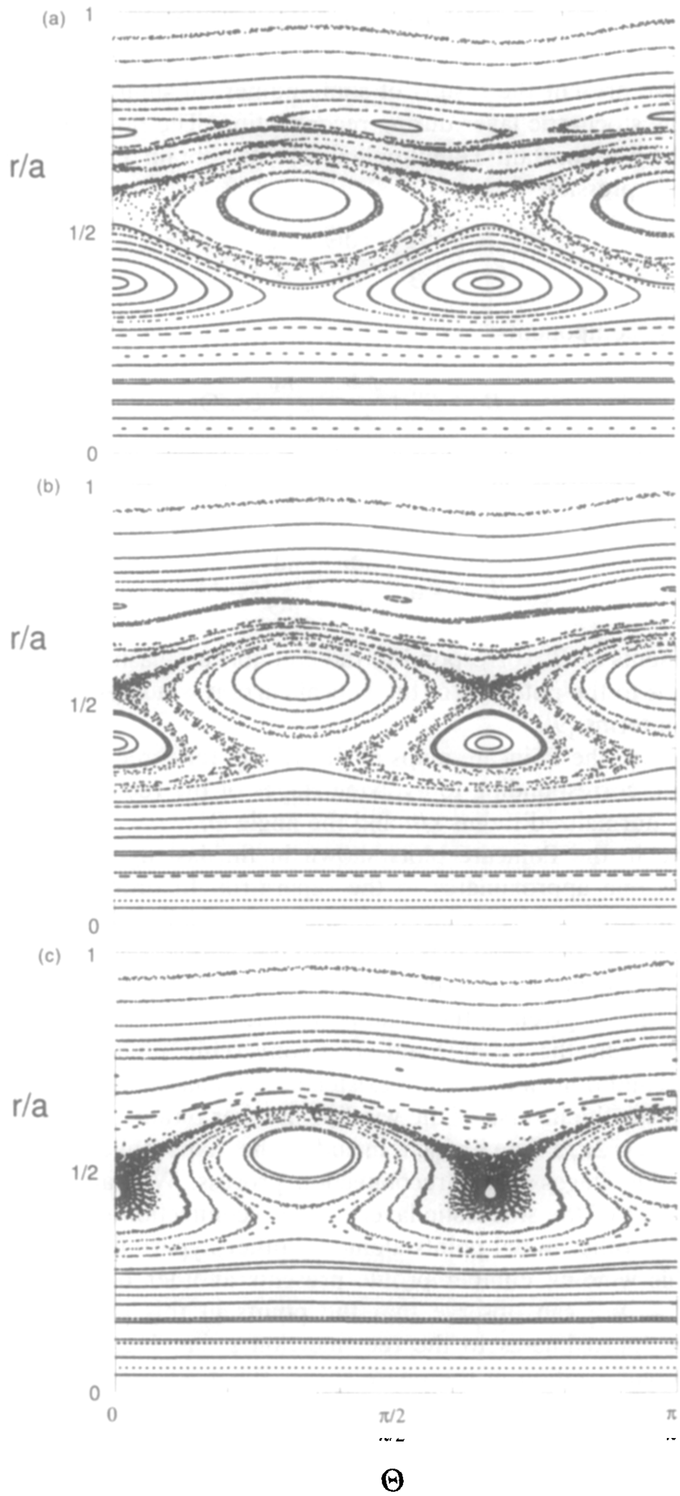

Fig. 3. Poincaré maps of the system for decreasing values of the total plasma current: (a) $I_p = 9300$ A, (b) $I_p = 9200$ A and (c) $I_p = 9120$ A.

