IMPROVING PARTICLE ACCELERATION IN PLASMAS

Meirielen C. de Sousa, Iberê L. Caldas

Institute of Physics, University of São Paulo

Workshop on Complex Dynamics of Plasmas

July, 26th 2018

MOTIVATION

- Wave-particle interactions are present in many areas

- They are used in several applications for particle heating and *particle acceleration*

  P. K. Shukla *et al.*, *Phys. Rep.* **138**, 1 (1986)
  G. Corso and F. B. Rizzato, *J. Plasma Phys.* **49**, 425 (1993)

- Chaos limits wave parameters for regular particle acceleration, as well as maximum energy achieved by particles

- New methods are required to control chaos and increase the maximum energy of particles

OBJECTIVES

- Improve acceleration from initial energies close to particles rest energy:

  - Control chaos in the low energy region

  - Reduce the initial energy of particles

  - Increase the maximum energy achieved by particles

ORIGINAL SYSTEM

- Relativistic low density beam: particles charge q and rest mass m

- Confined by external uniform magnetic field: $\mathbf{B} = B_0 \hat{z}$, with vector potential $\mathbf{A} = B_0 x \hat{y}$

- Interacting with stationary electrostatic wave given as series of periodic pulses propagating perpendicularly to $\mathbf{B}$: wave vector $\mathbf{k} = k\hat{x}$, period T, amplitude $\varepsilon / 2$

- Low density beam does not alter wave propagation, and its particles do not interact with each other

- Dimensionless Hamiltonian that describes dynamics transverse to $\mathbf{B}$:

$$H(x, p_x, t) = \sqrt{1 + p_x^2 + x^2} + \frac{\varepsilon}{2}\cos(kx)\sum_{n=-\infty}^{+\infty}\delta(t - nT)$$

- Between two consecutive wave pulses, Hamiltonian is integrable and time independent: action-angle variables $x = \sqrt{2I}\,\mathrm{sen}\,\theta$ and $p_x = \sqrt{2I}\cos\theta$

- Hamiltonian in the action-angle variables (I, θ):

$$H(I, \theta, t) = \sqrt{1 + 2I} + \frac{\varepsilon}{2}\cos(k\sqrt{2I}\,\mathrm{sen}\,\theta)\sum_{n=-\infty}^{+\infty}\delta(t - nT)$$

M. C. de Sousa *et al.*, *Phys. Rev.* E **82**, 026402 (2010)
M. C. de Sousa *et al.*, *Phys. Rev.* E **86**, 016217 (2012)

- Symplectic map preserves canonical aspects of Hamilton's equations:

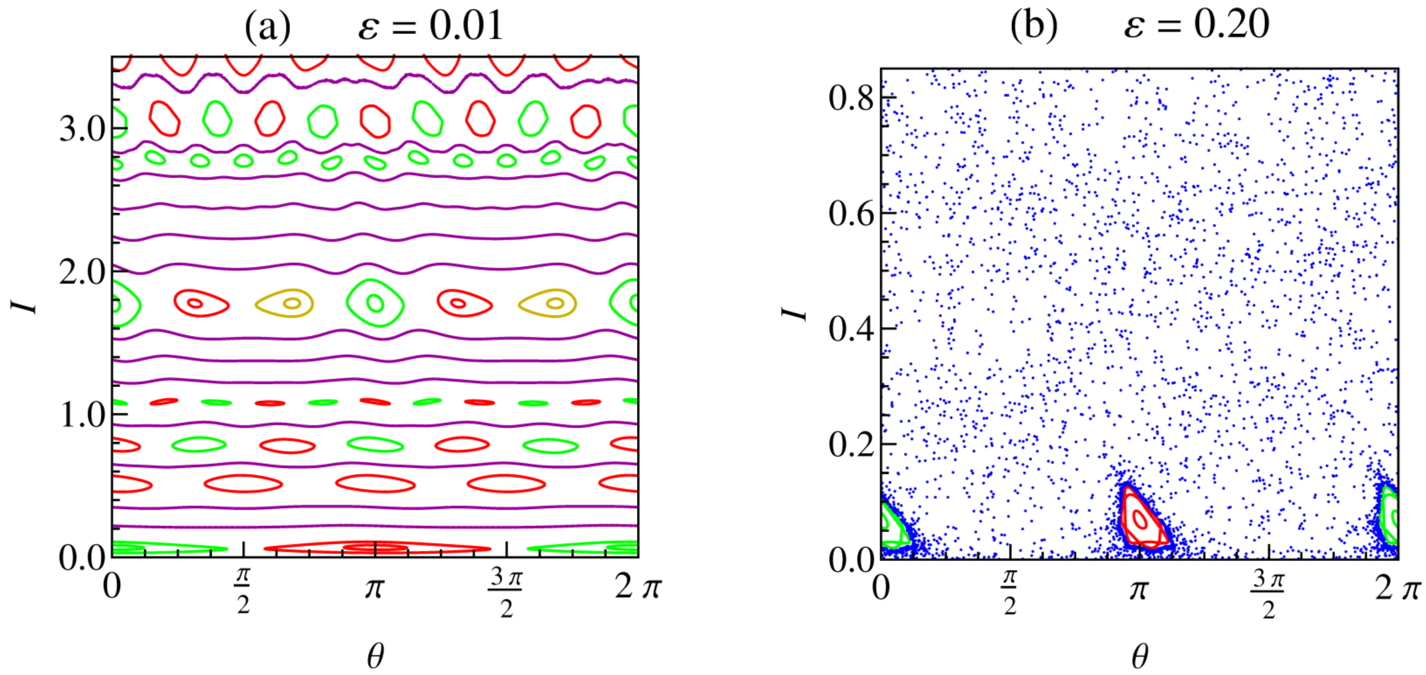
$$I_{n+1} = \frac{1}{2}\left\{2I_n \operatorname{sen}^2\theta_n + \left[\sqrt{2I_n}\cos\theta_n + \frac{1}{2}\varepsilon k \operatorname{sen}(k\sqrt{2I_n}\operatorname{sen}\theta_n)\right]^2\right\}$$

$$\theta_{n+1} = \operatorname{arctg}\left[\frac{2\sqrt{2I_n}\operatorname{sen}\theta_n}{2\sqrt{2I_n}\cos\theta_n + \varepsilon k \operatorname{sen}(k\sqrt{2I_n}\operatorname{sen}\theta_n)}\right] + \frac{T}{\sqrt{1+2I_{n+1}}}$$

