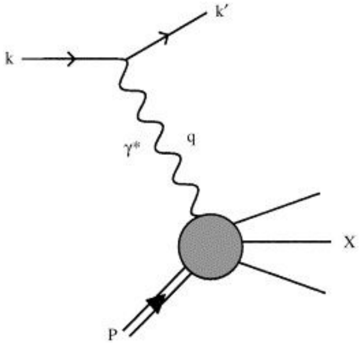
From the parton model to QCD

Parton evolution and scaling violation

Deep Inelastic Scattering (DIS)



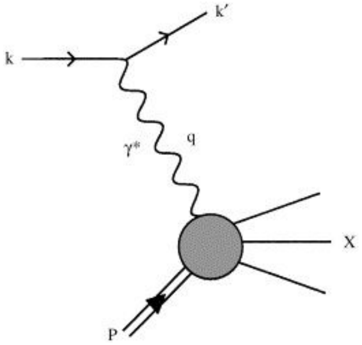
Note: virtual photon is space-like ($q^2<0$) $Q^2 = -q^2$

Lepton-Hadron scattering, e.g. electron scattering from a proton by exchanging a virtual photon

High momentum transfer ejects quark from the proton $\Rightarrow$ Proton shatters into a jet of hadrons

(Thus the name deep inelastic scattering)

Deep Inelastic Scattering (DIS)



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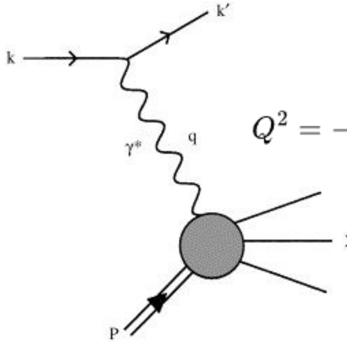
Used to probe the structure of hadrons

Cross section data $\Leftrightarrow$ Quarks and gluons distributions

In general the DIS cross section can be written as



$$\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

$$Q^2 = -q^2$$

Where $L_{\mu\nu}$ is the leptonic tensor and $W^{\mu\nu}$ is the hadronic tensor, which following Lorentz invariance and current conservation, can be written as

$$W^{\mu\nu}(q,p) = \tilde{g}^{\mu\nu} W_1(x,Q) + \tilde{p}^{\mu}\tilde{p}^{\nu} W_2(x,Q)$$

where

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

$$\tilde{p}^{\mu} = (p^\mu - \frac{q \cdot p}{q^2} q^\mu)/M$$

$$x = Q^2/2q \cdot p$$

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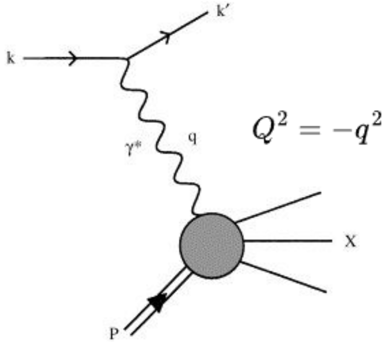

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$$\tilde{p}^\mu = (p^\mu - \frac{q \cdot p}{q^2} q^\mu)/M$$

And the structure functions are defined by

$$x = Q^2/2q \cdot p$$

$$F_1(x,Q^2) = W_1(x,Q^2) \qquad F_2(x,Q^2) = \nu W_2(x,Q^2)$$

$$\nu = q \cdot p/M$$

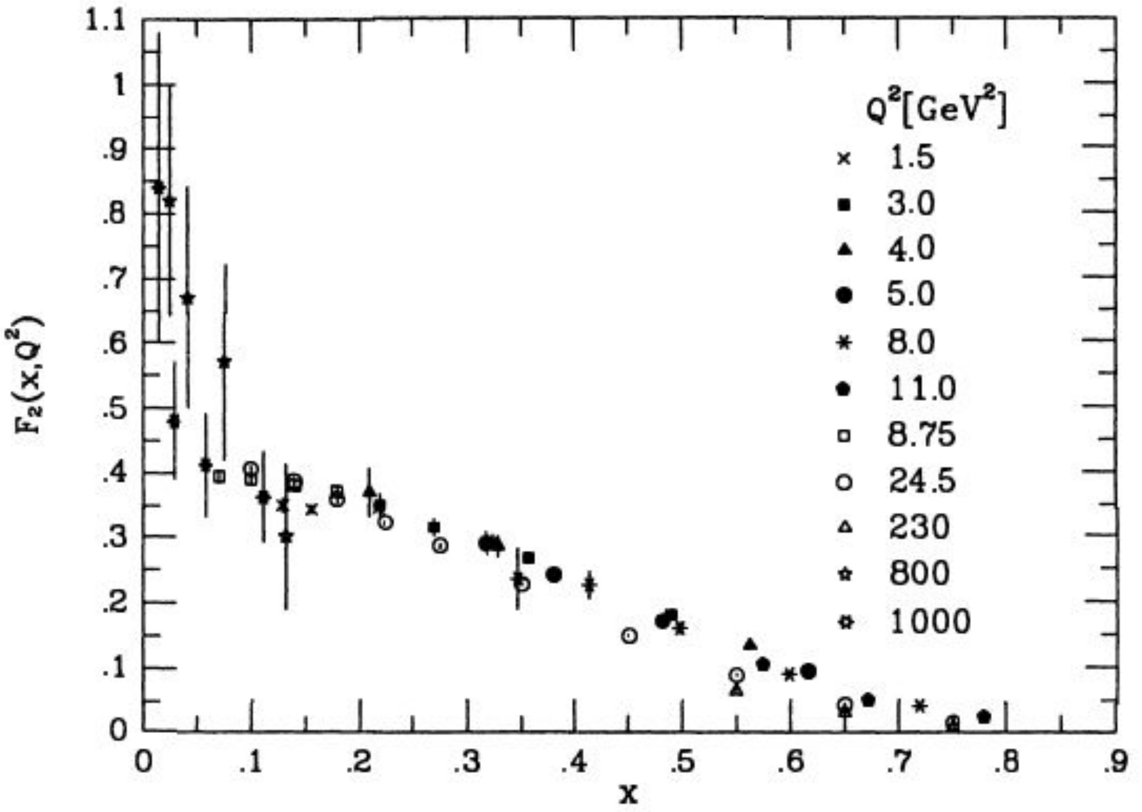
In 1968 Bjorken proposed that the structure functions may exhibit scaling behavior in the limit $Q^2 \to \infty$

