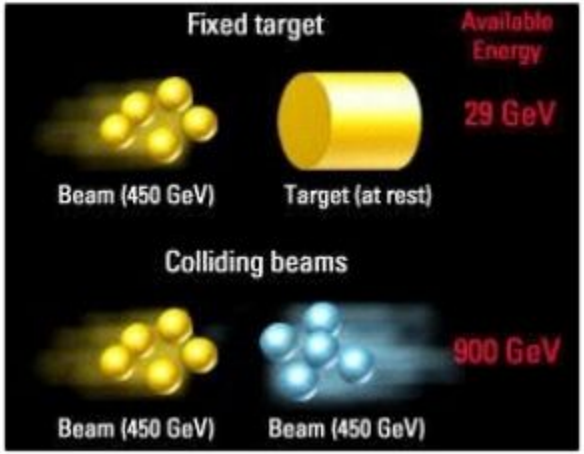
Quarks e Leptons

Funções de estrutura e equações de evolução

Sistemas de coordenadas

Consideremos a colisão de duas partículas de quadrimomentum

$$(E_a, \vec{p}_a) \quad (E_b, \vec{p}_b)$$



Na descrição destas colisões, dois sistemas de referência são usualmente utilizados:

- **Sistema de Centro de Massa (CM): é o sistema onde:**

$$\vec{p}_a + \vec{p}_b = 0$$

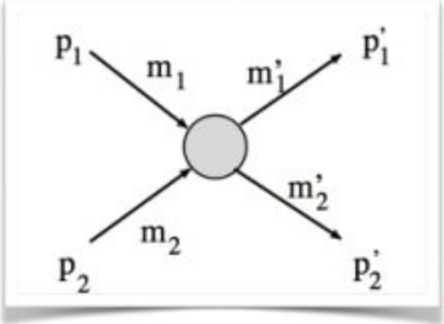
- **Sistema de Laboratório (LAB): é o sistema no qual são feitas as medidas.**
  - **Em experimentos de alvo fixo este sistema coincide com o sistema do alvo, onde uma das partículas encontra-se em repouso (e.g. b):**

$$\vec{p}_b = 0$$

  - **Nos experimentos de anéis de colisão, onde feixes de partículas idênticas colidem em direções opostas, este sistema coincide com o CM.**

http://www.physics.buffalo.edu/phy302/topic2/

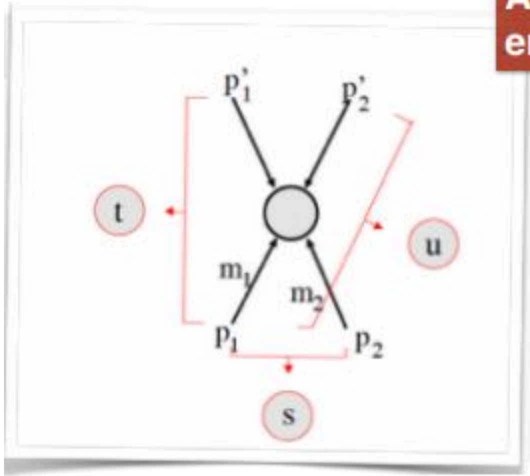
Variáveis de Mandelstam



Em Física de Altas Energias seção de choque e razão de decaimentos são descritos por variáveis cinemáticas que são invariantes relativistísticos.
Nos decaimentos de dois corpos existem de fato quatro invariantes disponíveis desde que a energia e momentum seja conservada de somente dois deles para definir a cinemática do evento.

https://en.wikipedia.org/wiki/Mandelstam_variables

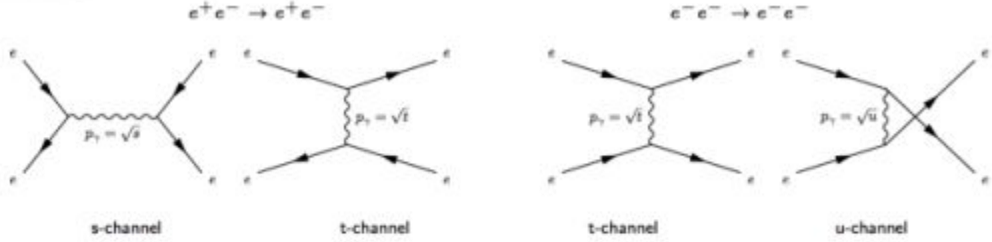
As variáveis de Mandelstam são invariantes de Lorentz em decaimentos de tipo 2->2



$$s = (p_1 + p_2)^2 = (p_1' + p_2')^2 = -(p_1 + p_2)(p_1' + p_2')$$

$$t = (p_1 + p_1')^2 = (p_2 + p_2')^2 = -(p_1 + p_1')(p_2 + p_2')$$

$$u = (p_1 + p_2')^2 = (p_2 + p_1')^2 = -(p_1 + p_2')(p_2 + p_1')$$



https://en.wikipedia.org/wiki/Mandelstam_variables

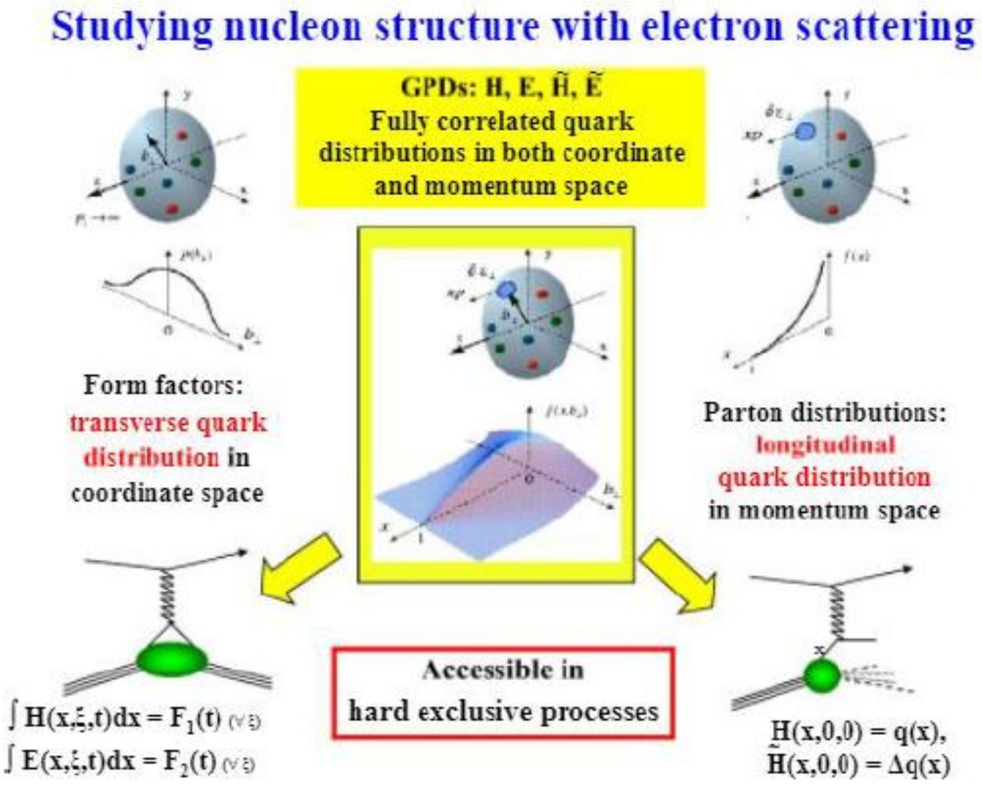
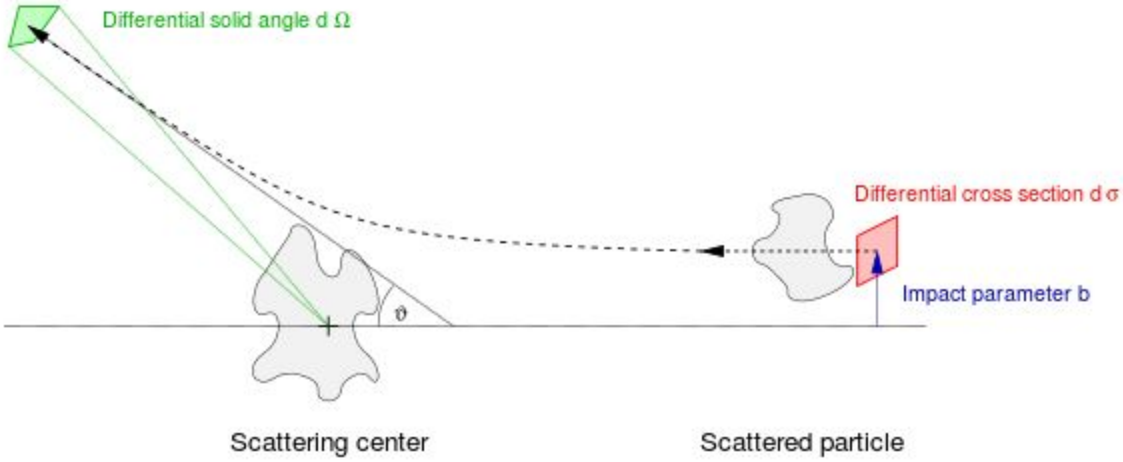


Figura 1 – Figura comparando os funções que podem ser estudadas com espalhamentos de elétrons, o fator de forma e a e a distribuição de partons. A figura foi obtida de uma apresentação de Silvia Niccolai, em (NICCOLAI, 2008)

Seção de choque



Vamos discutir de forma fenomenológica conceitos que envolvem processos de cálculo longo e com sutilezas conceituais.