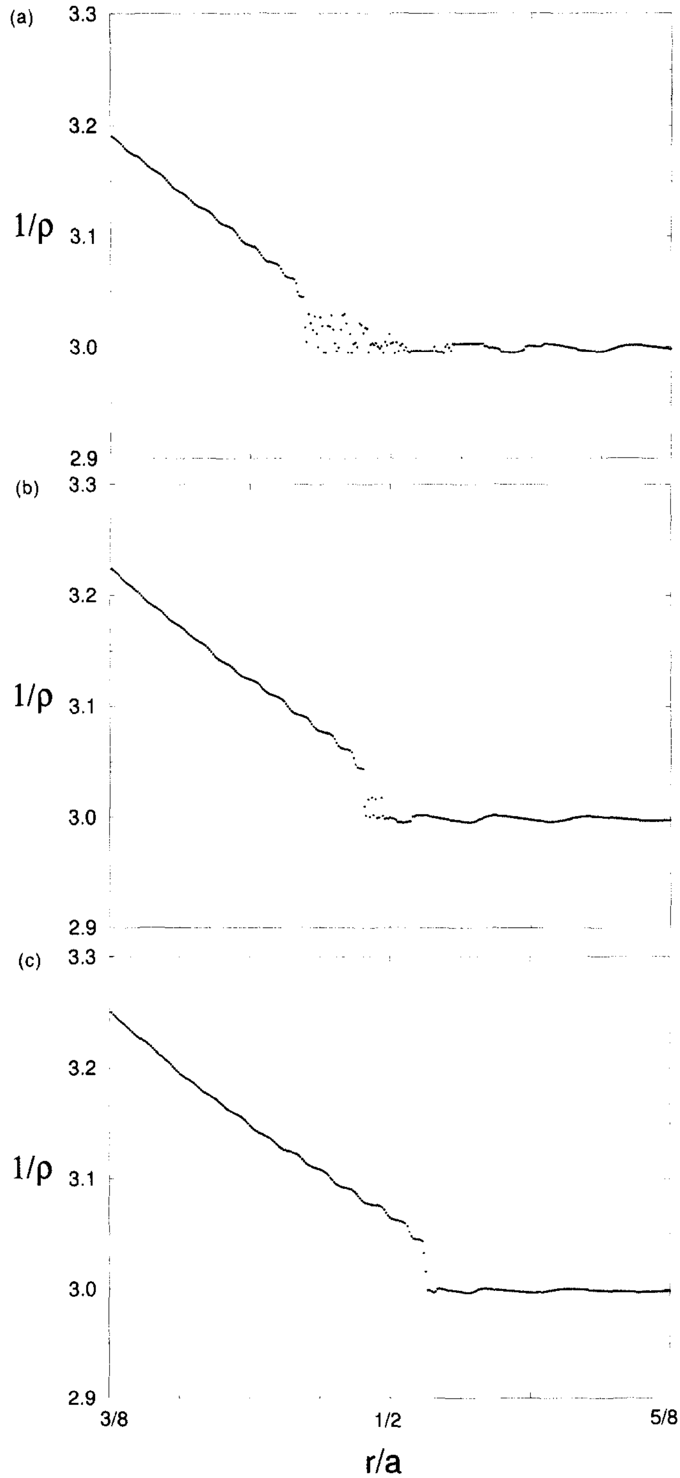

Fig. 4. $1/\rho$ Radial profiles of the system for decreasing values of the total plasma current: (a) $I_p = 9300$ A, (b) $I_p = 9200$ A and (c) $I_p = 9120$ A; $\Theta = 2\pi/3$.