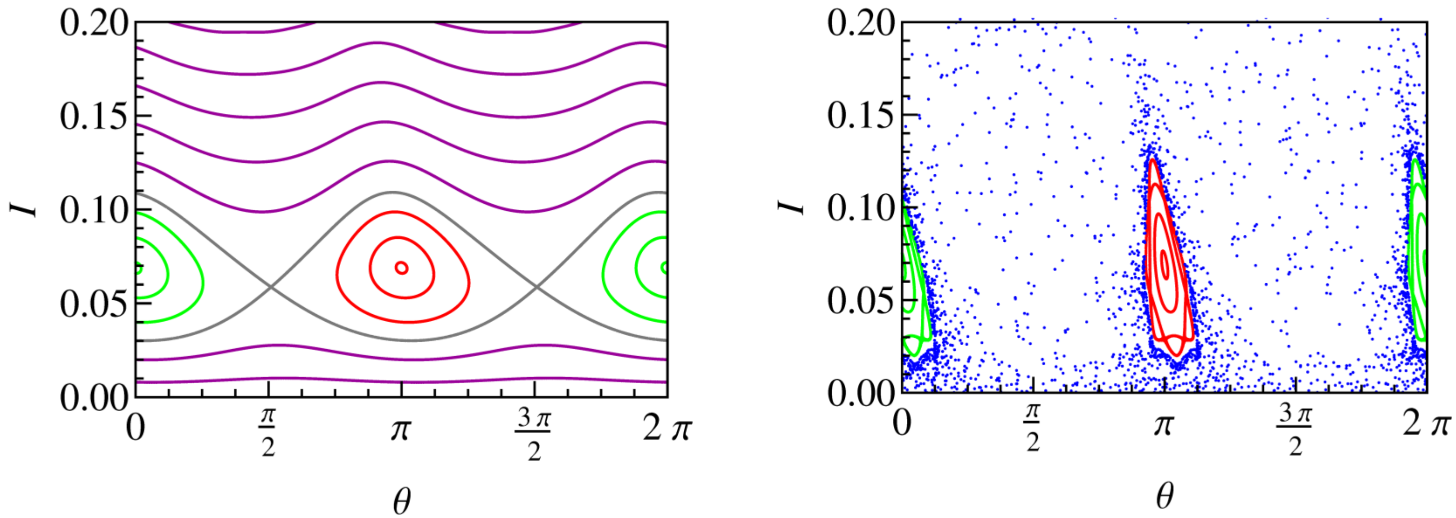
- Map is exact and explicit
- This map can be considered relativistic and magnetized version of classical standard map (Chirikov-Taylor map)

M. C. de Sousa *et al.*, *Phys. Rev.* E **82**, 026402 (2010)
M. C. de Sousa *et al.*, *Phys. Rev.* E **86**, 016217 (2012)

Phase Space



- Amplification around $I = 0$ (particles rest energy):



IMPROVING ACCELERATION

- To achieve condition for optimum acceleration:
  - Reduce initial energy of particles to their rest energy
  - Control chaos to prevent it from destroying resonant islands
- Method to improve particle acceleration: **addition of a perturbing invariant robust barrier (PRB) to the system**
- Perturbing barrier is a robust invariant curve in phase space generated by external perturbation $\rightarrow$ It is not affected by wave parameters, in contrast with KAM tori, which are destroyed by increasing wave amplitudes
- PRB does not alter main structures of phase space
- PRB reduces perturbation caused by the wave and controls chaos around it

- PRB investigated theoretically for fusion devices:
  - controls transport,
  - improves plasma confinement,
  - prevents plasma from reaching and damaging tokamak walls

  Martins *et al.*, *J. Phys.* A **43**, 175501 (2010)
  Martins *et al.*, *Physica* A **390**, 957 (2011)

- PRB observed experimentally in tokamaks:
  - barrier created by alterations in plasma electric field produced by electrodes placed at plasma edge
  - plasma response to resonant magnetic perturbation, generated by external current, produces effects similar to robust barrier

  Marcus *et al.*, *Phys. Plasmas* **15**, 112304 (2008)
  Frerichs *et al.*, *Phys. Plasmas* **19**, 052507 (2012)

- For low density beams, the addition of a typical perturbing robust barrier to the system produces a Hamiltonian:

$$H = \sqrt{1+2I} + \frac{\varepsilon}{2}(I - I_{\mathrm{PRB}})^2 \cos(k\sqrt{2I}\sin\theta)\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$

- I_{PRB}: position of perturbing robust barrier in phase space

- I close to I_{PRB}: effective wave amplitude is reduced

- $I = I_{\mathrm{PRB}}$: perturbation vanishes

M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

- Map that describes time evolution of perturbed system:

$$I_{n+1} = I_n + \frac{\varepsilon k}{2}(I_{n+1} - I_{\mathrm{PRB}})^2 \sqrt{2I_{n+1}} \cos\theta_n \sin(k\sqrt{2I_{n+1}} \sin\theta_n)$$

$$\theta_{n+1} = \theta_n - \frac{\varepsilon k}{2} \frac{(I_{n+1} - I_{\mathrm{PRB}})^2}{\sqrt{2I_{n+1}}} \sin\theta_n \sin(k\sqrt{2I_{n+1}} \sin\theta_n)$$

$$+ \varepsilon(I_{n+1} - I_{\mathrm{PRB}}) \cos(k\sqrt{2I_{n+1}} \sin\theta_n) + \frac{T}{\sqrt{1+2I_{n+1}}}$$

- What is the best position I_{PRB} for perturbing robust barrier?

