Lecture - Fractal Spaces
Scaling Cities

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Allometric Interactions

A General Model for the Origin of Allometric Scaling Laws in Biology

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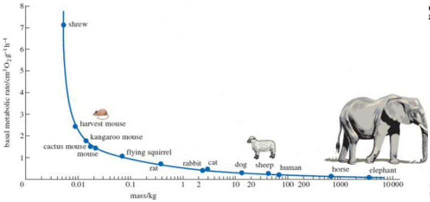
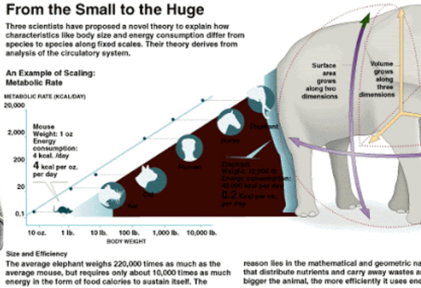
SCIENCE • 4 Apr 1997 • Vol 276, Issue 5309 • pp. 122-126 • DOI: 10.1126/science.276.5309.122





Metabolic rate/mass

Metabolic rate

Social Interactions

The Origins of Scaling in Cities

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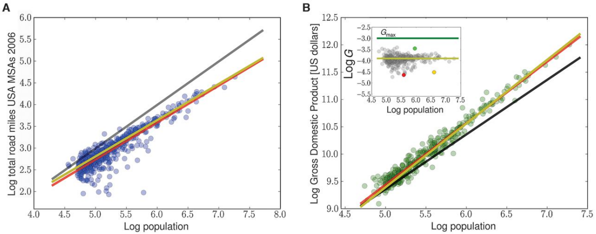
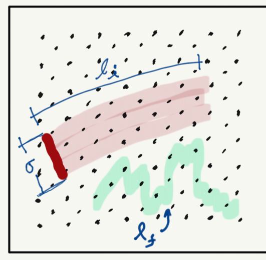
SCIENCE • 21 Jun 2013 • Vol 340, Issue 6139 • pp. 1438-1441 • DOI: 10.1126/science.1235823



Social Interactions



Individual social interaction: $S_i = \sigma_i l_i \frac{N}{A}$

Social output: $S = NS_i = \sigma_i l_i \frac{N^2}{A}$

Linear measure: $L = A^{1/D}$

Commutation Cost: $C = \varepsilon N L$

Area as a function of N: $A(N) = aN^{\alpha}$

Yield: $Y = \frac{S}{C} = \frac{\sigma_i l_i}{\varepsilon a^{(D+1)/D}} N^{[1-\alpha(1+1/D)]}$

Stability Condition: Y_{min} is independent on N: $\alpha = \frac{D}{1+D}$

Fractal Path

Linear measure of the city considering fractal path: $L_f = aA^{1/D_f}$

Fractal dimension: $D_f = D/\lambda$

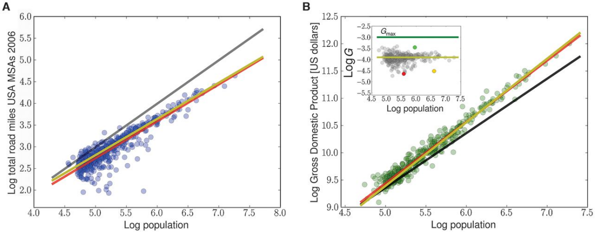
Fractal Yeld: $Y_f = yNA^{-(\lambda+\lambda/D)} = yN^{1-\alpha(\lambda+\lambda/D)}$

$$\alpha(\lambda + \lambda/D) = \lambda = 1 - \rho$$

$$Y_f = yN^{\rho} \text{ and } \begin{cases} A_f = A^{\lambda} = N^{1-\rho} \\ S_f = s\frac{N^2}{A_f} = sN^2 N^{-(1-\rho)} = sN^{1+\rho} \end{cases}$$

Social Interactions - real life

$$Y_f = yN^{\rho} \text{ and } \begin{cases} A_f = N^{1-\rho} \\ S_f = sN^{1+\rho} \end{cases}$$



$$\rho = 1/6 \text{ and } \begin{cases} A_f = N^{5/6} \\ S_f = sN^{7/6} \end{cases}$$