IMPROVING PARTICLE ACCELERATION IN PLASMAS

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• Wave-particle interactions are present in many areas

• They are used in several applications for particle heating and *particle acceleration*
  
  

• Chaos limits wave parameters for regular particle acceleration, as well as maximum energy achieved by particles

• New methods are required to control chaos and increase the maximum energy of particles
OBJECTIVES

- Improve acceleration from initial energies close to particles' rest energy:
  - Control chaos in the low energy region
  - Reduce the initial energy of particles
  - Increase the maximum energy achieved by particles
- Relativistic low density beam: particles charge $q$ and rest mass $m$

- Confined by external uniform magnetic field: $\mathbf{B} = B_0 \hat{z}$, with vector potential $\mathbf{A} = B_0 x \hat{y}$

- Interacting with stationary electrostatic wave given as series of periodic pulses propagating perpendicularly to $\mathbf{B}$: wave vector $\mathbf{k} = k \hat{x}$, period $T$, amplitude $\varepsilon / 2$

- Low density beam does not alter wave propagation, and its particles do not interact with each other
• Dimensionless Hamiltonian that describes dynamics transverse to $B$:

$$H(x, p_x, t) = \sqrt{1 + p_x^2 + x^2} + \frac{\varepsilon}{2}\cos(kx)\sum_{n=-\infty}^{+\infty}\delta(t - nT)$$

• Between two consecutive wave pulses, Hamiltonian is integrable and time independent: action-angle variables $x = \sqrt{2I}\sen\theta$ and $p_x = \sqrt{2I}\cos\theta$

• Hamiltonian in the action-angle variables $(I, \theta)$:

$$H(I, \theta, t) = \sqrt{1 + 2I} + \frac{\varepsilon}{2}\cos(k\sqrt{2I}\sen\theta)\sum_{n=-\infty}^{+\infty}\delta(t - nT)$$

• Symplectic map preserves canonical aspects of Hamilton’s equations:

\[
I_{n+1} = \frac{1}{2} \left\{ 2I_n \sin^2 \theta_n + \left[ \sqrt{2I_n \cos \theta_n} + \frac{1}{2} \varepsilon k \sin(k \sqrt{2I_n} \sin \theta_n) \right]^2 \right\}
\]

\[
\theta_{n+1} = \arctg \left[ \frac{2 \sqrt{2I_n \sin \theta_n}}{2 \sqrt{2I_n \cos \theta_n} + \varepsilon k \sin(k \sqrt{2I_n} \sin \theta_n)} \right] + \frac{T}{\sqrt{1 + 2I_{n+1}}}
\]

• Map is exact and explicit

• This map can be considered relativistic and magnetized version of classical standard map (Chirikov-Taylor map)

- Amplification around $I = 0$ (particles rest energy):
To achieve condition for optimum acceleration:
- Reduce initial energy of particles to their rest energy
- Control chaos to prevent it from destroying resonant islands

Method to improve particle acceleration: addition of a perturbing invariant robust barrier (PRB) to the system

Perturbing barrier is a robust invariant curve in phase space generated by external perturbation → It is not affected by wave parameters, in contrast with KAM tori, which are destroyed by increasing wave amplitudes

PRB does not alter main structures of phase space

PRB reduces perturbation caused by the wave and controls chaos around it
PRB investigated theoretically for fusion devices:
- controls transport,
- improves plasma confinement,
- prevents plasma from reaching and damaging tokamak walls

Martins et al., Physica A 390, 957 (2011)

PRB observed experimentally in tokamaks:
- barrier created by alterations in plasma electric field produced by electrodes placed at plasma edge
- plasma response to resonant magnetic perturbation, generated by external current, produces effects similar to robust barrier

For low density beams, the addition of a typical perturbing robust barrier to the system produces a Hamiltonian:

\[ H = \sqrt{1+2I} + \frac{\varepsilon}{2} (I - I_{\text{PRB}})^2 \cos(k\sqrt{2I}\sin \theta) \sum_{n=-\infty}^{+\infty} \delta(t-nT) \]

- \( I_{\text{PRB}} \): position of perturbing robust barrier in phase space
- \( I \) close to \( I_{\text{PRB}} \): effective wave amplitude is reduced
- \( I = I_{\text{PRB}} \): perturbation vanishes