Because increasing Q^2 implies higher spatial resolution, this suggests point-like substructure

Around the same time, the SLAC-MIT experiment on DIS revealed scaling behaviour

This led Feynman to propose the Parton Model, where the nucleon was composed of point-like free constituents, that later turned out to be quarks and gluon

The ideas of Bjorken and Feynman inspired the the idea of asymptotic freedom and the formulation of QCD



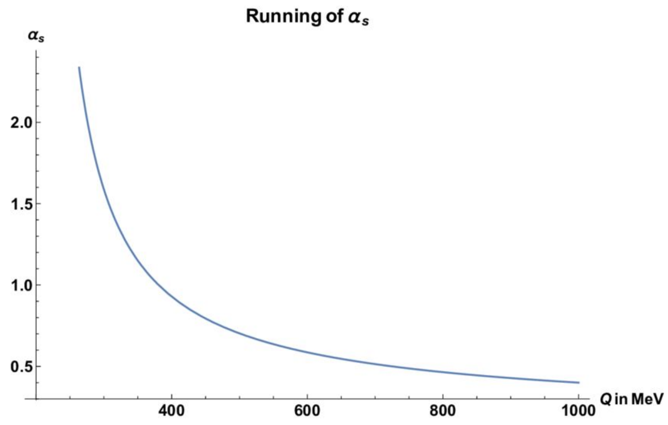
F_2 data from SLAC-MIT, BCDMS, H1 and Zeus collaborations lies approximately on a universal curve, showing scaling behaviour

The Parton Model

$$\sigma \propto 1 + c_1\alpha_s(Q) + c_2\alpha_s(Q)^2 + \ldots$$

Small α_s at high energies (> 1 GeV) - Nucleon as a collection of almost free constituents

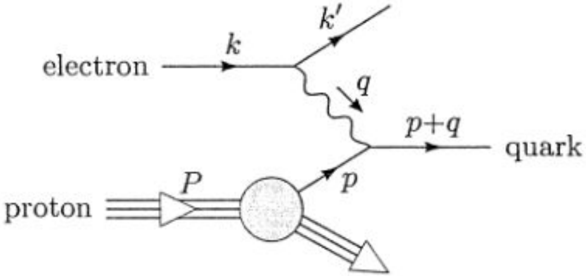
Suppression of gluon exchange at leading order $\Longrightarrow$ Quarks momentum collinear with the proton



$$p = \xi P$$

Where ξ is the longitudinal momentum fraction of the proton

We consider proton and constituents with lighlike momenta at center of mass frame (P>>M)



The probability of finding a constituent of the proton of type f at longitudinal fraction ξ $= f_f(\xi)d\xi$

$f_f(\xi)$ are the so called **Parton Distribution Functions**

Then in the parton model we can factorize the cross-section of the electron-proton DIS as:

$$\sigma(e^-(k)p(P) \to e^-(k) + X) = \int_0^1 d\xi \sum_f f_f(\xi)\sigma(e^-(k)q_f(\xi P) \to e^-(k) + q_f(p'))$$

Considering the outgoing quark approximately massless

$$0 \simeq (p+q)^2 = 2p\cdot q + q^2 = 2\xi P\cdot q - Q^2 \quad \Rightarrow \quad \xi = x \quad \text{Where} \quad x = \frac{Q^2}{2P\cdot q}$$