Seriam capítulos 6-8-9 e 10, livro quarks e Leptons, de Halzen-Martin.

Espalhamento e-p, fator de forma proton

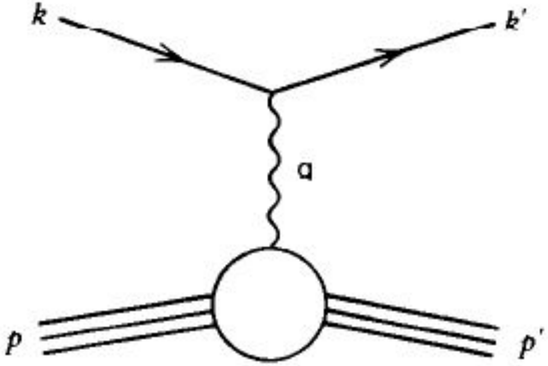


Fig. 8.2 Lowest-order electron–proton elastic scattering.

where $q = p' - p$ and the electron and proton transition currents are, respectively,

$$j^{\mu} = -e\,\bar{u}(k')\gamma^{\mu}u(k)\,e^{i(k'-k)\cdot x} \tag{8.11}$$

$$J^{\mu} = e\,\bar{u}(p')[\quad]u(p)\,e^{i(p'-p)\cdot x}, \tag{8.12}$$

[ ] é relacionado ao fator de forma

$$[\quad] = \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M} F_2(q^2)\, i\sigma^{\mu\nu} q_\nu \right] \tag{8.13}$$

Comparação seção de choque diferencial e-muon e e-p

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}, \qquad (8.10)$$

where the factor

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}},$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1 + \kappa F_2)^2\sin^2\frac{\theta}{2}\right\}, \qquad (8.15)$$

Espalhamento ep inelástico ep -> eX

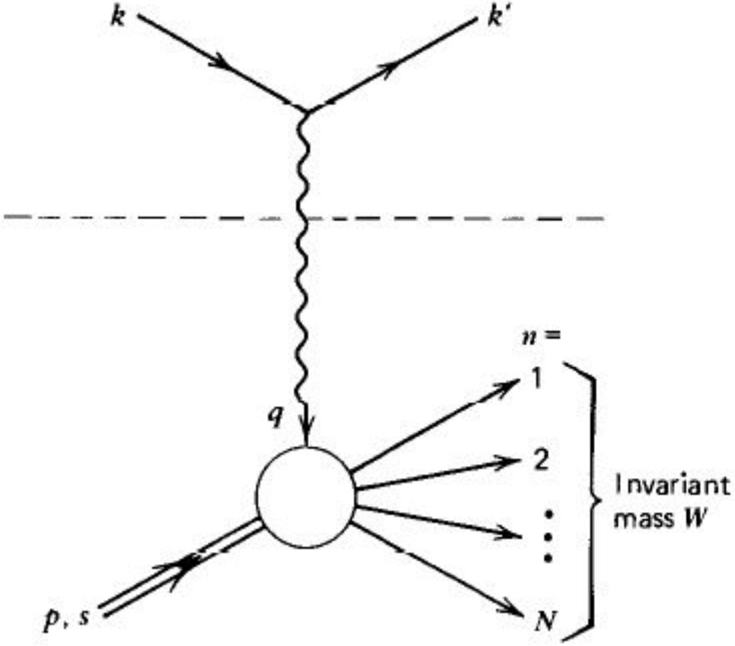


Fig. 8.5 Lowest-order diagram for ep → eX.

Ressonâncias

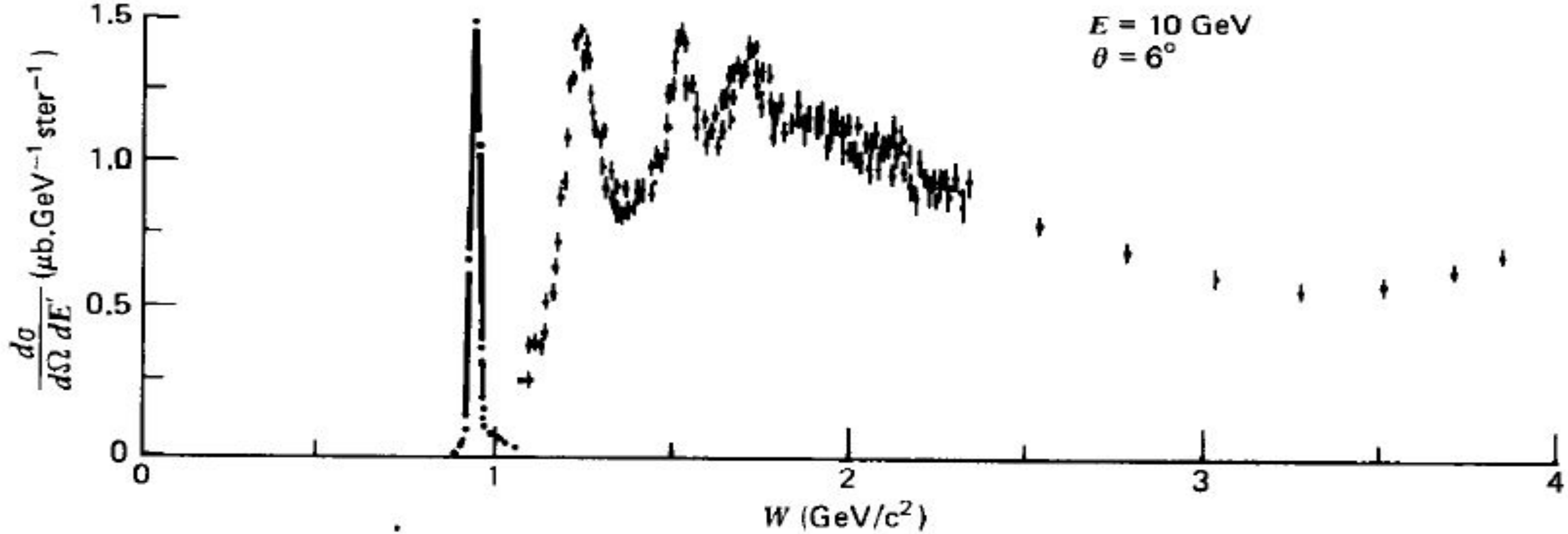


Fig. 8.6 The ep → eX cross section as a function of the missing mass W. Data are from the Stanford Linear Accelerator. The elastic peak at $W = M$ has been reduced by a factor of 8.5.

Agora a energia não fica só distribuída entre as duas partículas existentes inicialmente. A seção de choque diferencial é recalculada, considerando este detalhe :

$$d\sigma \sim L^{e}_{\mu\nu} W^{\mu\nu}$$

W de weigth,

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} \left(p^\mu q^\nu + q^\mu p^\nu \right). \quad (8.24)$$

Com a lei de conservação da corrente, temos uma relação entre os W (com indice)

$$W_5 = -\frac{p \cdot q}{q^2} W_2,$$

$$W_4 = \left(\frac{p \cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1.$$

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right), \tag{8.27}$$

there are two independent variables, and we choose

$$q^2 \qquad \text{and} \qquad \nu \equiv \frac{p\cdot q}{M}. \tag{8.28}$$

The invariant mass W of the final hadronic system is related to ν and q^2 by

$$W^2 = (p+q)^2 = M^2 + 2M\nu + q^2. \tag{8.29}$$

EXERCISE 8.11 It is common to replace ν and q^2 by the dimensionless variables

$$x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu}, \qquad y = \frac{p\cdot q}{p\cdot k}, \tag{8.30}$$

there are two independent variables, and we choose

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$$x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu}, \qquad y = \frac{p \cdot q}{p \cdot k}, \tag{8.30}$$

EXERCISE 8.12 Show that in the rest frame of the target proton,

$$\nu = E - E', \qquad y = \frac{E - E'}{E},$$

where E and E' are the initial and final electron energies, respectively.