Map that describes time evolution of perturbed system:

\[
I_{n+1} = I_n + \frac{\varepsilon k}{2} (I_{n+1} - I_{\text{PRB}})^2 \sqrt{2I_{n+1}} \cos \theta_n \sin(k \sqrt{2I_{n+1}} \sin \theta_n)
\]

\[
\theta_{n+1} = \theta_n - \frac{\varepsilon k}{2} \frac{(I_{n+1} - I_{\text{PRB}})^2}{\sqrt{2I_{n+1}}} \sin \theta_n \sin(k \sqrt{2I_{n+1}} \sin \theta_n)
\]

\[+ \varepsilon (I_{n+1} - I_{\text{PRB}}) \cos(k \sqrt{2I_{n+1}} \sin \theta_n) + \frac{T}{\sqrt{1 + 2I_{n+1}}}
\]

- What is the best position \( I_{\text{PRB}} \) for perturbing robust barrier?
We determine parameters that bring periodic points to $I = 0$

Hyperbolic points: $(I_{PRB})_{hp} = -\varepsilon + \sqrt{\varepsilon^2 - 2\varepsilon k^2(2\pi - T)} \varepsilon k^2$

$I_{PRB} \geq (I_{PRB})_{hp}$: hyperbolic points over $I = 0$. It is possible to accelerate particles from their rest energy

Elliptic points: $(I_{PRB})_{ep} = \frac{T - 2\pi}{\varepsilon}$

$I_{PRB} > (I_{PRB})_{ep}$: resonant islands are no longer in phase space

To accelerate particles: $I_{PRB} < (I_{PRB})_{ep}$
- Resonant islands also disappear from phase space when elliptic points undergo period doubling bifurcation

- Periodic points are elliptic (islands present in phase space) for:

\[
I_{\text{PRB}} > (I_{\text{PRB}})_{\text{pdb}} = \frac{T^2}{8\pi^2} - \frac{1}{2} - 2 \left[ \varepsilon k^2 \left( \frac{T^2}{8\pi^2} - \frac{1}{2} \right) \left( \frac{8\pi^3}{T^2} - \varepsilon \right) \right]^{-1/2}
\]

\[
I_{\text{PRB}} < (I_{\text{PRB}})_{\text{pdb}} = \frac{T^2}{8\pi^2} - \frac{1}{2} + 2 \left[ \varepsilon k^2 \left( \frac{T^2}{8\pi^2} - \frac{1}{2} \right) \left( \frac{8\pi^3}{T^2} - \varepsilon \right) \right]^{-1/2}
\]

with \( I_{\text{PRB}} \geq 0, \ \varepsilon T^2 < 8\pi^3 \)
Hatched area: it is possible to regularly accelerate particles from rest energy.

**PRB best position:** over solid green curve $I_{\text{PRB}} = (I_{\text{PRB}})_{\text{hp}}$. Initial energy close to rest energy. Final energy maximum.

<table>
<thead>
<tr>
<th>Wave amplitude $\varepsilon = 0.01$</th>
<th>Wave amplitude $\varepsilon = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original system:</td>
<td>Perturbed system:</td>
</tr>
<tr>
<td>Velocity</td>
<td>Velocity</td>
</tr>
<tr>
<td>Initial</td>
<td>Initial</td>
</tr>
<tr>
<td>Final</td>
<td>Final</td>
</tr>
<tr>
<td>$0.239c$</td>
<td>$0.064c$</td>
</tr>
<tr>
<td>$0.423c$</td>
<td>$0.453c$</td>
</tr>
<tr>
<td><strong>73% lower</strong></td>
<td><strong>98% lower</strong></td>
</tr>
<tr>
<td>$0.235c$</td>
<td>$0.0046c$</td>
</tr>
<tr>
<td>$0.414c$</td>
<td>$0.427c$</td>
</tr>
<tr>
<td><strong>3% higher</strong></td>
<td><strong>7% higher</strong></td>
</tr>
</tbody>
</table>

CONCLUSIONS

- Relativistic low density beam confined by uniform magnetic field, perturbed by stationary electrostatic wave

- Optimum condition for particle acceleration is only reached for limited values of wave amplitude

- We introduced a perturbing robust barrier (PRB) in the system to improve particle acceleration

- PRB preserves main structures in phase space, and controls chaos around it

- We obtained parameters interval for possible PRB positions

- We also determined the best PRB position in phase space
• For low amplitude waves, PRB reduces initial energy of particles to their rest energy

• For high amplitude waves, PRB controls chaos and restores particle acceleration from rest energy

• PRB also increases final energy of particles

• Perturbing robust barrier is efficient method to improve particle acceleration in plasmas

Thank you!