We can parametrize the differential cross section as

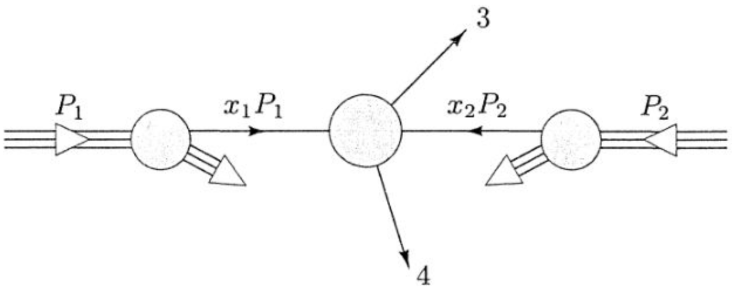
$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1(x) + \frac{1-y}{x}(F_2(x) - 2xF_1(x))\right]$$

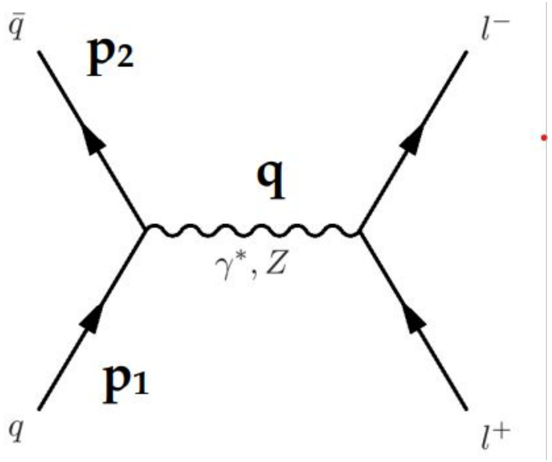
Where F_1 and F_2 are the proton **structure functions**, obeying

$$F_2 = 2xF_1 = \sum_f xf_f(x)Q_f^2$$ (Callan-Gross relation for fermions) Q_f is the charge of the quark of flavor f

$$\frac{d^2\sigma}{dxdy}(e^-p \to e^-X) = \left(\sum_f xf_f(x)Q_f^2\right)\frac{2\pi\alpha^2 s}{Q^4}\left[1+(1-y)^2\right] \qquad y = \frac{Q^2}{xs}$$

The left term, which concerns the internal structure of the nucleon, only depends on x and not Q^2





In the hadron center of mass frame

$$
\begin{aligned}
p_1 &= (x_1 E_{\text{beam}}/2, 0, 0, x_1 E_{\text{beam}}/2) \\
p_2 &= (x_2 E_{\text{beam}}/2, 0, 0, -x_2 E_{\text{beam}}/2) \\
q &= ((x_1 + x_2) E_{\text{beam}}/2, 0, 0, (x_1 - x_2) E_{\text{beam}}/2)
\end{aligned}
$$

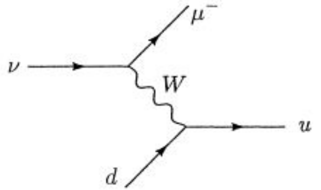
$$y = \frac{1}{2} \log \frac{E + |p|}{E - |p|} \quad \Rightarrow \quad y = \frac{1}{2} \log \frac{q_0 + q_z}{q_0 - q_z} = \frac{1}{2} \log \frac{x_1}{x_2} \quad \Rightarrow \quad e^{2y} = \frac{x_1}{x_2}$$

How to determine the distribution functions?

$$\frac{d^2\sigma}{dxdy}(e^- p \to e^- X) = \left(\sum_f x f_f(x) Q_f^2\right) \frac{2\pi\alpha^2 s}{Q^4}\left[1 + (1-y)^2\right]$$

The proton structure function obtained by photon exchange sums over all quarks distributions

Instead we can use neutrino scattering to determine the individual quark distributions, mediated by the weak interaction



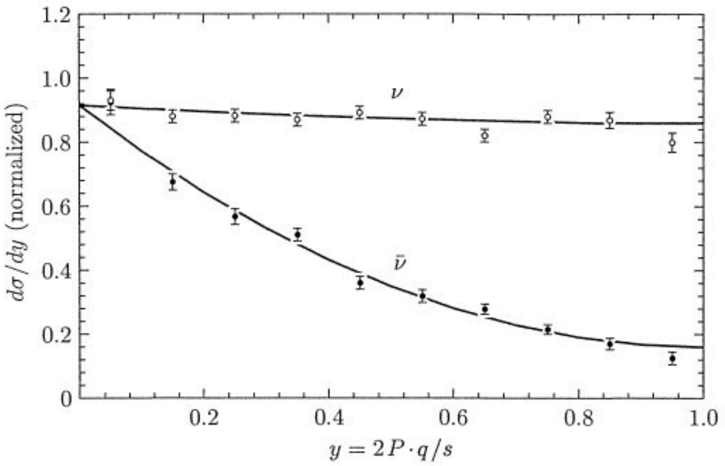
$d \to u$

$\bar{u} \to \bar{d}$

The neutrino/antineutrino differential cross sections are given by

$$\frac{d^2\sigma}{dxdy}(\nu p \to \mu^- X) = \frac{G_F^2 s}{\pi}\left[x f_d(x) + x f_{\bar{u}}(x)\cdot(1-y)^2\right]$$

$$\frac{d^2\sigma}{dxdy}(\bar{\nu} p \to \mu^+ X) = \frac{G_F^2 s}{\pi}\left[x f_u(x)\cdot(1-y)^2 + x f_{\bar{d}}(x)\right]$$



