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Suppressing grazing chaos in impacting system by structural nonlinearity

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Abstract

In this note we investigate the influence of structural nonlinearity of a simple cantilever beam impacting system on its dynamic responses close to grazing incidence by a means of numerical simulation. To obtain a clear picture of this effect we considered two systems exhibiting impacting motion, where the primary stiffness is either linear (piecewise linear system) or nonlinear (piecewise nonlinear system). Two systems were studied by constructing bifurcation diagrams, basins of attractions, Lyapunov exponents and parameter plots. In our analysis we focused on the grazing transitions from no impact to impact motion. We observed that the dynamic responses of these two similar systems are qualitatively different around the grazing transitions. For the piecewise linear system, we identified on the parameter space a considerable region with chaotic behaviour, while for the piecewise nonlinear system we found just periodic attractors. We postulate that the structural nonlinearity of the cantilever impacting beam suppresses chaos near grazing.

1. Introduction

Impacts between moving parts of a system or device occur frequently and are generally perceived as undesired effects compromising reliability and safety. Examples include ships colliding with fenders [1], offshore mooring buoys [2], rotor systems with clearances [3], heat exchangers pipes colliding with loose supports [4], rattling of gear boxes [5], print hammers [6], vibro-impact moling systems [7], and atomic force microscopy in tapping mode [8]. For all these systems a better understanding of their dynamics may help to reduce the negative effects of impacts and ultimately to improve practical designs.

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In recent years the impacting systems have attracted a significant attention. Notable contributions have been made by Feigin [9], Peterka [10], Blazejczyk-Okolewska and Kapitaniak [11], Nordmark [12], Wiercigroch [13], di Bernardo et al. [14], and Pavlovskaia et al. [15], to name a few.

In a real system it is very likely that not one but a few nonlinearities will interact and dictate its dynamic behaviour. These nonlinearities do not have to be large to lead to significant errors in both quantitative and qualitative predictions if one of the nonlinear terms is not accounted for. Consider for example a simple cantilever beam with a limit stop at the end. This was preliminary studied in [16,17]. It has been shown that the cumulative effect of both nonlinearities (impact and bending of the beam), out-weights the contributing effects from each nonlinearity [16]. At the end of this paper the grazing bifurcations were only mentioned. Therefore, in this work we aim to investigate some dynamical effects resulting from the interactions between nonlinearities around the grazing point. For this purpose, we consider the same oscillators as in [16], a cantilever beam system with impacts, comparing the dynamics of a piecewise linear system (PWL) to a piecewise nonlinear system (PWN).

This paper is organised as follows. In Section 2 we present the model and the equations of motion for the cantilever beam system with impacts [16]. Section 3 explores some aspects of the system dynamics by numerical simulation, investigating parameter space region around the grazing bifurcations.

2. Impacting cantilever beam system

The considered cantilever impacting beam system is shown schematically in Fig. 1a. It is comprised of a lump mass M supported by two leaf springs of length L and a linear spring of stiffness k. The mass and the linear spring are separated by a gap g. The excitation force acting on the mass M is described by a harmonic function, $F_0 \cos \omega t$. The restoring forces for piecewise linear (PWL) and piecewise nonlinear (PWN) systems are illustrated in Fig. 1b. The equation of motion is given by

$$M\ddot{x} + c\dot{x} + f(x) + kH(x)(x - g) = F_0 \cos \omega t, \tag{1}$$

where is $H(x)$ is the Heaviside step function described as

$$H(x) = \begin{cases} 0; & x < g, \\ 1; & x \geqslant g. \end{cases} \tag{2}$$

As was described in [16], the restoring force in the primary stiffness $f(x)$ for the piecewise linear and piecewise nonlinear systems are given by $\frac{12EI}{L^3}x$ and $\frac{12EI}{L^3}x + \frac{432EI}{35L^5}x^3$ respectively.

It is more convenient for purpose of numerical simulations to work with non-dimensional equations. Thus, one can rewrite the equation of motion in the following form:

$$\ddot{y} + 2\xi\dot{y} + f(y) + \alpha H(y)(y - \tilde{g}) = \beta \cos \Omega\tau, \tag{3}$$

where $y = \frac{x}{L}$, $\tau = \omega_0 t$, $\xi = \frac{c}{2\omega_0 M}$, $\beta = \frac{F_0}{\omega_0^2 ML}$, $\Omega = \frac{\omega}{\omega_0}$, $\alpha = \frac{kL^3}{12EI}$, $\tilde{g} = \frac{g}{L}$, and $\omega_0 = \sqrt{\frac{12EI}{ML^3}}$. The function $f(y)$ for the piecewise linear and nd for the piecewise nonlinear systems are given by y and $y + \frac{36}{35}y^3$ respectively.

