Fractal structure of gauge fields

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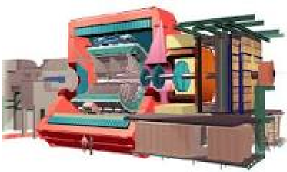
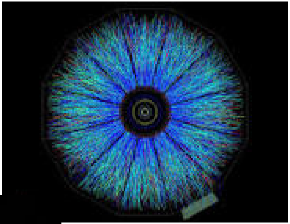
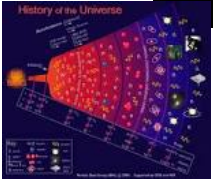
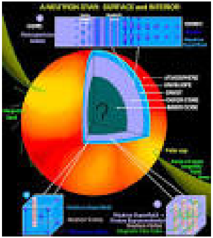
Grenac, Sept. 11, 2018

QGP: QCD and its applciations





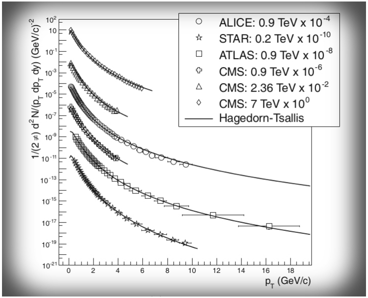




Thermodynamical approach

R. Hagedorn: thermodynamical approach to HEC

- exponential distributions of energy and momentum
- exponential hadron mass spectrum
- Hadron Resonance Gas models, conf,/deconf. phase-transition
- but disagrees from experimental data



Renormalization of gauge fields

Yang-Mills theory is renormalizable:

$$\Gamma(p, m, g) = \lambda^{-D}\Gamma(p, \mu, \bar{g})$$

F. Dyson, PR 75 (1949) 1736

M. Gell-Mann and F.E. Low, PR 95 (1954) 1300

Renormalization group equation:

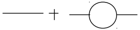
$$\left[M\frac{\partial}{\partial M} + \beta_g \frac{\partial}{\partial \bar{g}} + \beta_\mu \frac{\partial}{\partial \mu} + d\right]\Gamma = 0$$

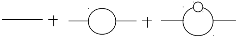
Callan-Symanzik Equation

C.G. Callan Jr., PRD 2 (1970) 1541

K. Symanzik, Comm. Math. Phys. 18 (1970) 227

n=0

n=1 

n=2 

Effective coupling constant $\bar{g}$

Effective mass μ

Renormalization of gauge fields

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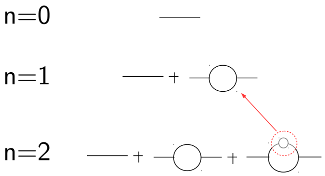
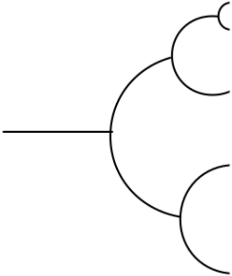


Effective coupling constant $\bar{g}$

Effective mass μ

Multiparticle production

Example of complex graphs in multiparticle production:

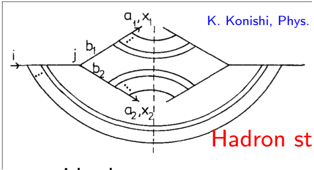


Summing up all diagrams → ideal gas of particles with different masses

R. Dashen, S. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345

R. Venugopalan and M. Prakash, Nucl. Phys. A546 (1992) 718

Particle production is complex, not chaotic



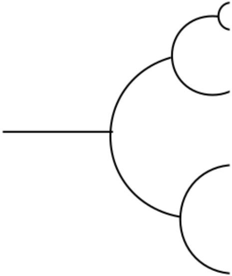
K. Konishi, Phys. Scr. 19 (1979) 195

Hadron structure is complex

Too many complex graphs to be considred.
Calculations limited to first leading orders or LQCD.

Including fractal structure in YM fields

At any scale: ideal gas of particles with different masses.



$Z = Tr < x|U(\mathrm{i}\beta H, 0)|x >= \int Dx < x|e^{-\beta H}|x >$

$Z = \prod_i \int dmd^3p < x_i|e^{-\beta H_i}|x_i >$

$Z = \prod_i \int dm\tilde{P}(m)d^3p < x_{i,m}|e^{-\beta H_i}|x_{i,m} >$

This partition function can be written as

$Z = \prod_i \int dm\tilde{P}(m)d^3p < x_{i,m}|e^{-\beta H_i}|x_{i,m} >$

i and μ are the particle index and effective mass.

