Emergence of non-extensive processes in QCD via fractal structures

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Summary

Basics of fractal and its geometry.

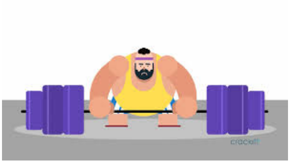
Fractal distributions in Thermodynamics - thermofractals

Thermofractals and Tsallis statistics.

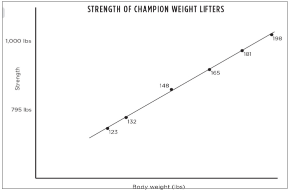
Yang-Mills fields and thermofractals.

Thermofractal in hadron structure? z-Scaling..

Scale and Self-Similarity



SCALING



SELF-SIMILARITY

A.D. - PRD 93 (2016) 054001

AD, T. Frederico, E. Megías, D.P. Menezes, Entropy 20 (2018) 633

Thermofractals

A thermofractal is a system defined by the following properties:

I- It is a system in thermodynamical equilibrium with total energy given by $U = K + E$, where K corresponds to the kinetic energy of N constituent subsystems and E describes the internal energy of those subsystems, which are endowed with an internal substructure.

II- The constituent particles are thermofractals which can be divided in two sublasses: Type I and Type II. For each subclass, the corresponding associated ratio, E/K or E/U, can vary according to a self-similar distribution, $P(U)$. This means the distribution of the internal energy is independent of the hierarchic level of the fractal structure.

III- At some level n in the hierarchy of subsystems the phase space is so narrow that one can neglect their internal structure and assume the following expression to the probability: $P(U_n)\, dU_n = \rho\, dU_n$

Thermofractal: Thermodynamics

For an ideal gas of elementary particles:

$$P(K)dK = (k_B T)^{-\frac{3N}{2}} K^{\frac{3N}{2}-1} \exp\left(-\frac{K}{k_B T}\right) dK$$

For a thermofractal:

$$P(U)dU = AK^{\frac{3N}{2}-1} \exp\left(-\frac{\alpha K}{k_B T}\right) dK [\tilde{P}(\varepsilon)]^{\nu} d\varepsilon$$

$$\alpha = 1 + \frac{\varepsilon}{NkT} \quad \varepsilon = \frac{E}{K} k_B T$$

Integration on K: $$P(U)\, dU = A\left[1 + \frac{\varepsilon}{NkT}\right]^{-3N/2} \tilde{P}(\varepsilon) d\varepsilon$$

Second property of thermofractals (self-similarity):

$$P(U) := \tilde{P}(\varepsilon) \Rightarrow P(\varepsilon) = A\left[1 + \frac{\varepsilon}{Nk_B T}\right]^{-\frac{3N}{2}\frac{1}{1-\nu}}$$

Introducing q and τ:

$$q - 1 = \frac{2}{3N}(1-\nu) \qquad \tau = \frac{2(1-\nu)}{3} T$$

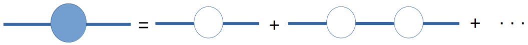
q-exponential distribution:

$$P(\varepsilon) = A\left[1 + (q-1)\frac{\varepsilon}{Nk_B \tau}\right]^{-\frac{1}{q-1}}$$

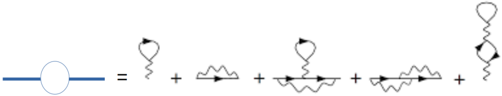
A.D., E. Megías,D.P. Menezes PRD (2020)

Fractals in Yang-Mill fields

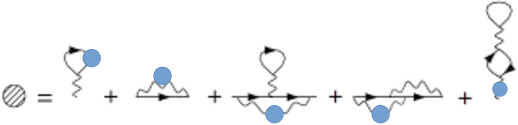
Self-energy interaction:



Self-energy diagrams:



Many-loops diagrams:



Complex structure of the effective parton:

A.D., E. Megías,D.P. Menezes PRD (2020)

