

Lecture - Fractal Spaces

Scaling Cities

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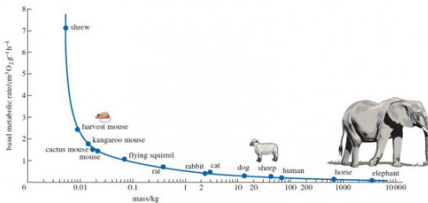
2020

Allometric Interactions

A General Model for the Origin of Allometric Scaling Laws in Biology

GEOFFREY B. WEST, JAMES H. BROWN, AND BRIAN J. ENQUIST [Authors Info & Affiliations](#)

SCIENCE • 4 Apr 1997 • Vol 276, Issue 5309 • pp. 122-126 • DOI:10.1126/science.276.5309.122

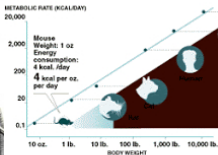


Metabolic rate/mass

From the Small to the Huge

Three scientists have proposed a novel theory to explain how characteristics like body size and energy consumption differ from species to species along fixed scales. Their theory derives from analysis of the circulatory system.

An Example of Scaling: Metabolic Rate



Size and Efficiency

The average elephant weighs 220,000 times as much as the average mouse, but requires only about 10,000 times as much energy in the form of food calories to sustain itself. The

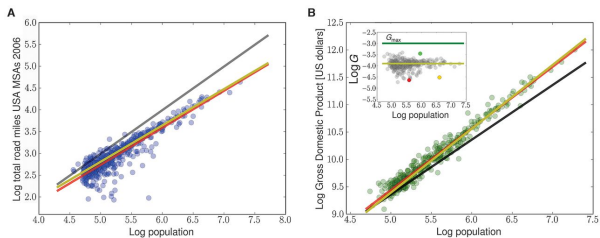
reason lies in the mathematical and geometric nature of the circulatory system that distributes nutrients and carry away wastes as the animal grows. The bigger the animal, the more efficiently it uses energy.

Metabolic rate

The Origins of Scaling in Cities

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Social Interactions

Individual social interaction: $S_i = \sigma_i l_i \frac{N}{A}$

Social output: $S = NS_i = \sigma_i l_i \frac{N^2}{A}$

Linear measure: $L = A^{1/D}$

Commutation Cost: $C = \varepsilon NL$

Area as a function of N: $A(N) = aN^\alpha$

Yield: $Y = \frac{S}{C} = \frac{\sigma_i l_i}{\varepsilon a^{(D+1)/D}} N^{[1-\alpha(1+1/D)]}$

Stability Condition: Y_{min} is independent on N : $\alpha = \frac{D}{1+D}$

Fractal Path

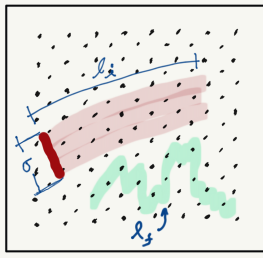
Linear measure of the city considering fractal path: $L_f = aA^{1/D_f}$

Fractal dimension: $D_f = D/\lambda$

Fractal Yield: $Y_f = yNA^{-(\lambda+\lambda/D)} = yN^{1-\alpha(\lambda+\lambda/D)}$

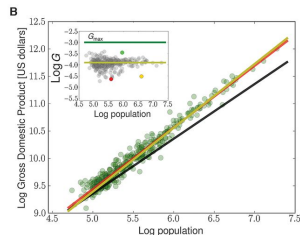
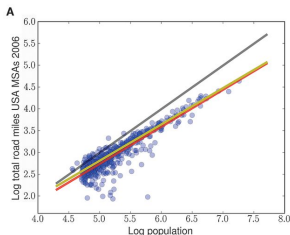
$$\alpha(\lambda + \lambda/D) = \lambda = 1 - \rho$$

$$Y_f = yN^\rho \text{ and } \begin{cases} A_f = A^\lambda = N^{1-\rho} \\ S_f = s \frac{N^2}{A_f} = sN^2 N^{-(1-\rho)} = sN^{1+\rho} \end{cases}$$



Social Interactions - real life

$$Y_f = yN^\rho \text{ and } \begin{cases} A_f = N^{1-\rho} \\ S_f = sN^{1+\rho} \end{cases}$$



$$\rho = 1/6 \text{ and } \begin{cases} A_f = N^{5/6} \\ S_f = sN^{7/6} \end{cases}$$