Therefore: $Z = \prod_i \int dm\tilde{P}(m)e^{-\beta\epsilon_i}d^3p_i$, $\epsilon^2 = p^2 + m^2$.

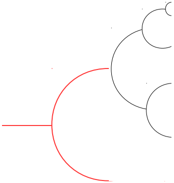
Including fractal structure in YM fields

$$Z = \prod_i \int dm\tilde{P}(\mu) e^{-\beta\epsilon_i} d^3p_i,\ \epsilon^2 = p^2 + \mu^2.$$

$$P(U)dU = \prod_i AkT\tilde{P}(\mu) e^{-\beta\epsilon_i} d\mu d^3\pi,$$

is the probability to find the system with energy between U and $U + dU$,

$$U = \sum_i \epsilon_i,\ \pi = p/(kT),\ \mu = m/(kT)\ .$$



Parent parton is also a parton $\rightarrow P(U) \propto \tilde{P}(\mu)$.

Self-symmetry in gauge fields!

It can be show that $P(\mu)$ must be such that:

$$P(\varepsilon) = A[1 + (q-1)\frac{\varepsilon}{k\tau}]^{-\frac{1}{q-1}}$$

AD, PRD (2016)

AD, E. Megías, D.P. Menezes, T. Frederico, arXiv:1801.01160

$\frac{\varepsilon}{kT} = \frac{\mu}{K}$, with K being the parton kinetic energy.

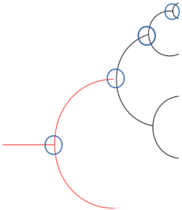
Non extensivity in gauge field theory

$$P(\varepsilon) = A[1 + (q-1)\frac{\varepsilon}{k\tau}]^{-\frac{1}{q-1}}$$ AD, PRD (2016)

Tsallis q-exponential function → Tsallis Statistics

$\frac{\varepsilon}{kT} = \frac{\mu}{K}$, with K being the parton kinetic energy.
$(q-1) = \frac{2}{3N}(1-\nu)$ and $\tau = (q-1)NT$

q and τ are completely determined interms of the fractal structure.



Suggest that at each vertex, momentum and effective masses are determined by the same scaled distribution

$$\bar{g}^2 = \prod_i \left[1 + (q-1)\frac{\varepsilon_i}{k\tau}\right]^{-\frac{1}{q-1}}$$

We can show that with this ansatz $\beta_{\bar{g}} \propto g^3$, same behavior as in pQCD

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Scales in YMtheory

Fractal structure of gauge fields

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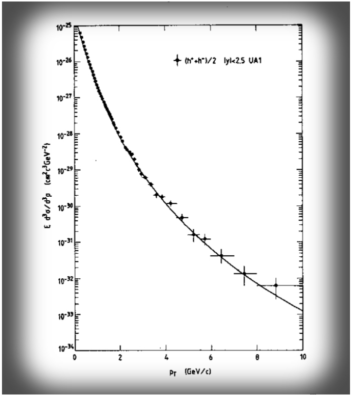
Comparison with experiments

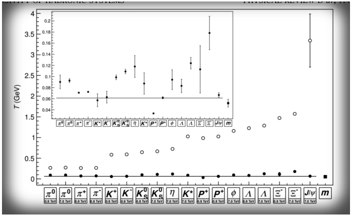
Extended Hagedorn theory to non extensive statistics: AD, Physica A 391 (2012) 6380

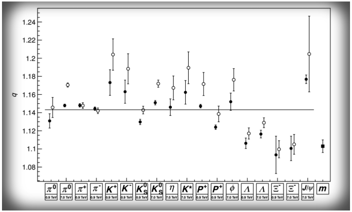
use of Tsallis factor: $P(\varepsilon) = A[1 + (q-1)\frac{\varepsilon}{k\tau}]^{-\frac{1}{q-1}}$

L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022

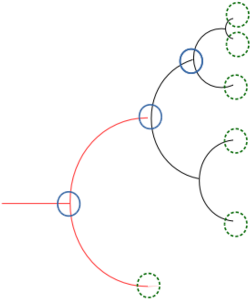
L. Marques, J. Cleymans, AD, PRD 91 (2015) 054025







Effective mass spectrum



$$\bar{g}(m,\varepsilon,T) = \left[\prod_{i=1}^{N'} \rho(m_i)[\tilde{P}(\varepsilon_i)]^{\nu}\right]$$

$$\left[M\frac{\partial}{\partial M} + \beta_{\bar{g}}\frac{\partial}{\partial \bar{g}} + d\right]\Gamma = 0$$