Renormalization of gauge fields

Yang-Mills theory is renormalizable:

$$\Gamma(p, m, g) = \lambda^{-D}\Gamma(p, \mu, \bar{g})$$

F. Dyson, PR 75 (1949) 1736

Stuekelberg and Petermann, Helv. Phys. Acta 26 (1953) 499

M. Gell-Mann and F.E. Low, PR 95 (1954) 1300

Renormalization group equation:

$$\left[M\frac{\partial}{\partial M} + \beta_g\frac{\partial}{\partial \bar{g}} + d\right]\Gamma = 0$$

Effective coupling constant $\bar{g}$

Effective mass μ

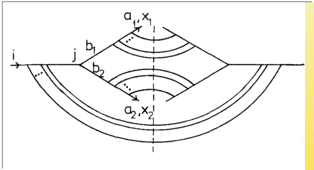
Callan-Symanzik Equation

C.G. Callan Jr., PRD 2 (1970) 1541

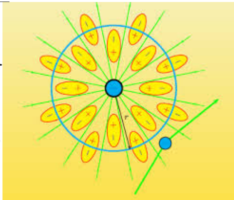
K. Symanzik, Comm. Math. Phys. 18 (1970) 227

Scaling properties are present in YMF

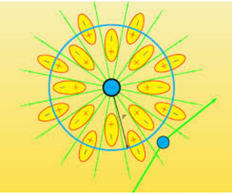
Is there internal structure?

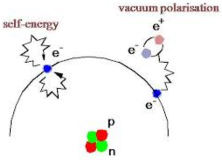


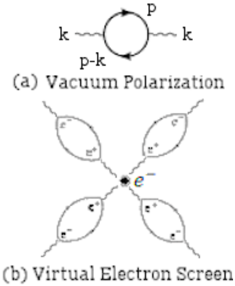
K. Konishi, Phys. Scr. 19 (1979) 195

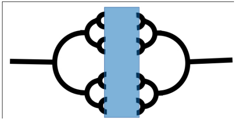


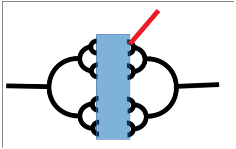
The effective parton











effective parton graph

proper vertex graph

Including fractal structure in YM fields

A.D., E..P.Menezes PRD 101, (2020) 034019

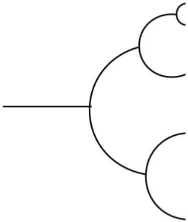
$$Z = Tr \langle \Psi_f | e^{-iHt} | \Psi_o \rangle$$

$$|\Psi\rangle = \sum_{\{n\}} \langle \Psi_n | \Psi \rangle \, |\Psi_n\rangle$$

n: order in pertubative calculation

$\{n\}$ the sum all graphs.

$$|\Psi_n\rangle = \frac{(-i)^n}{n!} e^{-iH_o(t_n - t_{n-1})} g \ldots e^{iH_o(t_2 - t_1)} g \, |\Psi_o\rangle$$



$$|\Psi_n\rangle = \sum_N \langle \psi_N | \Psi_n \rangle \, |\psi_N\rangle$$

N is the number of external lines

$$|\psi_N\rangle = \mathcal{S} \, |\gamma_1, m_1, p_1, \ldots, \gamma_N, m_N, p_N\rangle$$

$$\langle \psi_f | = \langle \gamma_o, m_o, p_o, \ldots |$$

$$\langle \gamma_o, m_o, p_o, \ldots | \Psi(t) \rangle = \sum_n \sum_N \langle \Psi_n | \Psi \rangle \, \langle \psi_N | \Psi_n \rangle \, \langle \gamma_o, m_o, p_o, \ldots | \psi_N \rangle$$