Fitting the curves with the form $A + B(1-y)^2$ where the parameters A and B are proportional to the structure functions

Parton Distribution Functions (PDF)

Proton internal structure = uud quarks + quark-antiquark pairs
So, to normalize the distributions we impose

$$\int_0^1 dx\,(f_u(x) - f_{\bar{u}}(x)) = 2 \qquad \int_0^1 dx\,\left(f_d(x) - f_{\bar{d}}(x)\right) = 1$$

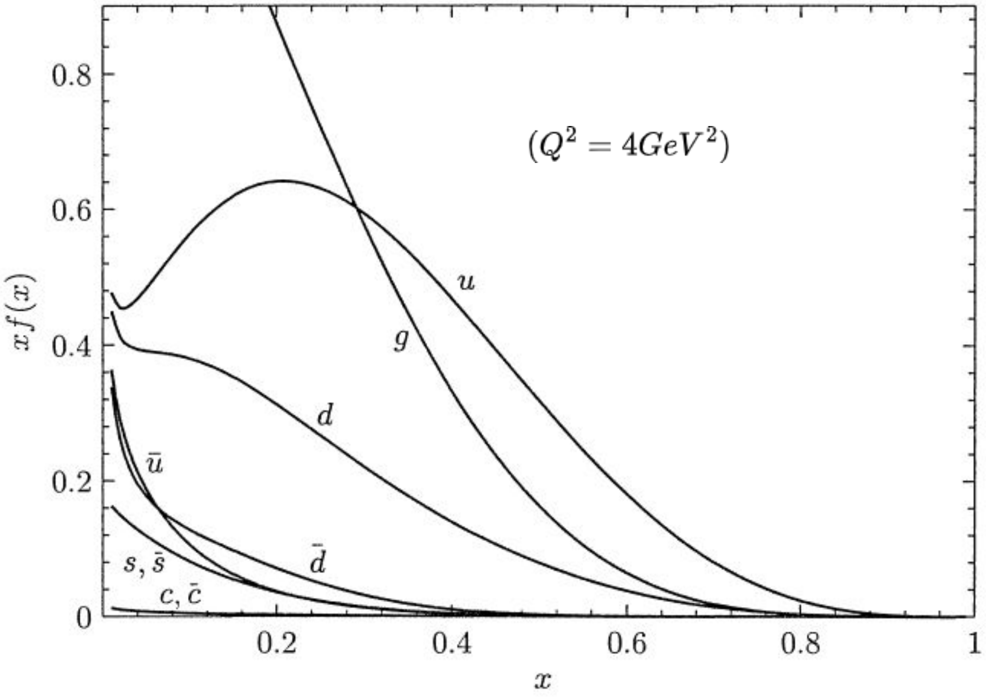
Similarly for the neutron we have $f_d^n(x) = f_u(x)$ $\quad f_u^n(x) = f_d(x)$ $\quad f_{\bar{u}}^n(x) = f_{\bar{d}}(x)$

$$\int_0^1 dx\,(f_u^n(x) - f_{\bar{u}}^n(x)) = 1 \qquad \int_0^1 dx\,\left(f_d^n(x) - f_{\bar{d}}^n(x)\right) = 2$$