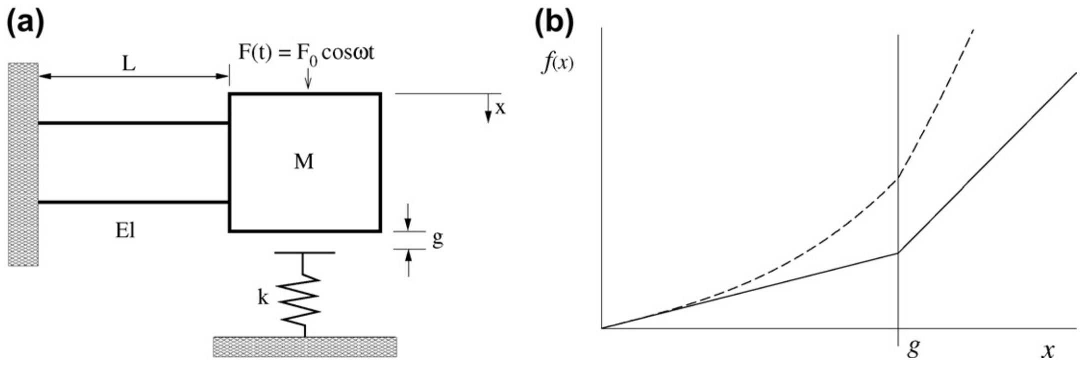

Fig. 1. (a) A cantilever beam system with impacts; (b) the restoring force of piecewise linear system is marked by solid line, $f(x) = \frac{12EI}{L^3}x + kH(x)(x - g)$; the restoring force of piecewise nonlinear system follows dash line, $f(x) = \frac{12EI}{L^3}x + \frac{432EI}{35L^5}x^3 + kH(x)(x - g)$.

3. Dynamics of piecewise linear and piecewise nonlinear systems

The system dynamics was explored using bifurcation diagrams, phase portraits, Lyapunov exponents, basins of attraction and parameter space plots. Numerical simulations were performed by using the fourth-order Runge–Kutta method with a fixed step but ensuring high computational accuracy. The system parameter values were fixed at $\xi = 0.02$, $\beta = 0.5$ and $\Omega = 0.417893$. As control parameter, the stiffness of linear spring, α, and the length gap, $\tilde{g}$, between the mass and the spring, were chosen.

Initially, considering the system without impacts, for the chosen parameters, both PWL and PWN exhibit a period-1 orbit with the same oscillation amplitude. As representative examples of the typical dynamics generated by the impacting cantilever beam system, we show in Fig. 2a–d bifurcation diagrams for the local maximum displacement versus gap $\tilde{g}$. By decreasing the value of parameter $\tilde{g}$, we can observe the grazing bifurcations occurring at $\tilde{g}_c = 0.605646$, which is shown in Figs. 2a and c. For the piecewise linear case (Fig. 2a), a period-1 orbit without impacts ($\tilde{g} > \tilde{g}_c$), just after the grazing bifurcation, becomes a chaotic orbit with impacts and for the piecewise nonlinear case, a period-1 orbit becomes period-3 with impacts (Fig. 2c). In Figs. 2b and d, increasing the control parameter and comparing these figures to the previous diagrams, we can note hysteresis zones close of the grazing bifurcations. Consequently, there is co-existence of attractors in these regions of the parameter space. Fig. 3a and b shows the phase portraits of two co-existing attractors (a period-1 orbit without impacts and a chaotic orbit with impacts) for the PWL, while in Fig. 3c and d (a period-1 orbit without impacts and a period-3 with impacts) the PWN system is considered. The corresponding basins of attraction for PWL and PWN are depicted in Fig. 4a and b, respectively. These figures are constructed using a grid of equally spaced 400×400 pixels as set of initial conditions for velocity, $\dot{y}_0$, and displacement, y_0, with initial time fixed at $\tau_0 = 0$. In both cases, the basins have smooth boundaries. However, the basins of attractors obtained for the PWL (Fig. 4a) are more complex with apparent band accumulation [18].

In order to obtain a further insight into the influence of the control parameters $\tilde{g}$ and α (α is the stiffness ratio) on the dynamics of the cantilever beam system around the grazing bifurcation region, we construct parameter space plots in Fig. 5a and b. In this case, we use a grid of 300×300 cells with the same initial conditions fixed at $(y_0, \dot{y}_0, \tau_0) = (0, 0, 0)$.