Initial Energy: $I = 0$

- We determine parameters that bring periodic points to $I = 0$
- Hyperbolic points: $(I_{\mathrm{PRB}})_{\mathrm{hp}} = \dfrac{-\varepsilon + \sqrt{\varepsilon^2 - 2\varepsilon k^2 (2\pi - T)}}{\varepsilon k^2}$
- $I_{\mathrm{PRB}} \geq (I_{\mathrm{PRB}})_{\mathrm{hp}}$: hyperbolic points over $I = 0$. It is possible to accelerate particles from their rest energy
- Elliptic points: $(I_{\mathrm{PRB}})_{\mathrm{ep}} = \dfrac{T - 2\pi}{\varepsilon}$
- $I_{\mathrm{PRB}} > (I_{\mathrm{PRB}})_{\mathrm{ep}}$: resonant islands are no longer in phase space
  To accelerate particles: $I_{\mathrm{PRB}} < (I_{\mathrm{PRB}})_{\mathrm{ep}}$

Period Doubling Bifurcation

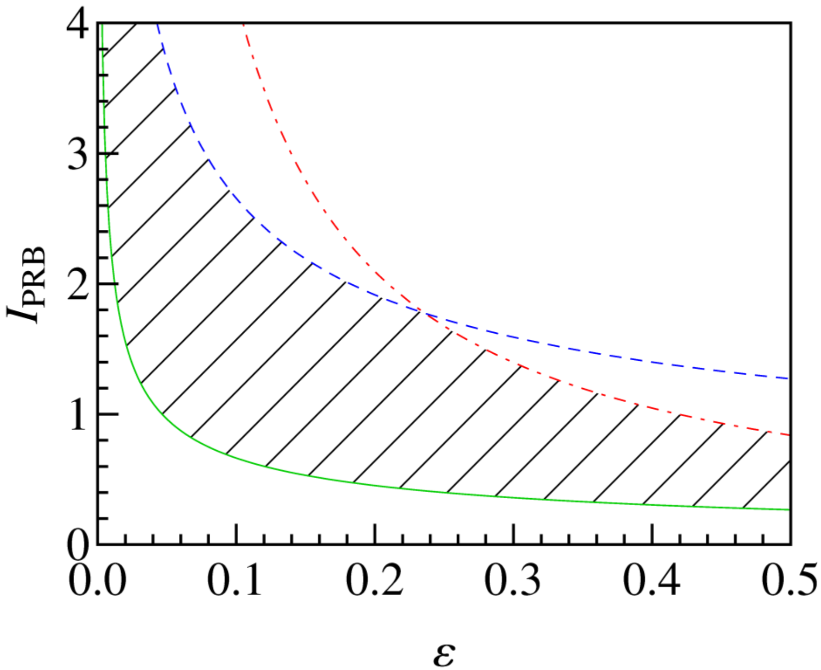
- Resonant islands also disappear from phase space when elliptic points undergo period doubling bifurcation