Each hadron obeys sum rules that reflects it’s particular quantum numbers

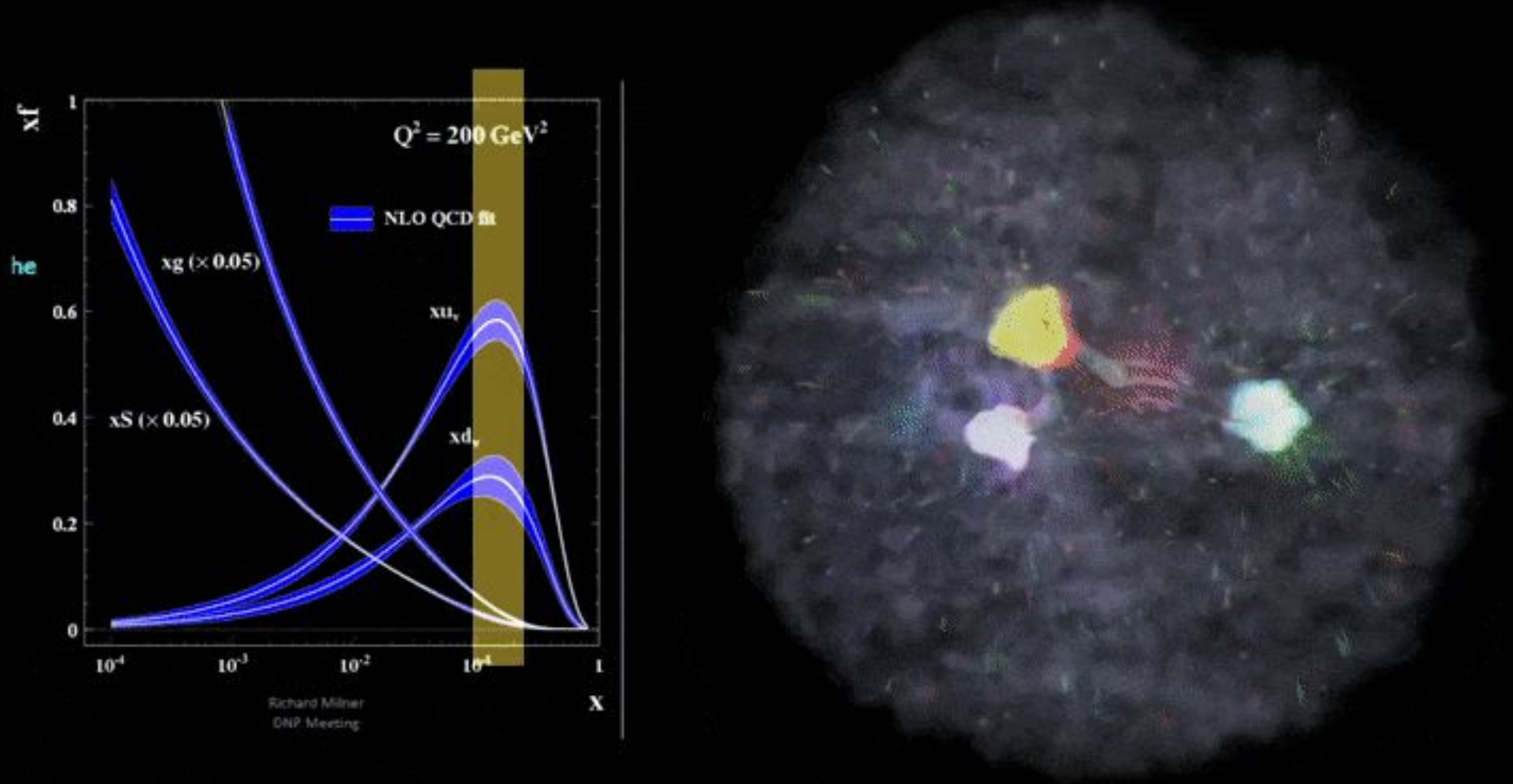
Total momentum carried by the partons = total momentum of the hadron

$$\Rightarrow \quad \int_0^1 dx\,x\left[f_u(x) + f_d(x) + f_{\bar{u}}(x) + f_{\bar{d}}(x) + f_g(x)\right] = 1$$



So far we've only talked about quarks

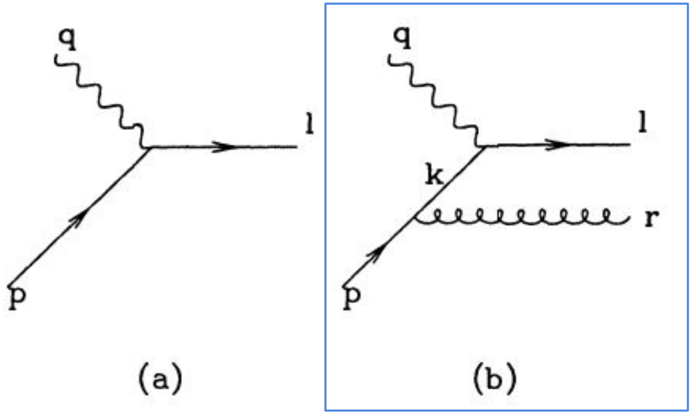
Quarks-antiquarks measured from DIS carry approx. half of the proton's momentum, the other half must be gluons https://scitechdaily.com/images/Visualizing-the-Proton-Animation.gif



Parton evolution and scaling violation

Until now we considered the parton model at leading order where we assumed the partons as free constituents moving collinearly with the proton.