Including fractal structure in YM fields

A.D., E..P.Menezes PRD 101, (2020) 034019

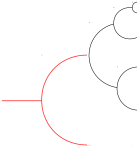
$$\langle \gamma_o, m_o, p_o, \dots | \Psi(t) \rangle = \sum_n \sum_N \langle \Psi_n | \Psi \rangle \langle \psi_N | \Psi_n \rangle \langle \gamma_o, m_o, p_o, \dots | \psi_N \rangle$$

$$\begin{cases} \langle \Psi_n | \Psi \rangle = G^n P(E) dE \\ \langle \psi_N | \Psi_n \rangle = A_N(n) \\ \langle \psi_f | = \langle \gamma_o, m_o, p_o, \dots | \end{cases}$$

$$\langle \gamma_o, m_o, p_o, \dots | \psi_N \rangle \rightarrow f(p_j) d^4 p_j$$

$$f(p_j) d^4 p_j = d^4 p_j \frac{1}{8\pi} \frac{\Gamma(4N)}{\Gamma(4(N-1))} E^{-4} \left(1 - \frac{p_j^0}{E} \right)^{4N-5}$$

$$\tilde{P}(p_o) = \langle \gamma_o, m_o, \dots | \Psi \rangle = \sum_n \sum_N G^n \left(\frac{N}{n\tilde{N}} \right)^4 \left(1 - \frac{\varepsilon_j}{M} \right)^{4N-5} d^4 \left(\frac{p}{M} \right) P(E) dE$$



Introducing self-similarity

Parent parton is also a parton $\rightarrow P(E) \propto \tilde{P}(p_o)$.

Self-symmetry in gauge fields!

Scaling factor: $P(\frac{E}{\mathcal{E}}) = \tilde{P}(\frac{p_o}{E})$ $\qquad \chi = \frac{\varepsilon}{\lambda} = \frac{p_j^o}{E} = \frac{E}{\mathcal{E}}$

It can be show that $P(\mu)$ must be such that:

$$P(\varepsilon) = A[1 - (q-1)\frac{\varepsilon}{\lambda}]^{\frac{1}{q-1}}$$

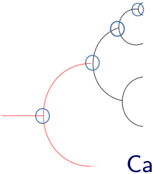
A.D. - PRD 93 (2016) 054001

AD, T. Frederico, E. Megías, D.P. Menezes, Entropy 20 (2018) 633

Non extensivity in gauge field theory

$$P(\varepsilon) = G^n[1-(q-1)\frac{\varepsilon}{\lambda}]^{\frac{1}{q-1}}$$

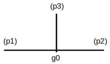
q - the number of internal degrees of freedom in the fractal structure that are relevant in the process of energy transfer to the effective parton

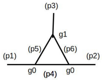


Describes how momentum and energy are distributed at each vertex:

$\bar{g} = \prod_i G\left[1-(q-1)\frac{\varepsilon_i}{k\tau}\right]^{\frac{1}{q-1}}$ effective coupling

Calculation of q from gauge field parameters: 1-loop approx. (QCD)





Fractal method: $\beta_g = -\frac{1}{q-1} g^{N'+1}$

CS equation: $\frac{1}{q-1} = d - \gamma$ $\Rightarrow \frac{1}{q-1} = \frac{11}{3}N_c - \frac{4}{3}N_f \Rightarrow q = 1.14$

QCD: $d - \gamma = \left[\frac{11}{3}c_1 - \frac{4}{3}c_2\right]$

Comparison with experiments

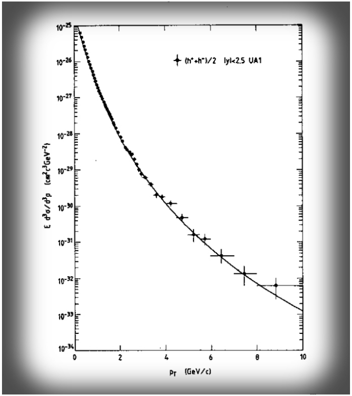
Extended Hagedorn theory to non extensive statistics: AD, Physica A 391 (2012) 6380