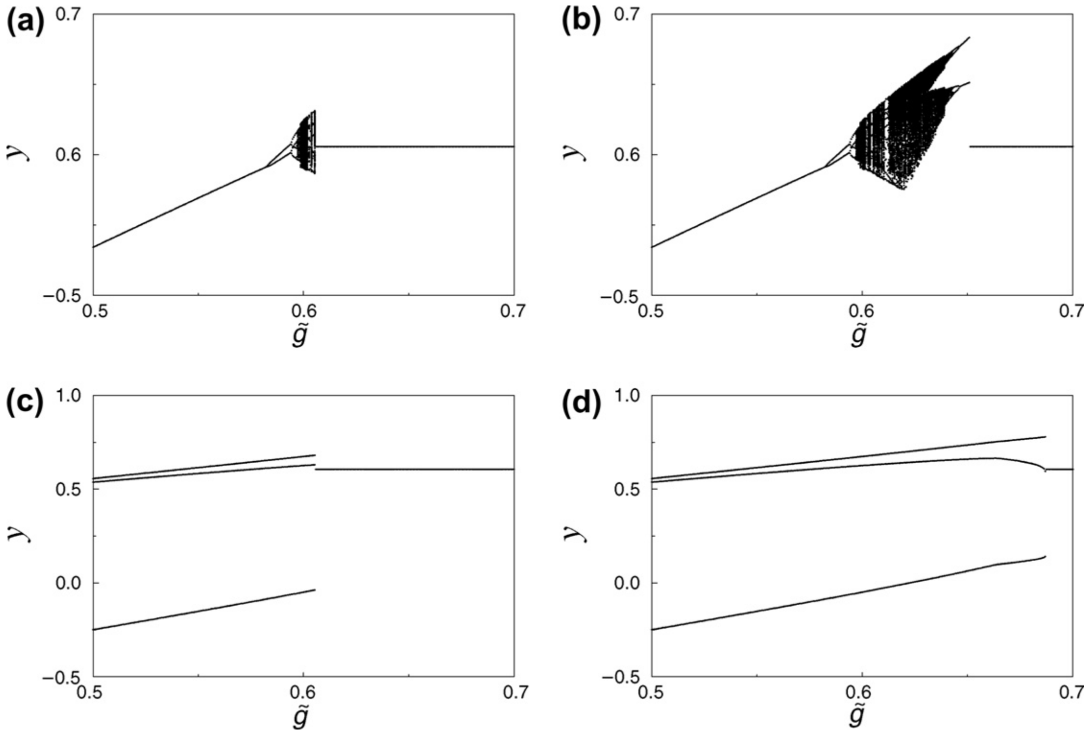

Fig. 2. Bifurcation diagrams of the local maximum of the displacement, y, as a function the non-dimensional gap, $\tilde{g}$, for the value of stiffness $\alpha = 20.0$. (a) and (b) show the diagrams of the piecewise linear system for decrease and increase values of the parameter control, respectively, while (c) and (d) depict diagrams of the piecewise nonlinear system.