However, when we consider the $O(\alpha_s)$ corrections, the partons can emit a gluon acquiring transverse momentum, getting contributions proportional to $\alpha_s log(Q^2)$



$\Rightarrow$ Scaling violation

Defining $f_q(x) = q(x)$

We can write q(x) as a distribution

$$q(x) = \int dy q(y)\delta(x-y) = \int \frac{dy}{y} q(y)\delta(1 - x/y)$$

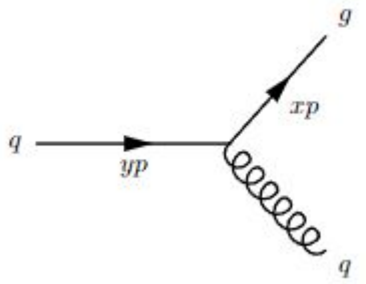
Considering the process of a quark interacting with a photon and emitting a gluon, then $x/y \neq 1$

The cross section is modified by changing the probability density

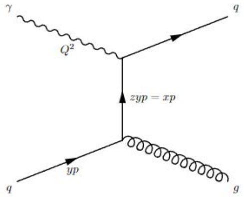
$$\delta(1 - x/y) \Rightarrow \delta(1 - x/y) + f(x/y, Q^2)$$

$$q(x) \longrightarrow q(x, Q^2) = q(x) + \Delta q(x, Q^2) = q(x) + \int_x^1 \frac{dy}{y} q(y) f(x/y, Q^2)$$

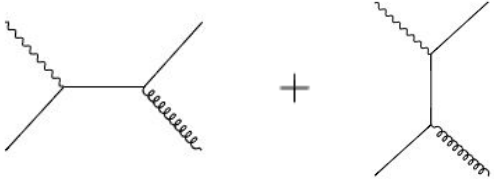
Where $f(x/y, Q^2)$ is the probability density of a quark with momentum yp becoming a quark with momentum xp by emitting a gluon



The new distribution is related to the amplitude of the process $\gamma q \longrightarrow gq$



$M =$



$$z = \frac{x}{y}$$

$$\frac{d\sigma}{dt}(\gamma q \longrightarrow gq) = -\frac{|\bar{M}|^2}{16\pi}\frac{z^2}{Q^4} = \sigma_0 Q_q^2 \left(\frac{\alpha_s}{2\pi}\right)\frac{4}{3}\left[\frac{1}{t}\left(\frac{1+z^2}{1-z}\right) + \frac{z^2(t+2Q^2)}{(1-z)Q^4}\right]$$

$$q(x,Q^2) = q(x) + \left(\frac{\alpha_s}{2\pi}\right)\int_x^1 \frac{dy}{y} q(y) \int_0^{-Q^2} dt \frac{4}{3}\left[\frac{1}{t}\left(\frac{1+z^2}{1-z}\right) + \frac{z^2(t+2Q^2)}{(1-z)Q^4}\right]$$

We notice 2 IR divergences

- A soft divergence for $z \to 1$, where the gluon momentum tends to 0
- When t=0, where the emitted gluon is massless and collinear with the quark

The soft divergence is cancelled by the contribution of the vertex correction diagram

The collinear divergence comes from the assumption that perturbative QCD is valid at all scales, but below ~1GeV it becomes non-perturbative. So we introduce a cut off in the integral $t = -\mu^2$

Then at high Q^2 , we have

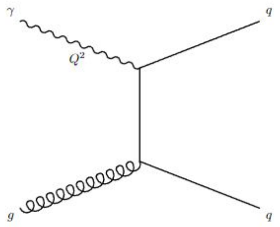
$$\Delta q = \frac{\alpha_s}{2\pi} t \int_x^1 \frac{dy}{y} q(y) P_{qq}\left(\frac{x}{y}\right)$$ Where $$t = ln\left(\frac{Q^2}{\mu^2}\right) \Rightarrow O(ln(Q^2))$$ Correction to the pdf

And

$$p_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

Where P(z) is the function associated with the probability of a quark with momentum yp becoming a quark with momentum xp by emitting a gluon

For the total cross section of the electron-proton scattering we also need to consider the process $\gamma g \longrightarrow q\bar{q}$



Which will add a term $P_{qg}(z)$ related to the probability of a gluon with momentum yp creating a quark antiquark pair

So the full quark distribution function can be written as

$$q(x,t) = q(x) + \frac{\alpha_s}{2\pi} t \int_x^1 \frac{dy}{y} \left[q(y,t) P_{qq}\left(\frac{x}{y}\right) + g(y,t) P_{qg}\left(\frac{x}{y}\right) \right]$$

Splitting functions

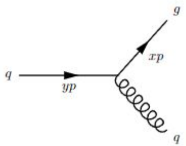
Probability of a parton with momentum yp becoming another parton with momentum xp by emission of a gluon or $q\bar{q}$ pair creation

Calculable as a series in α_s

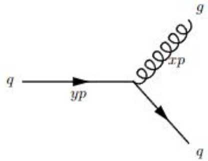
$$P(z,\alpha_s) = P^0(z) + \frac{\alpha_S}{2\pi}P^1(z) + \ldots$$

At leading order they are

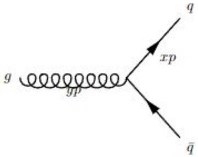
$$p_{qq}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$



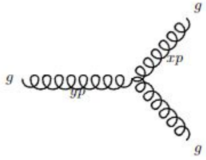
$$p_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$



$$p_{qg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$



$$p_{gg}(z) = \frac{4}{3}\left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$