Comparação de Resultados

First, for a muon target of mass m (or a quark target of mass m after substitution $\alpha^2 \to \alpha^2 e_q^2$ where e_q is the quark's fractional charge),

$$\left\{ \quad \right\}_{e\mu \to e\mu} = \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2m} \right). \tag{8.41}$$

For elastic scattering from a proton target,

$$\left\{ \quad \right\}_{ep \to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2M} \right), \tag{8.42}$$

where $\tau = -q^2/4M^2$ and M is the mass of the proton. Finally, for the case when the proton target is broken up by the bombarding electron,

$$\left\{ \quad \right\}_{ep \to eX} = W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2}. \tag{8.43}$$

Escala de Bjorken

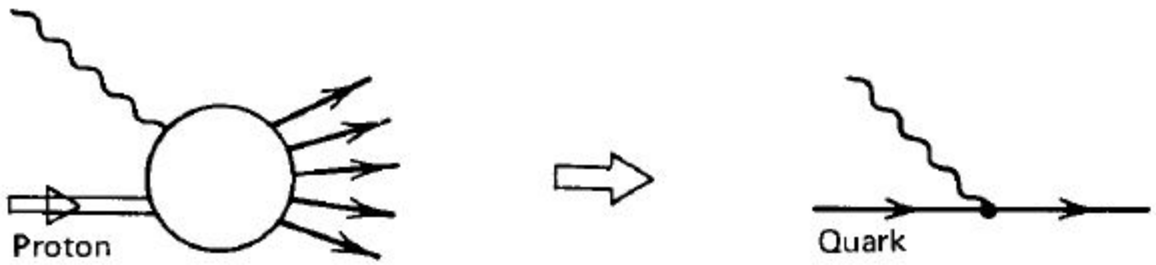
free Dirac particle (a quark) and (8.43) turns into (8.41). The proton structure functions thus become simply

$$2W_1^{\text{point}} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right),$$

$$W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right). \tag{9.1}$$

For convenience, we have introduced the positive variable

$$Q^2 \equiv -q^2.$$



$$(9.2)$$

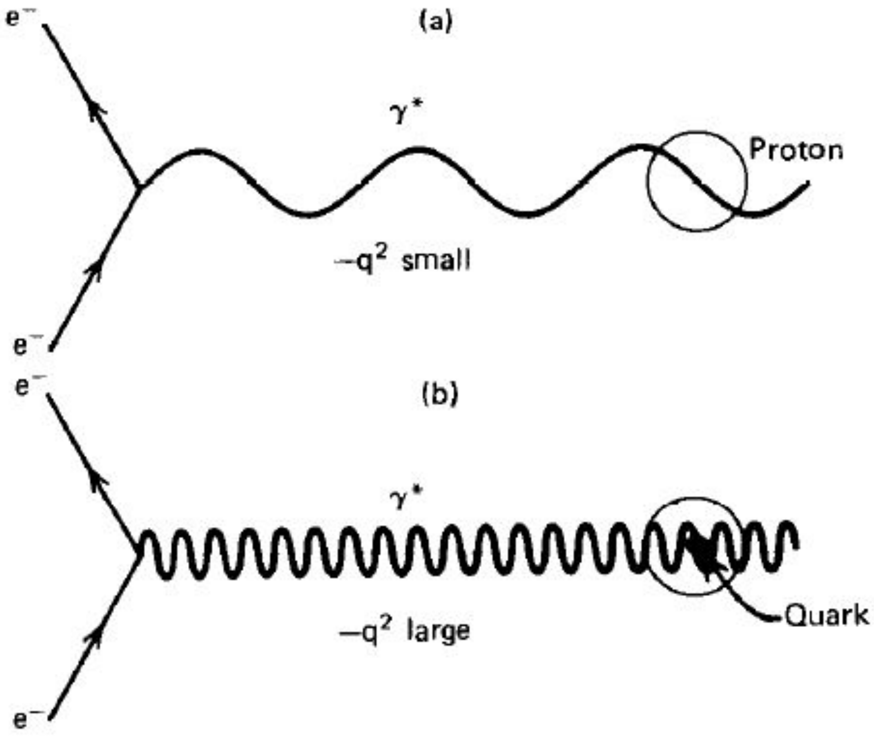


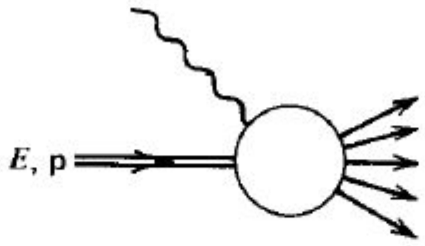
Fig. 9.1 (a) Elastic ep → ep scattering in which a large-wavelength "photon beam" measures the size of the proton through the elastic form factor analysis. (b) In deep inelastic scattering a short-wavelength "photon beam" resolves the quarks within the proton provided $\lambda\left(\approx 1/\sqrt{-q^2}\right) \ll 1\mathrm{F}$.

$\delta(x/a) = a\,\delta(x)$, (9.1) may be rearranged to introduce dimensionless structure functions

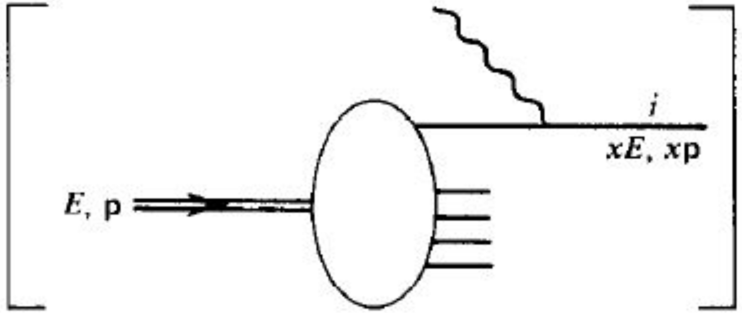
$$2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right),$$

$$\nu W_2^{\text{point}}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right). \tag{9.3}$$

Partons e a Escala de Bjorken

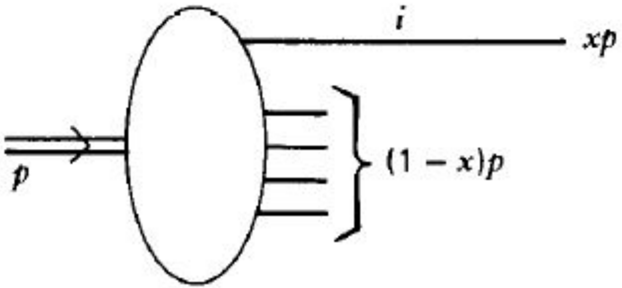


$$= \sum_i \int dx\, e_i^2$$



(9.7)

$$f_i(x) = \frac{dP_i}{dx} =$$



(9.8)

which describes the probability that the struck parton i carries a fraction x of the proton's momentum p. All the fractions x have to add up to 1; therefore,

$$\sum_{i'} \int dx\, x f_{i'}(x) = 1. \tag{9.9}$$

Here, i' sums over all the partons, not just the charged ones i which interact with the photon. The kinematics can be summarized as follows:

$$\begin{array}{llll}
 & \textbf{Proton} & \textbf{Parton} & \\
 & \downarrow & \downarrow & \\
\text{Energy} & E & xE & (9.10) \\
\text{Momentum} & p_L & xp_L & \\
 & p_T = 0 & p_T = 0 & \\
\text{Mass} & M & m = (x^2E^2 - x^2p_L^2)^{1/2} = xM. &
\end{array}$$

For an electron hitting a parton with momentum fraction x and unit charge, we see from (9.3) and (9.5) that the dimensionless structure functions are

$$F_1(\omega) = \frac{Q^2}{4m\nu x}\delta\left(1 - \frac{Q^2}{2m\nu}\right) = \frac{1}{2x^2\omega}\delta\left(1 - \frac{1}{x\omega}\right),$$

$$F_2(\omega) = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta\left(1 - \frac{1}{x\omega}\right). \tag{9.11}$$