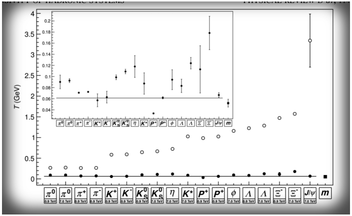
use of Tsallis factor: $P(\varepsilon) = A[1 + (q-1)\frac{\varepsilon}{k\tau}]^{-\frac{1}{q-1}}$

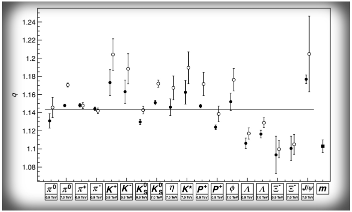
L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022

L. Marques, J. Cleymans, AD, PRD 91 (2015) 054025

Experimental value $q = 1.14 \pm 0.01$





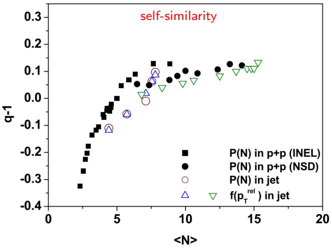


Experimental verification

Scale invariance of gauge theory

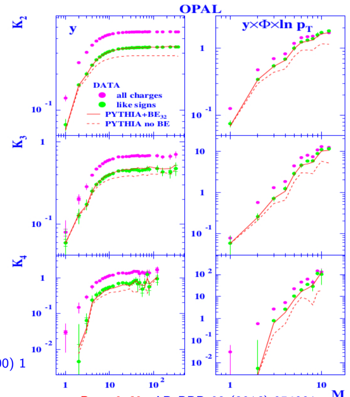
leads to fractal structure

fractal dimension in multiparticle production



G. Wilk and Z. Wlodarczyk, PLB 727 (2013) 163

fractal dimension - from intermittecy analysis



E. Sarkisyan-Grinbaum, PLB 477 (2000) 1

$D \sim 0.69$ AD PRD 93 (2016) 054001

Multiplicity and energy

Multiplicity as a manifestation of fractal aspects: AD, PRD 93 (2016) 054001

r is the scale in wich energies are measured.

$\varepsilon \sim r^{-D}$ is the scaling behavior of the individual parton energy.

$E \sim r^{-1}$ is the scaling behavior of the total energy.

$\mathcal{N}$ is the multiplicity.

R is the ratio between parton energy ε and its immediate parent energy

$\mathcal{N} r^{-D} \propto r^{-1} \to \mathcal{N} \propto r^{-1+D}$

$\mathcal{N} \propto E^{1-D}$

$D \sim 0.69$ from fractal dimension analysis and intermittence analysis

Theory: $1 - D \sim 0.31$

Experiment: $1 - D \sim 0.302$

E. Sarkisyan-Grinbaum et al. PRD 93 (2016) 054046

Experimental verification

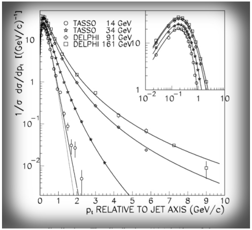
Scale invariance of gauge theory

leads to fractal structure

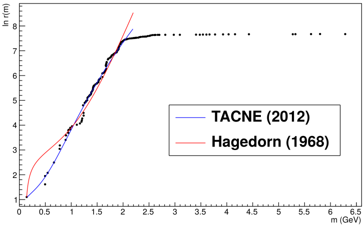
fractal dimension in multiparticle production

Tsallis statistics

non extensive self-consistent thermodynamics



power-law distributions



non extensive mass spectrum

Generalized Hagedorn Self-Consistent Thermodynamics

AD Physica A 391 (2012) 6380

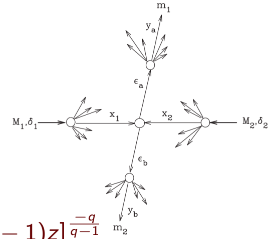
Fractal structure of hadrons (in development)