DGLAP equations

(...or GLAP or AP)

$$\frac{dq(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,t)P_{qq}\left(\frac{x}{y}\right) + g(y,t)P_{qg}\left(\frac{x}{y}\right)\right]$$

$$\frac{dg(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_j q_j(y,t)P_{gq}\left(\frac{x}{y}\right) + g(y,t)P_{gg}\left(\frac{x}{y}\right)\right]$$

Usually solved using mellin moments

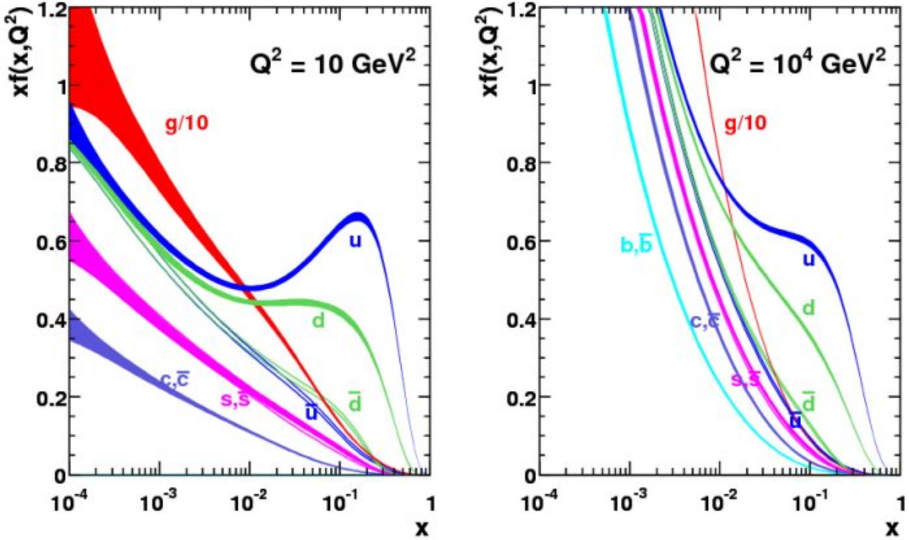
$$f(j,t) = \int_0^1 dx\; x^{j-1}\, f(x,t)\;, \qquad f = q_i, g$$

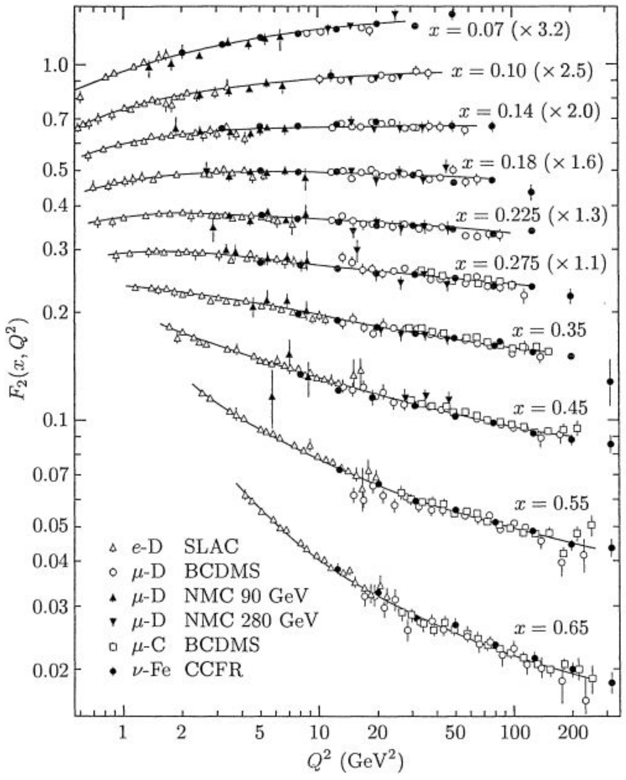
$$\gamma_{qq}(j,\alpha_s) = \int_0^1 dx\; x^{j-1} P_{qq}(x,\alpha_s)$$

$$t\frac{\partial}{\partial t}\begin{pmatrix}\Sigma(j,t)\\ g(j,t)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\gamma_{qq}(j,\alpha_s(t)) & 2n_f\gamma_{qg}(j,\alpha_s(t))\\ \gamma_{gq}(j,\alpha_s(t)) & \gamma_{gg}(j,\alpha_s(t))\end{pmatrix}\begin{pmatrix}\Sigma(j,t)\\ g(j,t)\end{pmatrix}$$

DGLAP equations

Bjorken scaling violation





Applications

Using the DGLAP equations we can evolve the measured parton distribution into higher energies and get a *renormalization equation* for PDF's

We can factorize qcd cross sections into convolutions of:

- hard (perturbative), process-dependent partonic subprocess
- non-perturbative, process-independent parton distribution functions