We have used the kinematics of (9.10); ω is the dimensionless variable defined in (9.6). Summing our results for $F_{1,2}$ for one parton, (9.11), over the partons making up a proton, (9.7) and (9.8), we obtain

$$F_2(\omega) = \sum_i \int dx\, e_i^2 f_i(x) x\, \delta\left(x - \frac{1}{\omega}\right),$$

$$F_1(\omega) = \frac{\omega}{2} F_2(\omega). \tag{9.12}$$

It is conventional to redefine $F_{1,2}(\omega)$ as $F_{1,2}(x)$ and to express the results in terms of x. Recalling the identification (9.5), we see that (9.12) become, at large Q^2,

$$\boxed{\begin{aligned} \nu W_2(\nu, Q^2) &\to F_2(x) = \sum_i e_i^2 x\, f_i(x), \qquad (9.13)\\ M W_1(\nu, Q^2) &\to F_1(x) = \frac{1}{2x} F_2(x), \qquad (9.14) \end{aligned}}$$

with

$$x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}. \qquad (9.15)$$

Os quarks dentro do proton

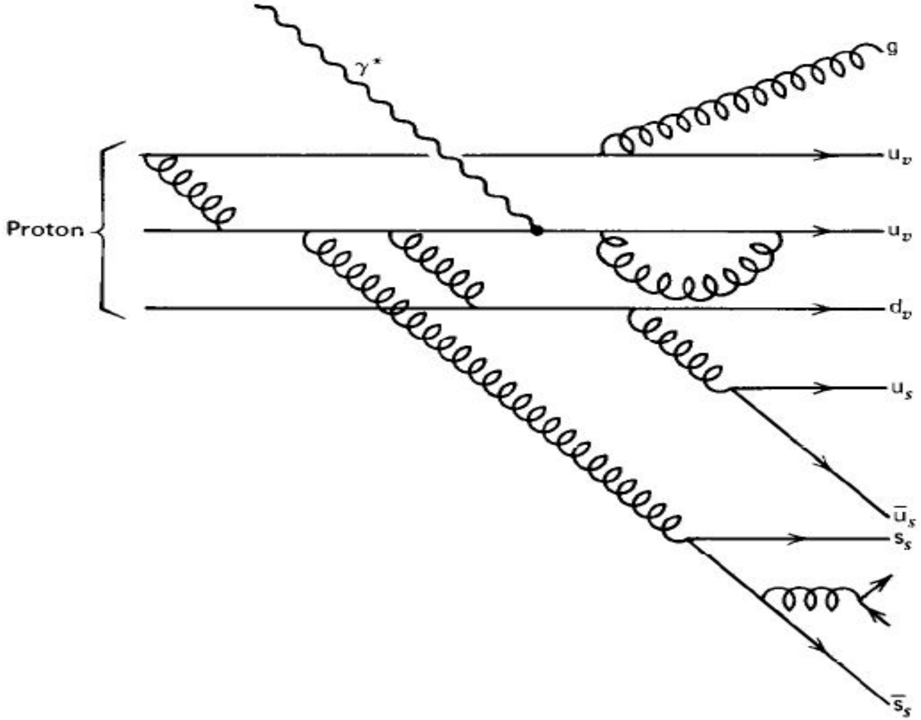


Fig. 9.5 A proton made up of valence quarks, gluons, and slow debris consisting of quark–antiquark pairs.

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2[u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2[d^p(x) + \bar{d}^p(x)]$$

$$+ \left(\frac{1}{3}\right)^2[s^p(x) + \bar{s}^p(x)], \tag{9.27}$$

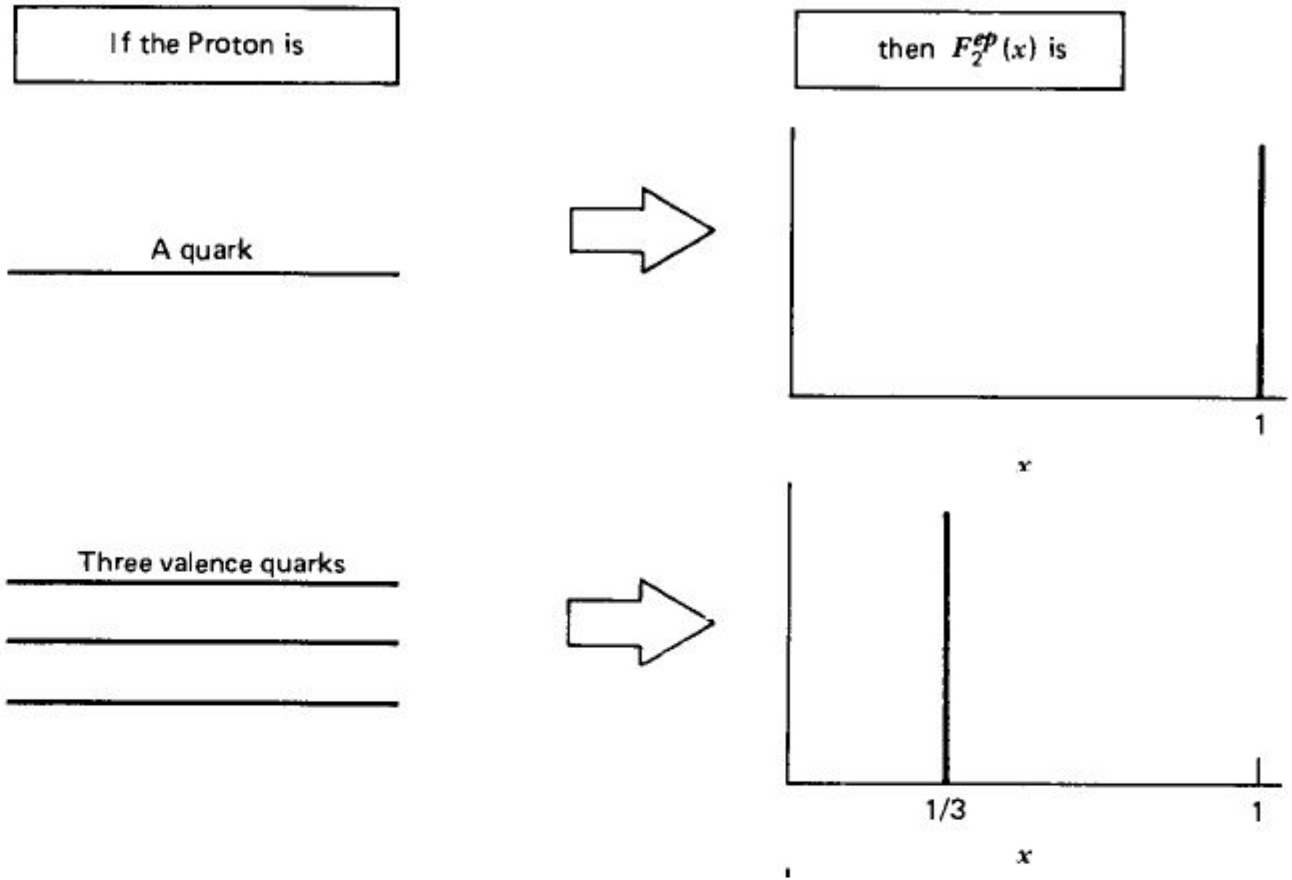
$$\frac{1}{x}F_2^{en} = \left(\frac{2}{3}\right)^2[u^n + \bar{u}^n] + \left(\frac{1}{3}\right)^2[d^n + \bar{d}^n] + \left(\frac{1}{3}\right)^2[s^n + \bar{s}^n]; \tag{9.28}$$

regras de soma:

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx = 2,$$

$$\int_0^1 [d(x) - \bar{d}(x)]\, dx = 1,$$

$$\int_0^1 [s(x) - \bar{s}(x)]\, dx = 0.$$



If the Proton is
then $F_2^{ep}(x)$ is
A quark
1
x
Three valence quarks
1/3
1
x

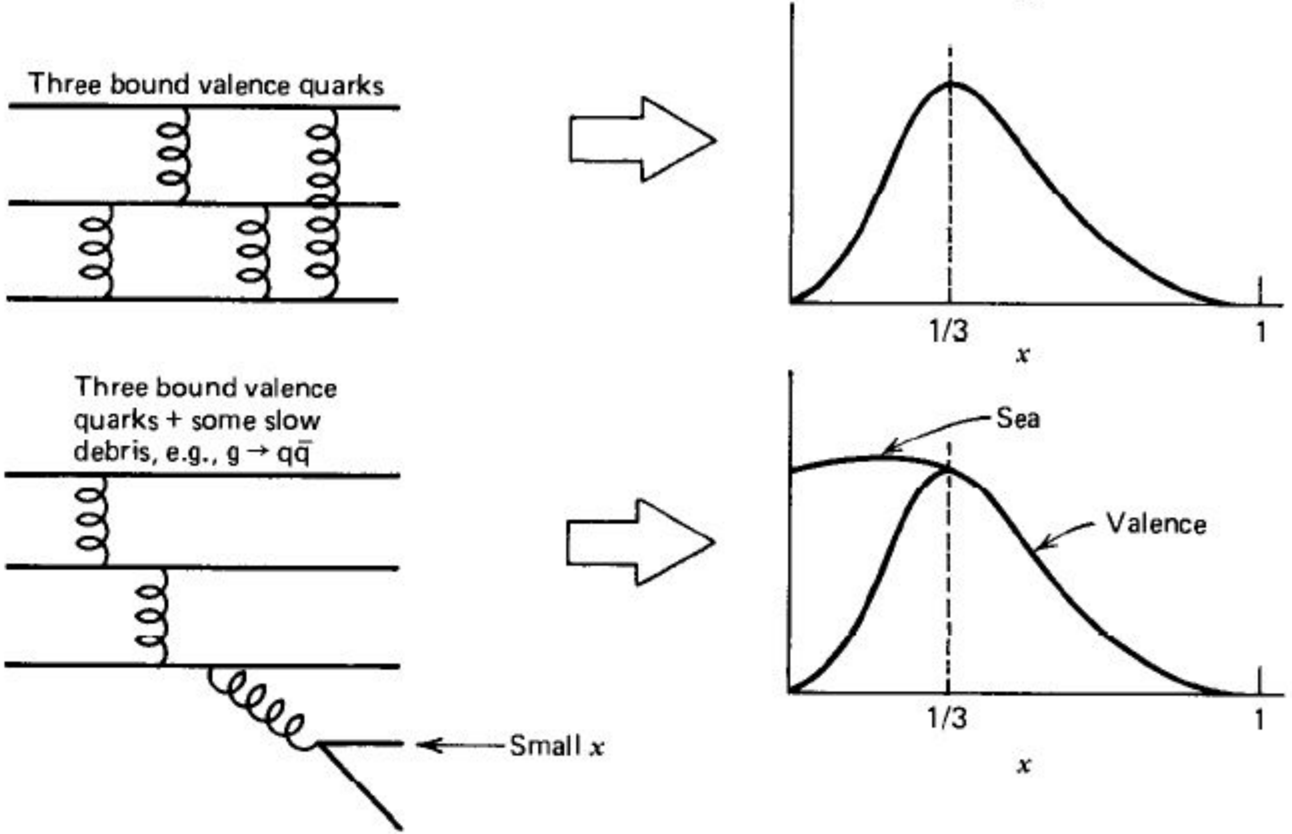


Fig. 9.7 The structure function pictured corresponding to different compositions assumed for the proton.

9.4 Where Are the Gluons?

If we sum over the momenta of all the partons, we must reconstruct the total momentum p of the proton [see (9.8)],

$$\int_0^1 dx\,(xp)[u + \bar{u} + d + \bar{d} + s + \bar{s}] = p - p_g,$$

or, dividing by p, (9.37)

$$\int_0^1 dx\, x(u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1 - \varepsilon_g.$$

The momentum fraction $\varepsilon_g \equiv p_g/p$ carried by the gluons is not directly exposed

Dados experimentais (bem antigos!)

$$\int dx\, F_2^{ep}(x) = \tfrac{4}{9}\varepsilon_u + \tfrac{1}{9}\varepsilon_d = 0.18,$$

$$\int dx\, F_2^{en}(x) = \tfrac{1}{9}\varepsilon_u + \tfrac{4}{9}\varepsilon_d = 0.12, \qquad (9.38)$$

where

$$\varepsilon_u \equiv \int_0^1 dx\, x(u + \bar{u})$$

is the momentum carried by u quarks and antiquarks, and similarly for ε_d.

$$\varepsilon_u = 0.36, \qquad \varepsilon_d = 0.18, \qquad \varepsilon_g = 0.46.$$

comentário

Resultado pode depender de Q^2, e, nos nossos estudos , da temperatura.

Temos a intenção de relacionar Q^2 com a temperatura.

Diagramas com Gluons

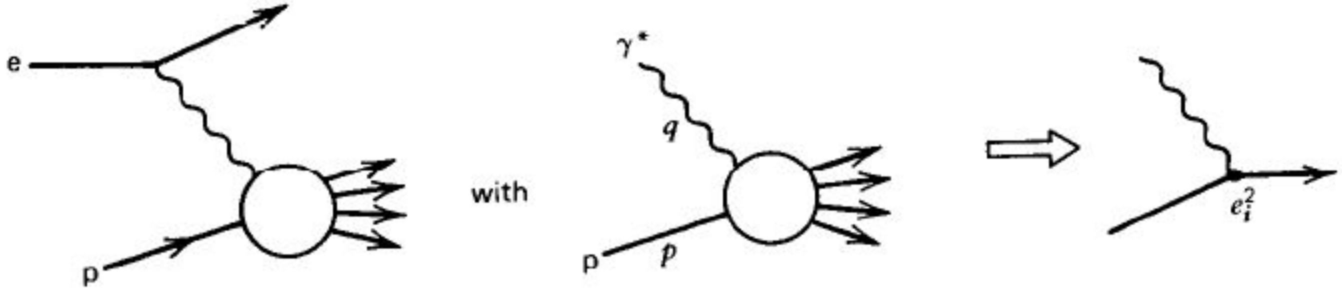


Fig. 10.1 Pictorial representation of the parton model for ep → eX.

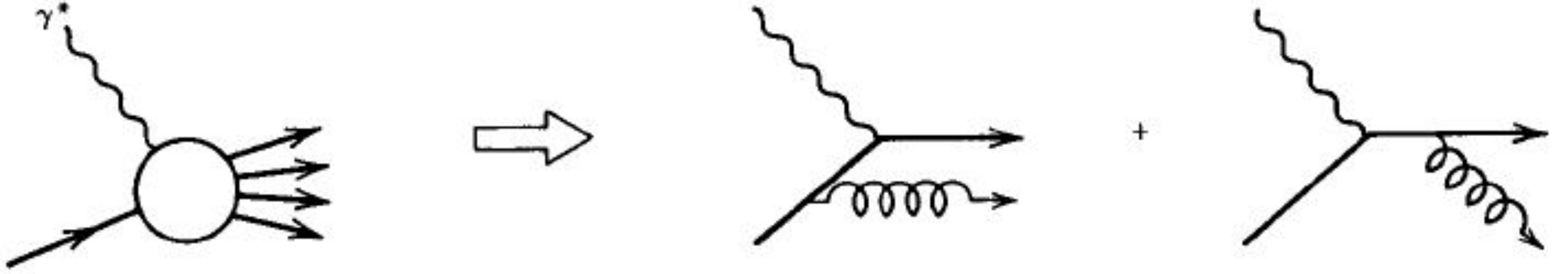


Fig. 10.2 $O(\alpha\alpha_s)$ contributions ($\gamma^* q \to qg$) to $ep \to eX$.