- Periodic points are elliptic (islands present in phase space) for:

$$I_{\mathrm{PRB}} > (I_{\mathrm{PRB}})_{\mathrm{pdb}} = \frac{T^2}{8\pi^2} - \frac{1}{2} - 2\left[\varepsilon k^2 \left(\frac{T^2}{8\pi^2} - \frac{1}{2}\right)\left(\frac{8\pi^3}{T^2} - \varepsilon\right)\right]^{-1/2}$$

$$I_{\mathrm{PRB}} < (I_{\mathrm{PRB}})_{\mathrm{pdb}} = \frac{T^2}{8\pi^2} - \frac{1}{2} + 2\left[\varepsilon k^2 \left(\frac{T^2}{8\pi^2} - \frac{1}{2}\right)\left(\frac{8\pi^3}{T^2} - \varepsilon\right)\right]^{-1/2}$$

with $I_{\mathrm{PRB}} \geq 0,\ \varepsilon T^2 < 8\pi^3$

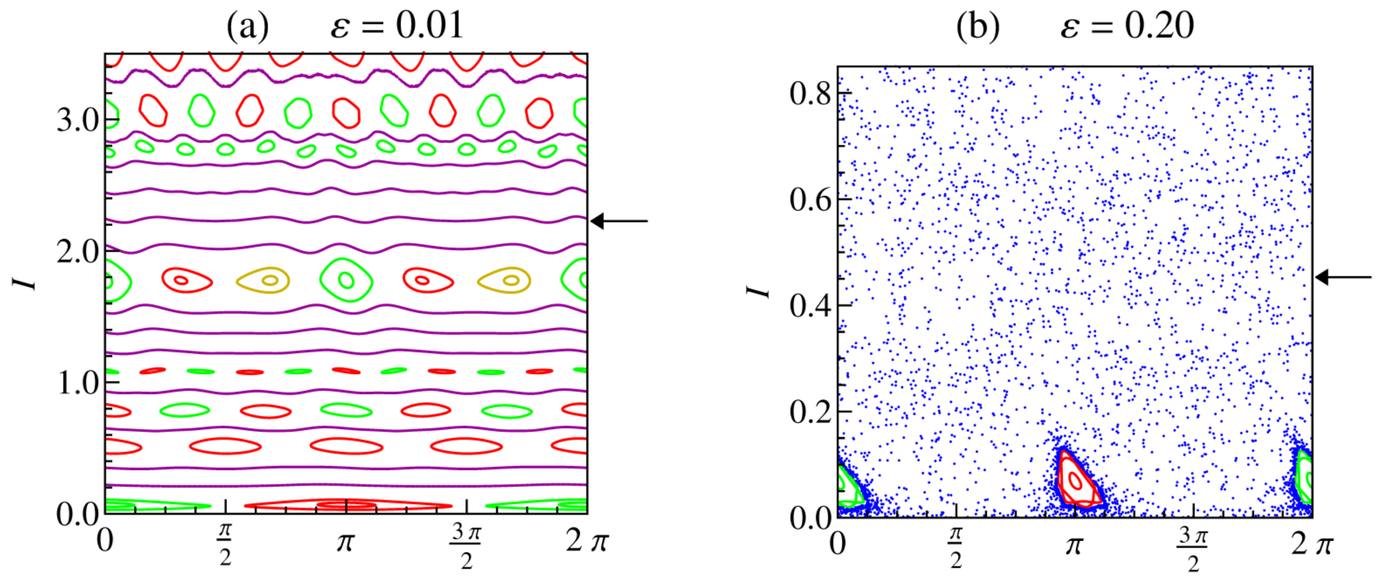


- Solid green curve: hyperbolic points are located on axis $I = 0$
- Dot-dashed red curve: elliptic points move down to $I = 0$
- Dashed blue curve: elliptic points undergo period doubling bifurcation