Thermofractal structure of hadrons:

$$\frac{d^4\sigma}{dq^4} = |\langle\varphi\phi|\, g(p)\, |\phi_o\rangle|^2 \delta(q^2 - m_q)$$

$$E\frac{d^2\sigma}{2\pi q_t dq_t\ dq_z} = \frac{1}{2}|\langle\varphi\phi|\, g(E)\, |\phi_o\rangle|^2 \propto e_q(\varepsilon/\lambda)^2$$

M. Tokarev, I. Zborovsky: PRD 56 (1996) 5548

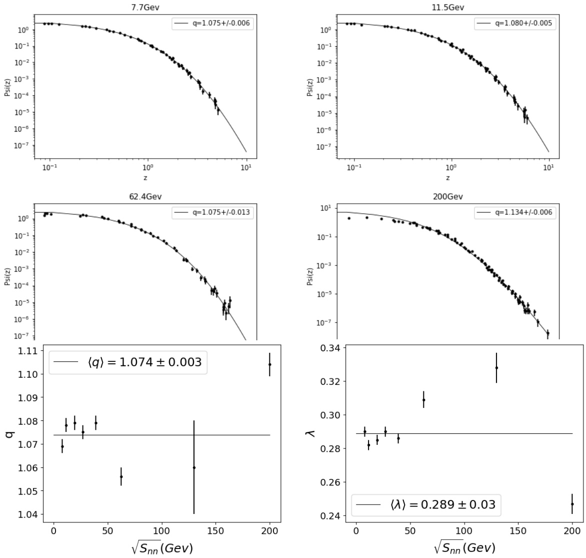


$$E\frac{d^2\sigma}{dq^2} = \frac{\sigma_0}{\pi s}\frac{d^2\sigma}{dx_1 dx_2}$$

z-scaling: $\psi(z) = \frac{d\sigma(z)}{dz}$

$$z = \frac{s_x^{1/2}}{Q} \quad \Rightarrow \psi(z) = [1 + (q-1)z]^{\frac{-q}{q-1}}$$

z-Scaling (in development)



Dynamics of quarks in the medium

Boltzmann Equation:

$$\frac{\partial f(\mathrm{p},t)}{\partial t} = -\nabla_{\mathrm{p}} f(\mathrm{p},t)\cdot \mathrm{F} + C[f]$$

$$h[f(\mathrm{p}),f(\mathrm{q})] = f(\mathrm{p})f(\mathrm{q})$$

Non-Additive Boltzmann Equation:

Correlation functional

$$h[f(\mathrm{p}),f(\mathrm{q})] = \left[f(\mathrm{p})^{1-q} + f(\mathrm{q})^{1-q} - 1\right]^{\frac{1}{1-q}}$$

Fokker-Planck Equation

$$\frac{\partial f}{\partial t} - \frac{\bar{\partial}}{\partial p_i}\left[A_i f + \frac{\bar{\partial}(B_{ij} f)}{\partial p_j}\right] = 0$$

Plastino-Plastino Equation

$$\frac{\partial f}{\partial t} - \frac{\bar{\partial}}{\partial p_i}\left[A_i f + \frac{\bar{\partial}(B_{ij} f^{2-q})}{\partial p_j}\right] = 0$$

Plastino-Plastino Equation was proved for the 1st time in
A.D., E. Megias, A. Golmankhaneh and R. Pasechnik in
PLB (2023) accept - online soon

Conclusions:

Scale invariance and complex structure leads to:

- Self-consistency and fractal struture
- Recursive calculations at any order
- Non extensive statistics
- Reconciles Hagedorn's theory with QCD
- Agreement with experimental data
- Short review in A.D, E. Megìas and D.P. Menezes, Physics (2020)
- Thermofractal transformation group: scale and complexity transf.
- Scale transformation algebra $\rightarrow$ q-algebra
- Complexity transformation algebra extends the q-algebra

  A.D, Physics 3 (2021) 290
- Hints of fractal hadrons through z-scaling

Thank you