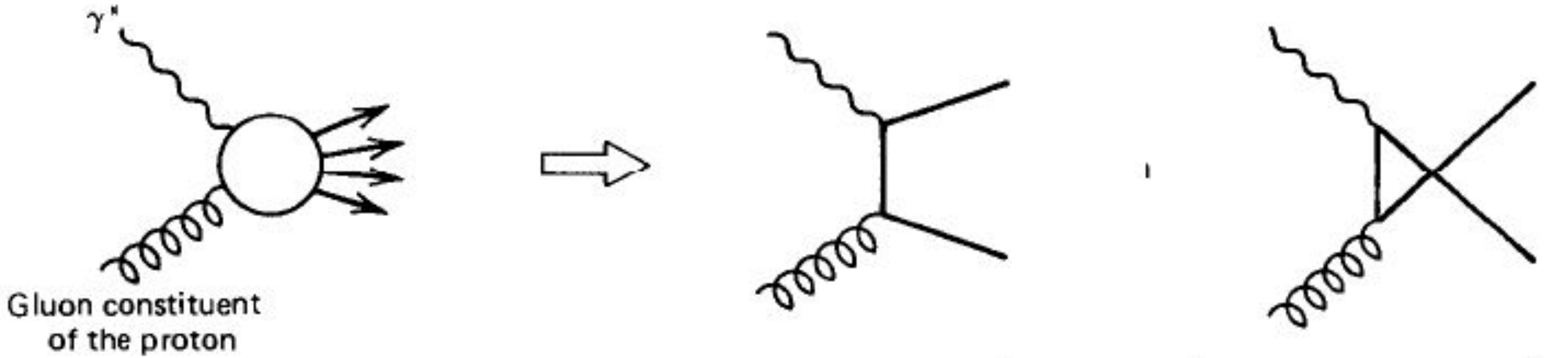


Fig. 10.3 $O(\alpha\alpha_s)$ gluon-initiated "hard" scattering contributions ($\gamma^* g \to q\bar{q}$) to $ep \to eX$.

mudança de sistema e de variáveis

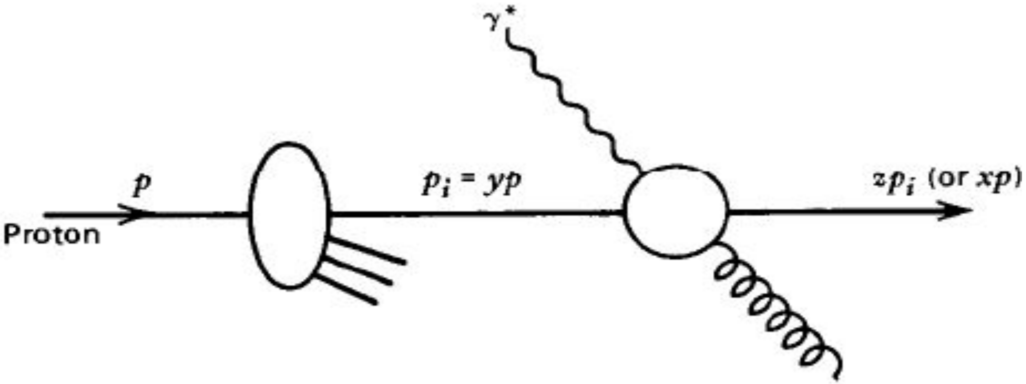


Fig. 10.5 Embedding $\gamma^* q \to qg$ in $\gamma^* p \to X$.

in Fig. 10.5. The relation between the two frames is given by

γ*-Proton Frame		**γ*-Parton Frame**
p	$\longrightarrow$	$p_i = yp$
$x = \dfrac{Q^2}{2p \cdot q}$	$\longrightarrow$	$z = \dfrac{Q^2}{2p_i \cdot q} = \dfrac{x}{y}$

Seção de choque emissao gluon - resumo

Mudanças nos processos inelásticos

Temos de incluir os diagramas de gluons na função de estrutura

$$F_1 = MW_1(\nu, Q^2),$$
$$F_2 = \nu W_2(\nu, Q^2), \tag{10.1}$$

see (8.34). Since we are going beyond the parton model, we shall find $F_{1,2}$ no longer scale; that is, they are functions of both

$$\nu = \frac{p \cdot q}{M} \quad \text{and} \quad Q^2 = -q^2, \tag{10.2}$$

Não depende somente da variavel

$$x = \frac{-t}{s + u} = \frac{Q^2}{2M\nu}.$$

No espalhamento profundamente inelástica há uma relação entre as seções de choque e F1 e F2

$$2F_1 = \frac{\sigma_T}{\sigma_0} \tag{10.3}$$

$$\frac{F_2}{x} = \frac{\sigma_T + \sigma_L}{\sigma_0} \tag{10.4}$$

where σ_T and σ_L are the $\gamma^* p$ total cross sections for transverse and longitudinal virtual photons, respectively, and

$$\sigma_0 \equiv \frac{4\pi^2\alpha}{2MK} \simeq \frac{4\pi^2\alpha}{s}. \tag{10.5}$$

O eletron interge com um quark (via foton) que pode ter emitido um gluon.

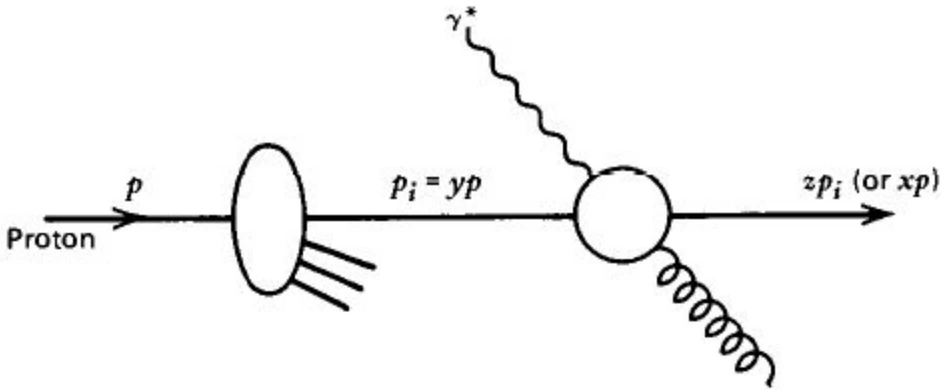


Fig. 10.5 Embedding $\gamma^* q \to qg$ in $\gamma^* p \to X$.

Temos de usar o sistema de coordenadas do parton, não do proton

γ*-Proton Frame		**γ*-Parton Frame**
p	$\longrightarrow$	$p_i = yp$
$x = \dfrac{Q^2}{2p \cdot q}$	$\longrightarrow$	$z = \dfrac{Q^2}{2p_i \cdot q} = \dfrac{x}{y}$

$$\left(\frac{\sigma_T}{\sigma_0}(x, Q^2)\right)_{\gamma^* p} = \sum_i \int_0^1 dz \int_0^1 dy\, f_i(y)\, \delta(x - zy) \left(\frac{\hat{\sigma}_T}{\hat{\sigma}_0}(z, Q^2)\right)_{\gamma^* i} \tag{10.7}$$

$$\boxed{\left(\frac{\sigma_T}{\sigma_0}(x, Q^2)\right) = \sum_i \int_x^1 \frac{dy}{y} f_i(y) \left(\frac{\hat{\sigma}_T}{\hat{\sigma}_0}(x/y, Q^2)\right).}$$

Amplitudes para o cálculo de seções de choque

Na QED

$$\overline{|\mathfrak{M}|^2} = 32\pi^2\alpha^2\left(-\frac{u}{s} - \frac{s}{u} + \frac{2tQ^2}{su}\right), \tag{10.16}$$

Na QCD muda a constante

$$\overline{|\mathfrak{M}|^2} - 32\pi^2(e_i^2\alpha\alpha_s)\frac{4}{3}\left(-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right). \tag{10.17}$$

Var

$$\hat{s} = 2k^2 + 2kq_0 - Q^2 = 4k'^2, \tag{10.20}$$

$$\hat{t} = -Q^2 - 2k'q_0 + 2kk'\cos\theta = -2kk'(1 - \cos\theta), \tag{10.21}$$

$$\hat{u} = -2kk'(1 + \cos\theta), \tag{10.22}$$

where k, k' are the magnitudes of the center-of-mass momenta $\mathbf{k}, \mathbf{k}'$. Note that for the virtual photon, $q_0^2 = k^2 - Q^2$. A useful result is

$$4kk' = -\hat{t} - \hat{u} = \hat{s} + Q^2. \tag{10.23}$$

The interesting quantity is the transverse momentum of the outgoing quark, $p_T = k'\sin\theta$. Show that

$$p_T^2 = \frac{\hat{s}\hat{t}\hat{u}}{(\hat{s} + Q^2)^2} \tag{10.24}$$

or, in the limit of small-angle scattering, $-\hat{t} \ll \hat{s}$, that

$$p_T^2 = \frac{\hat{s}(-\hat{t})}{\hat{s} + Q^2}. \tag{10.25}$$

$$\frac{d\hat{\sigma}}{dp_T^2} \simeq \frac{1}{16\pi\hat{s}^2}\overline{|\mathfrak{M}|^2}. \tag{10.27}$$

EXERCISE 10.4 Derive (10.27). Make use of (10.26) and (10.20), together with (4.34). Show that the γ^*q flux factor is given by $2\hat{s}$ using the convention of (8.48).