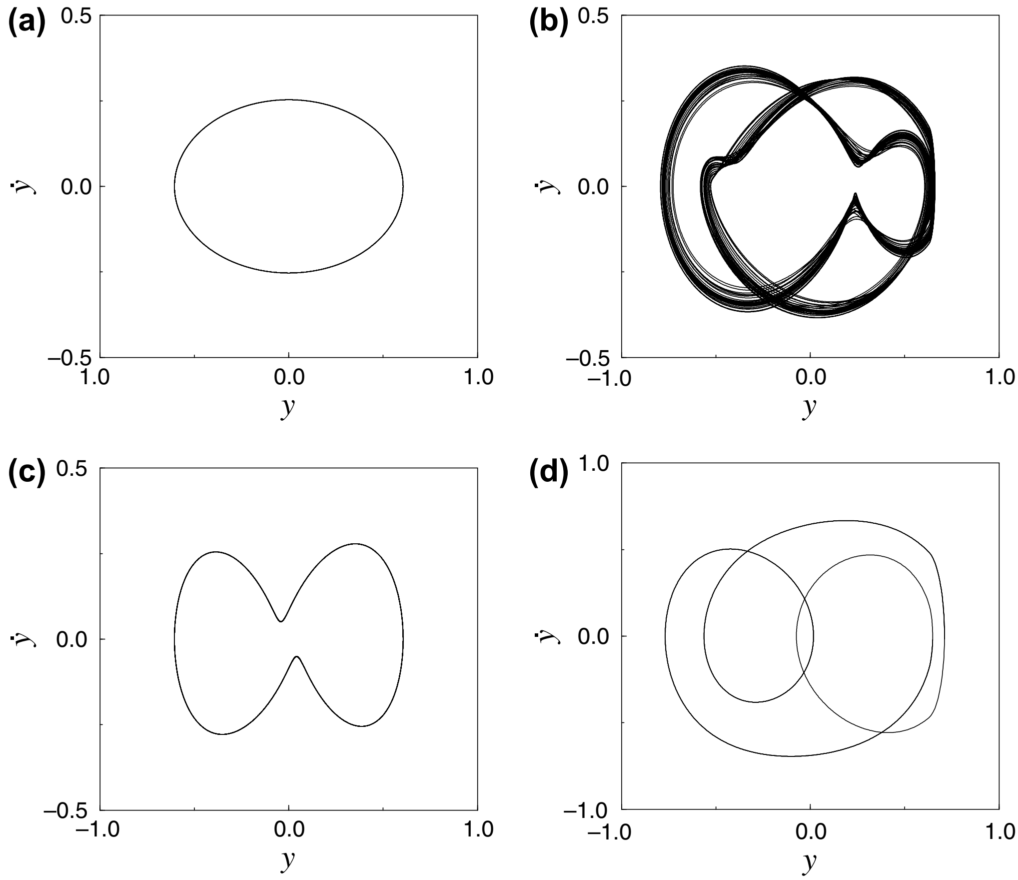

Fig. 3. Phase portraits for control parameters $\tilde{g} = 0.63$ and $\alpha = 20.0$. For piecewise linear system we have two co-existing attractors, a periodic without impacts (a) and an chaotic with impacts (b). For piecewise nonlinear system two co-existing periodic attractors occur, without impacts (c) and with impacts (d).

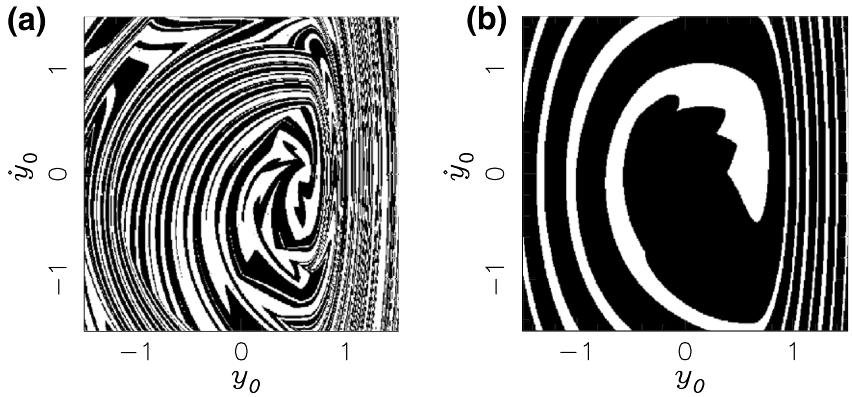

Fig. 4. (a) Basins of attraction of the attractors shown in Fig. 3a (white) and b (black); (b) for the attractors shown in Fig. 3c (white) and d (black).

For each point the largest Lyapunov exponent is calculated and plotted with the appropriately allocated colour from linear grade grey scale. The vertical bars on the of Fig. 5a and b mark the strength of the Lyapunov exponents, which were computed using the algorithm described in the Ref. [19].

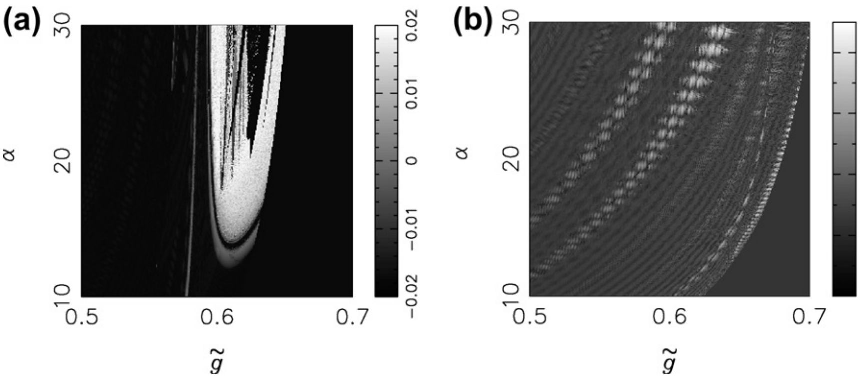

Fig. 5. The largest Lyapunov exponents of the piecewise linear system (a) and the piecewise nonlinear system (b) for the stiffness α versus the gap $\tilde{g}$ with $(y_0, \dot{y}_0, \tau_0) = (0, 0, 0)$.