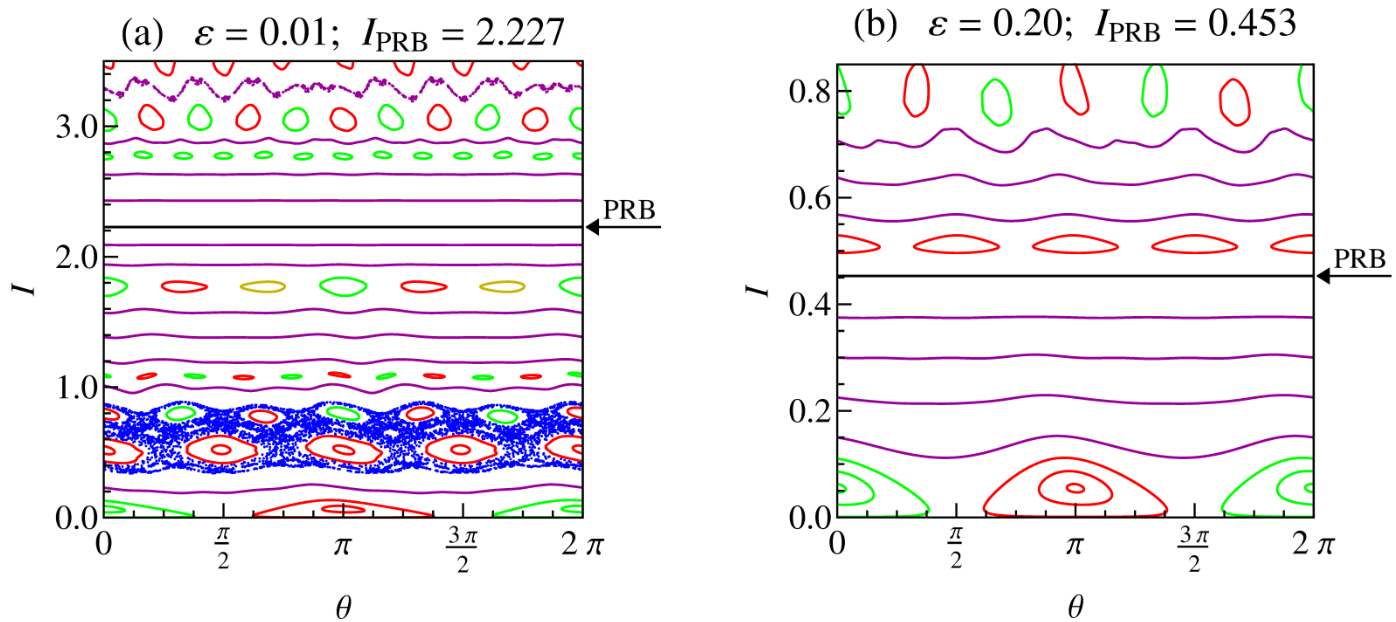
- Hatched area: it is possible to regularly accelerate particles from rest energy
- **PRB best position**: over solid green curve $I_{\mathrm{PRB}} = (I_{\mathrm{PRB}})_{\mathrm{hp}}$. Initial energy close to rest energy. Final energy maximum

M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

Original system:



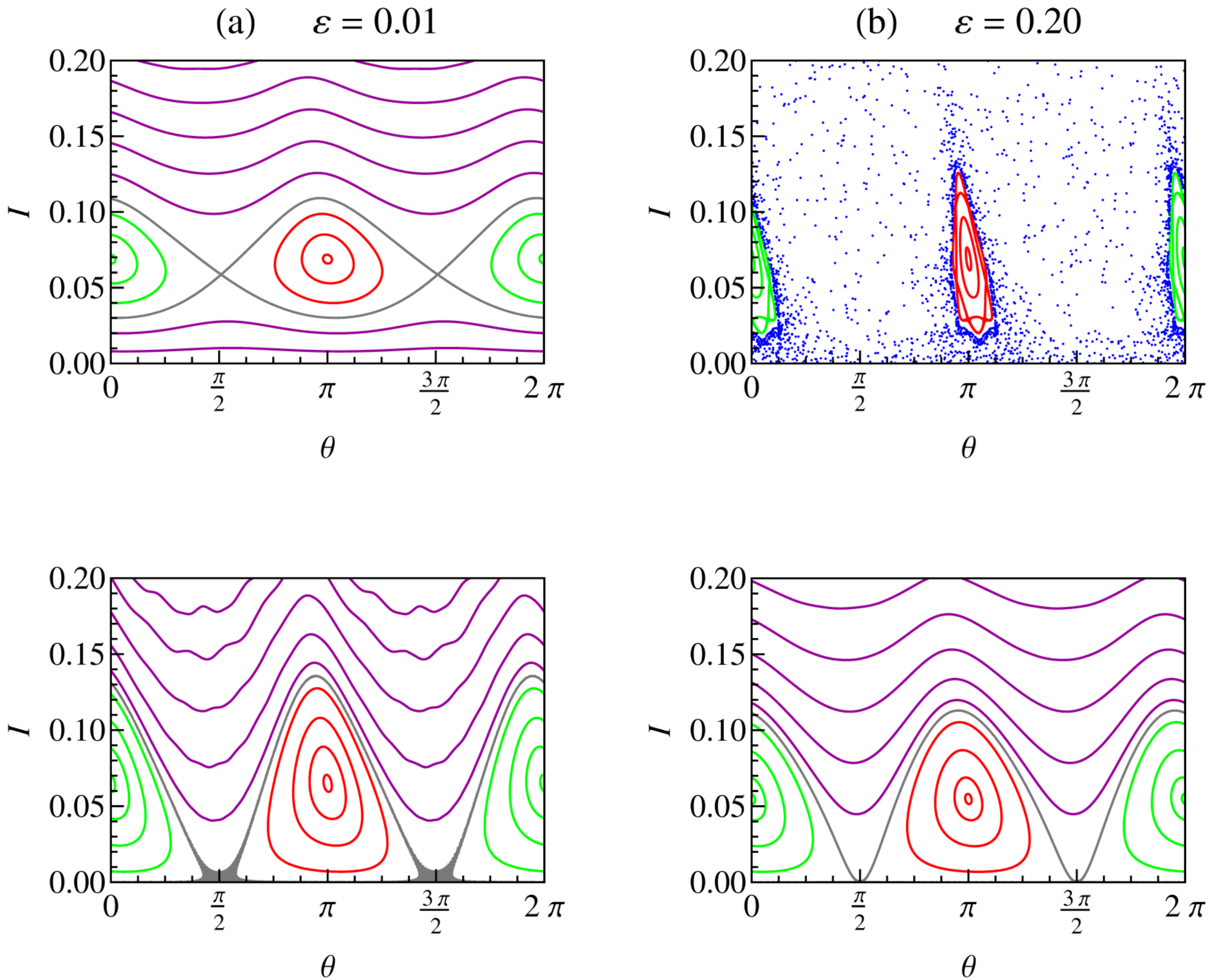
Perturbed system:



M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

Original system:

Perturbed system:



M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

	Wave amplitude $\varepsilon = 0.01$		Wave amplitude $\varepsilon = 0.20$	
	Velocity		Velocity	
	Initial	Final	Initial	Final
Original system:	$0.239c$	$0.423c$	$0.235c$	$0.414c$
Perturbed system:	$0.064c$ 73% lower	$0.453c$ 7% higher	$0.0046c$ 98% lower	$0.427c$ 3% higher

M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

CONCLUSIONS

- Relativistic low density beam confined by uniform magnetic field, perturbed by stationary electrostatic wave
- Optimum condition for particle acceleration is only reached for limited values of wave amplitude
- We introduced a perturbing robust barrier (PRB) in the system to improve particle acceleration
- PRB preserves main structures in phase space, and controls chaos around it
- We obtained parameters interval for possible PRB positions
- We also determined the best PRB position in phase space

- For low amplitude waves, PRB reduces initial energy of particles to their rest energy
- For high amplitude waves, PRB controls chaos and restores particle acceleration from rest energy
- PRB also increases final energy of particles
- Perturbing robust barrier is efficient method to improve particle acceleration in plasmas

M. C. de Sousa and I. L. Caldas, *Phys. Plasmas* **25**, 043110 (2018)

Thank you!