On substituting (10.17) into (10.27), the $\gamma^*q \to qg$ cross section becomes

$$\frac{d\hat{\sigma}}{dp_T^2} \simeq \frac{8\pi e_i^2\alpha\alpha_s}{3\hat{s}^2}\left(\frac{1}{-\hat{t}}\right)\left[\hat{s} + \frac{2(\hat{s}+Q^2)Q^2}{\hat{s}}\right], \tag{10.28}$$

where we have used $-\hat{t} \ll \hat{s}$. Using (10.25) and

$$z \equiv \frac{Q^2}{2p_i\cdot q} = \frac{Q^2}{(p_i+q)^2 - q^2} = \frac{Q^2}{\hat{s}+Q^2}, \tag{10.29}$$

equation (10.28) can finally be rewritten as

$$\boxed{\frac{d\hat{\sigma}}{dp_T^2} \simeq e_i^2\hat{\sigma}_0\frac{1}{p_T^2}\frac{\alpha_s}{2\pi}P_{qq}(z),} \tag{10.30}$$

where $\hat{\sigma}_0 = 4\pi^2\alpha/\hat{s}$, see (10.5), and where

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right) \tag{10.31}$$

represents the probability of a quark emitting a gluon and so becoming a quark with momentum reduced by a fraction z. The $z \to 1$ singularity is associated with

Diagr

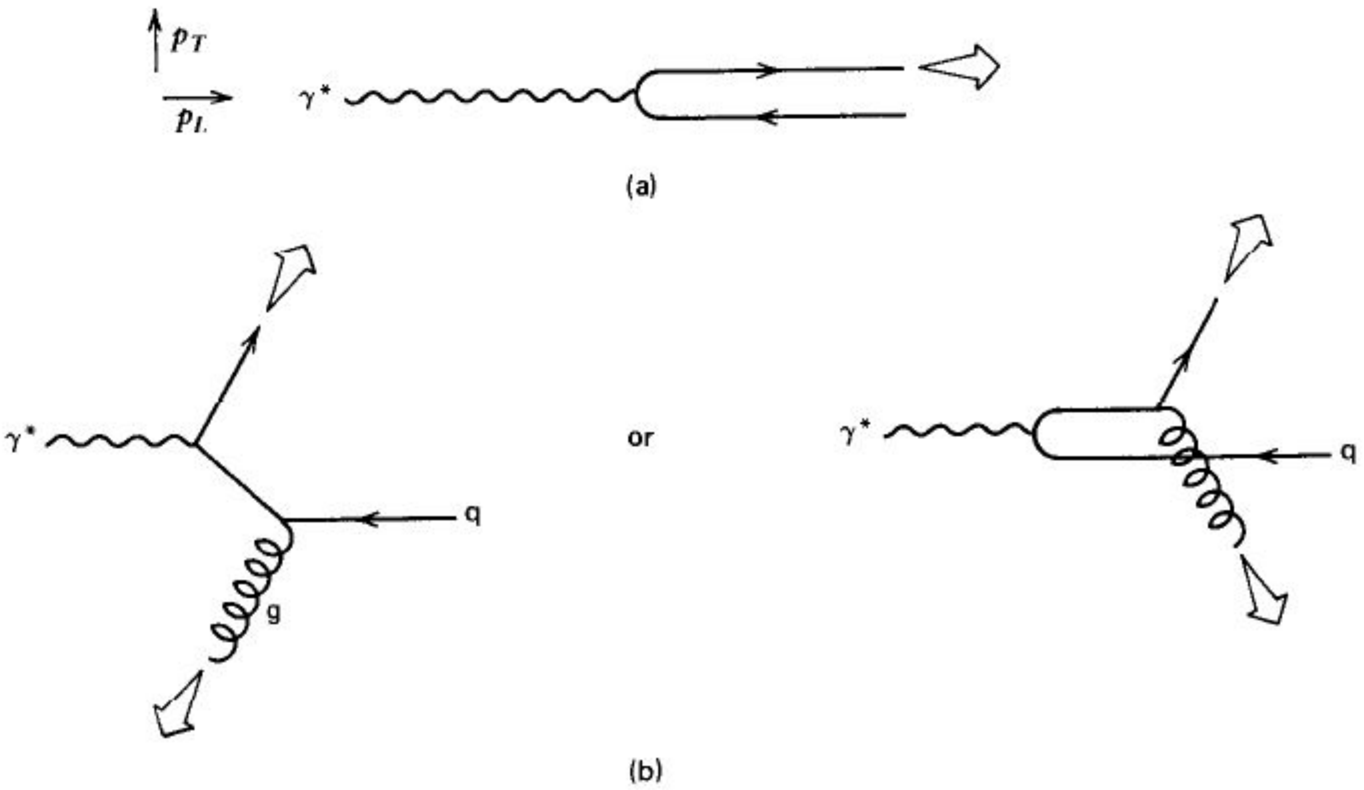


Fig. 10.4 (a) Parton model diagram for $\gamma^* q \to q$, producing a jet with $p_T = 0$. (b) Gluon emission diagrams which produce jets with $p_T \neq 0$.

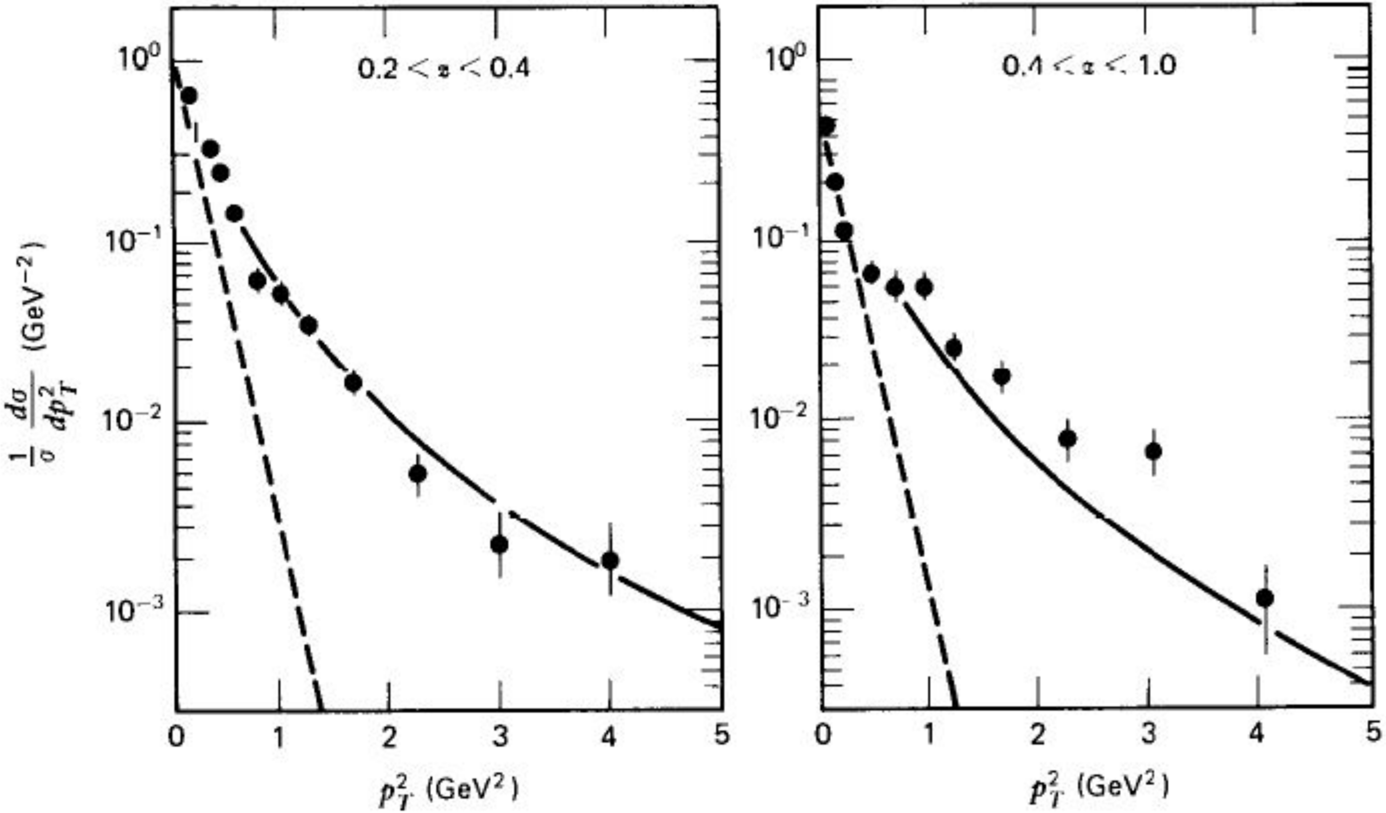


Fig. 10.8 The p_T^2 distribution of hadrons produced in μN interactions relative to the direction of the virtual photon. The dashed line is the expectation in the absence of gluon emission. Data are from the EMC collaboration at CERN. (μN and eN interactions give the same curves.)

