PGF 5005 - Mecânica Clássica

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Respostas - Primeira Lista de Exercícios

1.

(a)

$$L = \frac{m}{2}[(v_0 + at)(v_0 + at + 2l\dot{\theta}\cos\theta) + l^2\dot{\theta}^2] + mgl\cos\theta$$
$$H = \frac{p^2}{2ml} + (v_0 + at)\left[\frac{m}{2}\left((v_0 + at)\cos^2\theta - 1\right) - \frac{p}{l}\cos\theta\right] - mgl\cos\theta$$

(b)

$$\ddot{\theta} + \frac{g}{l}\sin\theta + \frac{a}{l}\cos\theta = 0$$

(c)

$$\theta_e = \tan^{-1}\left(-\frac{a}{g}\right)$$

(e)

$$\ddot{\theta} + \frac{(g+a)}{l}\sin\theta = 0$$

3.

(a)

$$L = \frac{m\dot{q}^2}{2} - \frac{\omega^2 q^2}{2} + \frac{Aq^3}{3}$$
$$H = \frac{p^2}{2m} + \frac{\omega^2 q^2}{2} - \frac{Aq^3}{3}$$

(b)

$$\frac{dH}{dt} = 0$$

4.

(a)

$$\{Q, P\} = 1 \quad \Rightarrow \alpha = \frac{1}{2},\ \beta = 2$$

(b)

$$F_3 = -\frac{Q^2\tan(2p)}{2}$$

5.

Usando uma função geratriz do tipo,

$$F(q_1, p_2, P_1, P_2) = F_1 + q_2 p_2 - Q_1 P_1 - Q_2 P_2$$

encontra-se

$$F = q_1(P_1 + 2p_2) + P_2 p_2$$

8.

(b)

$$I = \frac{2a\sqrt{2mE}}{\pi}, \quad \theta = \begin{cases} -\dfrac{\pi}{2} + \dfrac{\pi^2 I}{4ma^2}t, \ 2nT < t < (2n+1)T \\ \\ \dfrac{\pi}{2} - \dfrac{\pi^2 I}{4ma^2}t, \ (2n+1)T < t < (2n+2)T \end{cases} \quad \text{onde } \ T = 2a/v_0 \ \text{ e } \ n = 0, 1, ...$$

(d)

$$H = \frac{p^2}{2m}, \qquad H = \frac{\pi^2 I^2}{8ma^2}$$

(e)

$$x = \frac{2a}{\pi}\theta, \qquad p = \begin{cases} \dfrac{\pi}{2a}I, \ 2nT < t < (2n+1)T \\ \\ -\dfrac{\pi}{2a}I, \ (2n+1)T < t < (2n+2)T \end{cases} \quad \text{onde } \ T = 2a/v_0 \ \text{ e } \ n = 0, 1, ...$$

11.
(a)

$$\theta_1(t) = \theta_1^0 + (1 - 2I_1^0 - 3I_2^0)t, \qquad \theta_2(t) = \theta_2^0 + (1 - 3I_1^0 + 2I_2^0)t$$