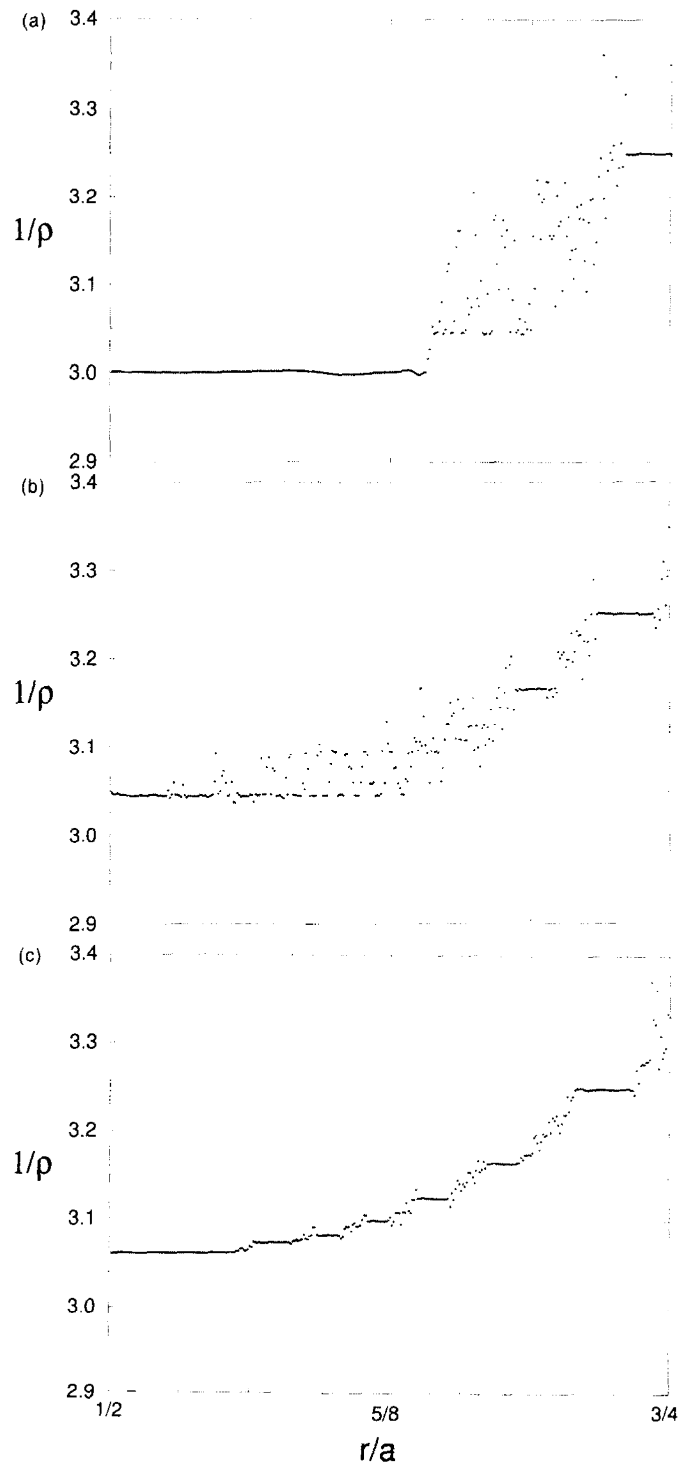

Fig. 5. $1/\rho$ Radial profiles of the system for decreasing values of the total plasma current: (a) $I_p = 9300$ A, (b) $I_p = 9200$ A and (c) $I_p = 9120$ A; $\Theta = \pi/3$.

during the process. Two distinct numerical simulations were done in order to detect the phenomenon of regularization during the reconnection. First we integrate the equations of movement and construct a Poincaré plot. Also, we estimate the winding number, a tool that permits one to observe closely the stochastic layer.

During the reconnection process the amplitude of the chains remains constant, but ρ, the density of chains, diminishes. In the reconnection process a diminution of the neighbor chains works out as a decrease of the resonance overlap among the main resonant chains and their neighborhood. The decrease of the width of the stochastic layer is due to the decrease of overlapping among the resonant chains adjacent to the main resonances that suffer reconnection.

Numerical simulations were performed, and a decrease of chaos was found during the reconnection process in the reversed magnetic shear region. Even if no evident reconnection process takes place, the region of small magnetic shear, minimum for the safety factor profile, presents less resonant chains and develops less chaos than in regions of higher magnetic shear, as can be concluded by eqn (3). So the configuration of non-monotonic profiles suggests an improvement of the confinement in Tokamaks. Some experimental evidence for this point may exist [10, 11]. In fact, in the new improved confinement Tokamak regimes, obtained with reversed magnetic shear, a transport barrier was identified in the small shear region, improving the confinement parameters in the plasma center.