Violação de escala- Equação de Altarelli-Parisi

$$\hat{\sigma}(\gamma^* \mathrm{q} \to \mathrm{qg}) = \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2}$$

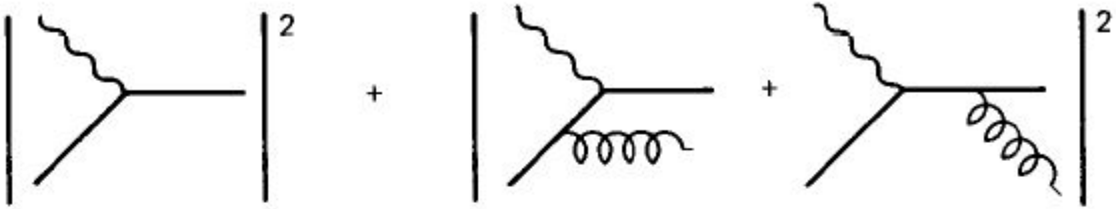
$$\simeq e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{\hat{s}/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

$$\simeq e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right). \qquad (10.32)$$

$$(p_T^2)_{\max} = \frac{\hat{s}}{4} = Q^2 \frac{1-z}{4z}.$$

Adding $\hat{\sigma}(\gamma^* q \rightarrow qg)$ to the parton model cross section, (10.14), we find QCD modifies (10.15) to

$$\frac{F_2(x, Q^2)}{x} =$$



$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left(\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right), \tag{10.34}$$

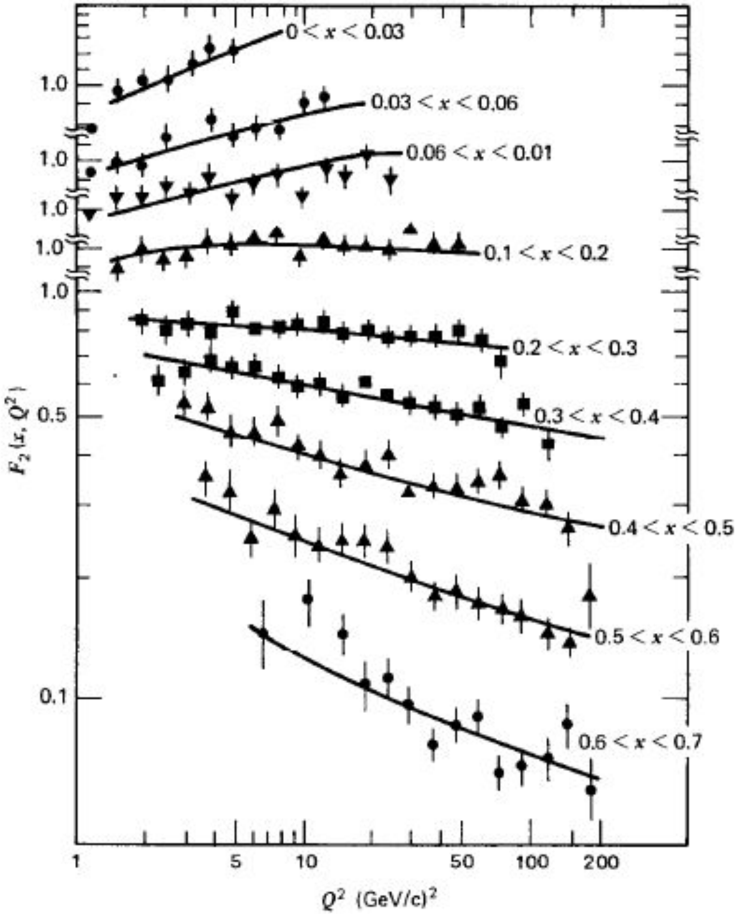


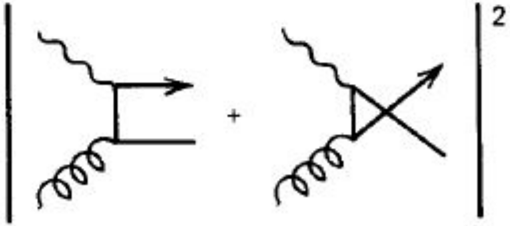
Fig. 10.10 Deviations from scaling. With increasing Q^2, the structure function $F_2(x, Q^2)$ increases at small x and decreases at large x. The data are from the CDHS counter experiment at CERN.

Produção de pares q(anti-q) por gluons

$$\overline{|\mathcal{M}|^2} = 32\pi^2\left(e_q^2\alpha\alpha_s\right)\frac{1}{2}\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}}\right), \qquad (10.38)$$

using $\sum \epsilon_\mu^* \epsilon_\nu = -g_{\mu\nu}$ for the γ^*-polarization sum. Hence, show that (10.34) for the proton structure function contains the additional contribution

$$\left.\frac{F_2(x,Q^2)}{x}\right|_{\gamma^* g \to q\bar{q}} =$$



(10.39)

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, g(y)\, \frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log\frac{Q^2}{\mu^2},$$

where $g(y)$ is the gluon density in the proton and where

$$P_{qg}(z) = \tfrac{1}{2}\left(z^2 + (1 - z)^2\right) \tag{10.41}$$

represents the probability that a gluon annihilates into a $q\bar{q}$ pair such that the quark has a fraction z of its momentum. Detailed measurements of the

If we include the pair production contribution to the evolution of the quark density, (10.37) becomes

$$\frac{dq_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left(q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right) \tag{10.42}$$

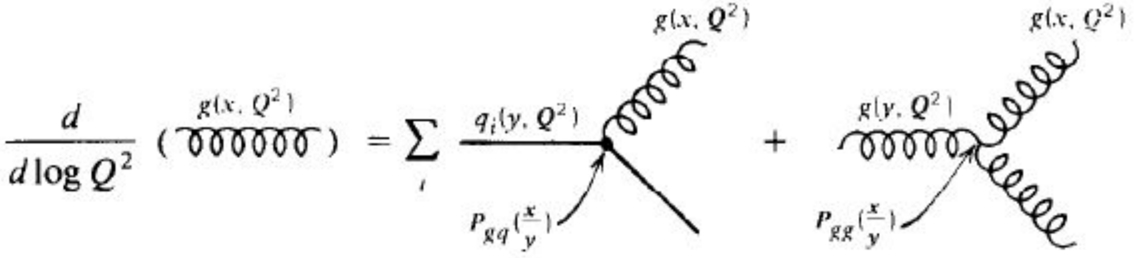
for each quark flavor i. The second term, previously omitted, considers the

$$\boxed{\frac{d}{d\log Q^2}q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}q(y,Q^2)\,P_{qq}\left(\frac{x}{y}\right).} \tag{10.37}$$

As equações 10.42 e 10.37 mostram um quark que tem menos energia porque emitiu um gluon, e um quark originário da divisão de um gluon.

Depende de uma informação fenomenológica (ou teórica) inicial q(x,Q^2) e g(x,Q^2).

incluindo gluons



(10.43)

which tells us that

$$\frac{dg(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left(\sum_i q_i(y, Q^2)\, P_{gq}\left(\frac{x}{y}\right) + g(y, Q^2)\, P_{gg}\left(\frac{x}{y}\right)\right), \tag{10.44}$$

EXERCISE 10.10 How would you set about verifying that

$$P_{gq}(z) = \frac{4}{3}\,\frac{1 + (1 - z)^2}{z}, \tag{10.45}$$

$$P_{gg}(z) = 6\left(\frac{1 - z}{z} + \frac{z}{1 - z} + z(1 - z)\right)? \tag{10.46}$$

Conclusões

A variável x de Bjorken foi definida primeiro “dinamicamente”, depois como uma fração do momento total. Com as funções delta de Dirac podemos verificar a equivalência.

As funções de distribuições de probabilidade de quarks e gluons dependem de x (de forma direta) e Q^2 (de forma logaritimica).