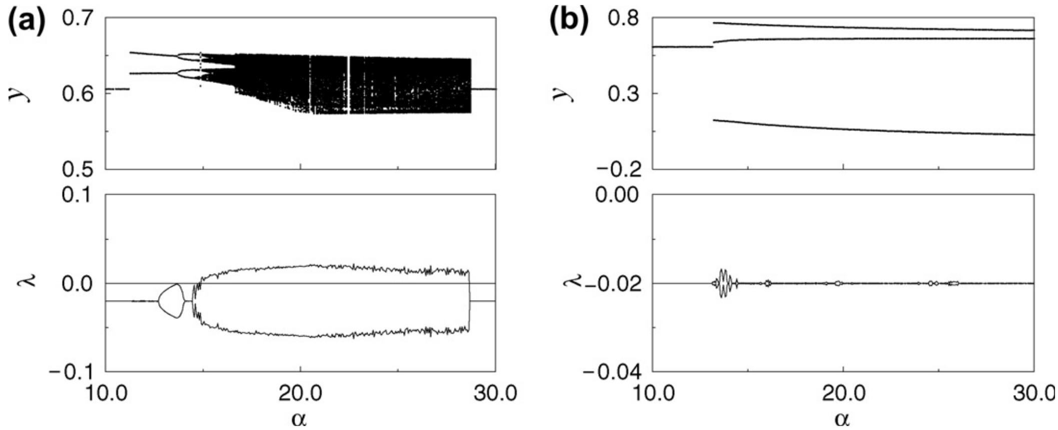

Fig. 6. Bifurcation diagrams (above) and the corresponding Lyapunov exponents (below): (a) for piecewise linear system calculated for the same initial conditions as Fig. 5a and $\tilde{g} = 0.62$; (b) piecewise nonlinear system calculated for the same initial conditions as Fig. 5b and $\tilde{g} = 0.65$.

In Fig. 5a computed for the PWL, as shown in the parameter space, the chaotic regimes (white region) are predominant around the grazing bifurcation point, $\tilde{g} = 0.605646$. Some interesting feature of the parameter space diagram can be found by examining bifurcation diagrams and Lyapunov exponents. For example, varying α with $\tilde{g} = 0.62$, we determined bifurcation diagrams and Lyapunov exponents in Fig. 6a, respectively. From Fig. 6a, we clearly see the period-1 orbit without impacts. By increasing the value of control parameter α, there is a period-2 orbit with impacts and a sequence of period-doubling bifurcations leading to chaos. Then, we have again the period-1 orbit without impacts.

For the PWN, by examining the parameter space shown in Fig. 5b, we can identify only periodic responses. The figure is comprised of a homogeneous region corresponding to the period-1 orbit without impacts (Fig. 3c), and of a heterogeneous region, that corresponds to a period-3 orbit with impacts. In Fig. 6b, we present a bifurcation diagram and the Lyapunov exponents of these period-1 and period-3 orbits.

4. Conclusions

The work reported in [16,17] was followed in the current study, where we investigated by a means of numerical simulations the influence of structural nonlinearity of a simple cantilever beam impacting system on its dynamic responses close to grazing incidence. We were particularly interested how impacts and a small continuous nonlinearity interact with each other. Hence to obtain a clear picture of this effect we considered two systems exhibiting impacting motion, where the primary stiffness is linear (piecewise linear system) and nonlinear (piecewise nonlinear system). Both systems were modelled by piecewise linear and piecewise nonlinear ordinary differential equations and solved numerically using the fourth-order Runge–Kutta method. Two systems were studied by constructing corresponding bifurcation diagrams, basins of attractions, Lyapunov exponents and parameter plots. We focussed our analysis on the grazing transitions from no impact to impact motions. We observed that dynamic responses of these two similar systems are qualitatively different around the grazing transitions. Both systems experienced hysteretic behaviour. For the piecewise linear system, we identified on the parameter space a considerable region with chaotic behaviour, while for the piecewise nonlinear system we found just periodic attractors. We have postulated that the structural nonlinearity of the cantilever impacting beam suppresses chaos near grazing.

Acknowledgements

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