We can show that to satisfy CS Equation the mass spectrum must be given by:

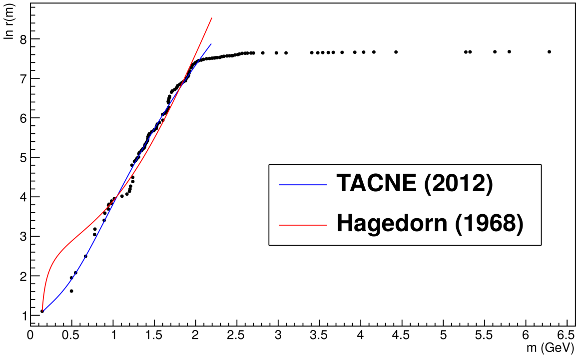
$$\rho(m) = \rho_o\,[1+(q-1)m/M]^{1/(q-1)}$$

Such result was already obtained by extending Hagedorn's Self-Consistent Thermodynamics to the non extensive case.

Effective mass spectrum and observed data

$$\rho(m) = \rho_o \left[1 + (q-1)m/M\right]^{1/(q-1)}$$

Obtained in Non Extensive Self-Consistent Thermodynamics, by a completely different approach AD, Phys. A 391 (2012) 6380



L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022

Summary of theory results

Scale invariance of gauge theory leads to

fractal structure

fractal dimension in multiparticle production

Tsallis statistics

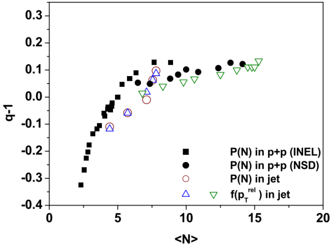
non extensive self-consistent thermodynamics

Experimental verification

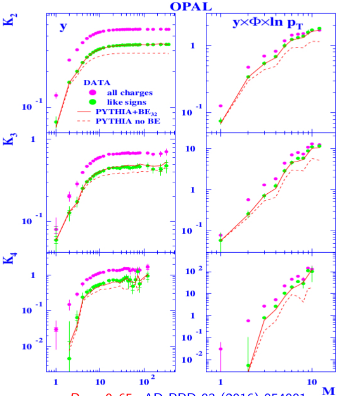
Scale invariance of gauge theory

- leads to fractal structure
- fractal dimension in multiparticle production
- Tsallis statistics
- non extensive self-consistent thermodynamics

self-similarity



fractal dimension - from intermittecy analysis



$D \sim 0.65$ AD PRD 93 (2016) 054001

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Scales in YMtheory

Fractal structure of gauge fields

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Experimental verification

Scale invariance of gauge theory

leads to fractal structure

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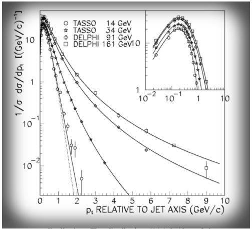
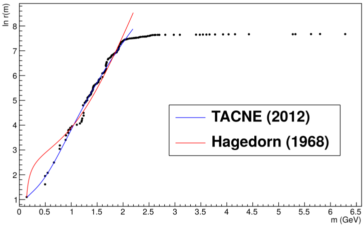


power-law distributions



non extensive mass spectrum

Applications

High energy collisions:

J. Cleymans; D.J. Worku. Phys. G Nucl. Part. Phys. 2012, 39, 025006
C.-Y. Wong; G. Wilk, G.; Tsallis, C. Phys. Rev. D 2015, 91, 11402
L. Marques, J. Cleymans, and AD, PRD 91 (2015) 054025

Hadron models:

P.H.G Cardoso; T.N. da Silva; AD; D.P. Menezes, EPJA 51 (2015) 155

Hadron mass spectrum:

L. Marques; E. Andrade-II; AD, Phys. Rev. D 2013, 87, 114022

Neutron stars:

D. P. Menezes, AD, E. Megias, and L. B. Castro, EPJA 51, (2015) 155

LQCD:

AD PG 41 (2014) 055108

Non extensive statistcs:

E. Megias, AD, D.P. Menezes, Physica A 421 (2015) 15
AD, Physica A 391 (2012) 6380
AD, E. Megias, D.P. Menezes, T. Frederico, (2018) to be published in Entropy

Conclusions:

Scale invariance in gauge fields leads to:

- Self-consistency and fractal struture
- Recursive calculations at any order
- Non extensive statistics
- Reconciles Hagedorn's theory with QCD
- Agreement with experimental data

Thank you