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APPENDIX A

The Chirikov criterion can be found in Refs [4] and [13], and a clear application for a continuous system, analogous to that treated here, in Ref. [16].

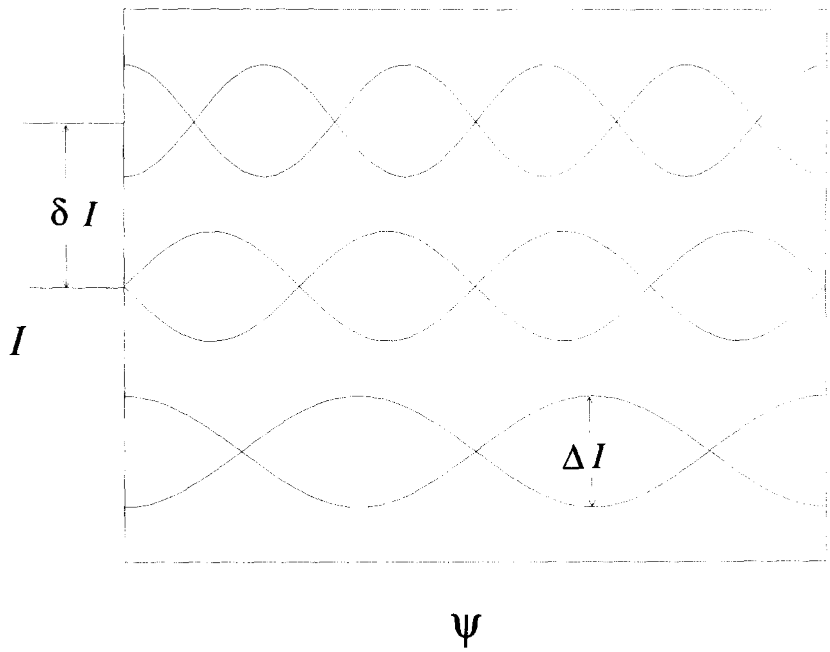

Fig. A1. A sketch of the phase space with three resonant chains, the amplitude of the islands is ΔI and they are separated by δI.

In Fig. A1 we can visualize the most important aspects of the criterion from the situation of two resonant chains with amplitude ΔI and separated by a distance δI. The stochastic parameter η is defined by

$$\eta = \frac{2\Delta I}{\delta I}.$$

One says that the system is strongly chaotic when $\eta > 1$.

Let us estimate the value of η for a Hamiltonian that presents the form

$$H = H_0(I) + \epsilon \sum_{l=-\infty}^{+\infty} H_l(I)\cos(l\phi - \omega t)$$

where $H_0(I)$ is the integrable part of H and $H_l(I)$ are the Fourier modes of the perturbative part. The local Hamiltonian H_p, that describes a single resonance $q = 1$ and any p in eqn (2), is written as

$$H_p = \frac{G}{2}J^2 - F\cos(p\psi) \tag{4}$$

where $G \equiv d^2H_0/dI^2 = d\omega_0/dI$, J is the deviation of the action from the fixed point $I = I_{p/q}$, $F \equiv \epsilon H_p(I)$, and $\psi \equiv (p\phi - \omega t)$.

The Hamiltonian (4) represents a resonant chain, the amplitude of each island is found from [13]

$$\Delta I = 2\sqrt{\frac{F}{G}}.$$

If we suppose that the values of p are large, the distance between two adjacent resonant chains can be found from

$$\delta I = \frac{dI}{d\omega_0}\delta\omega.$$

We will take $\delta\omega = 1$, choosing as adjacent chains the resonances p and $p + 1$.

Using the relations cited above, we finally achieve an estimate for the stochastic parameter

$$\eta = 4\sqrt{\frac{F\,d\omega_0}{dI}\omega}.$$

From this relation we can see that if $d\omega_0/dI \to 0$, then $\eta \to 0$ and the system will never become chaotic. The interpretation is simple: the condition $d\omega_0/dI \to 0$ is equivalent to an increase in the distance between adjacent resonances, that makes difficult the overlapping of resonances.