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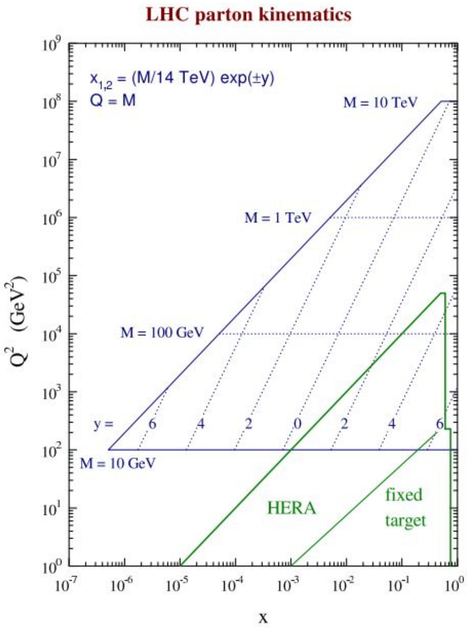
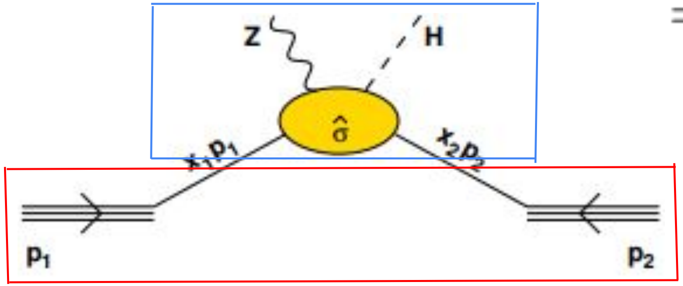


E.g.: Hadron colliders



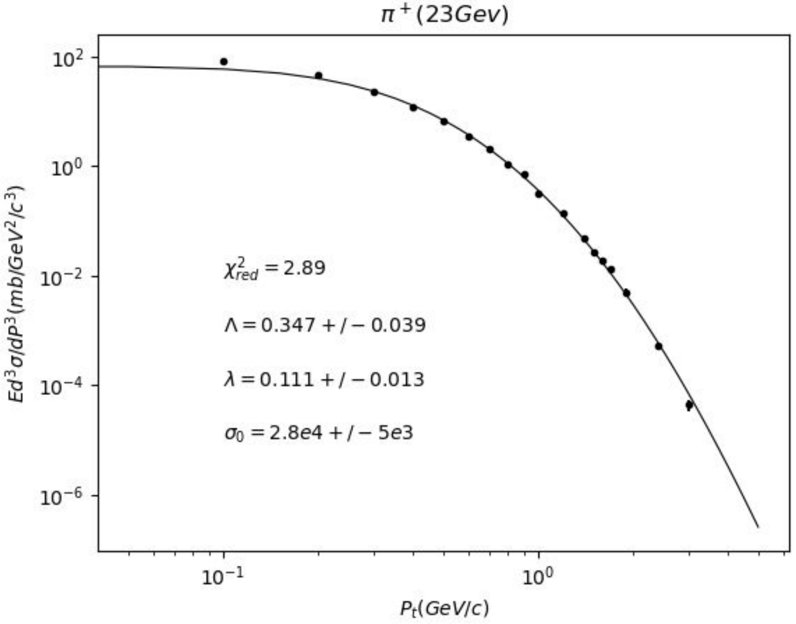
$\Rightarrow$ Very high energies

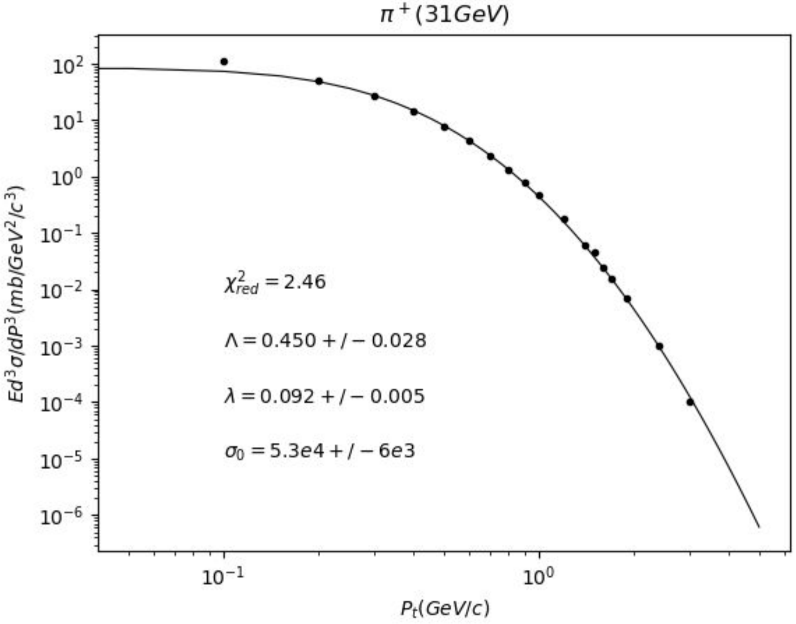
$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$$

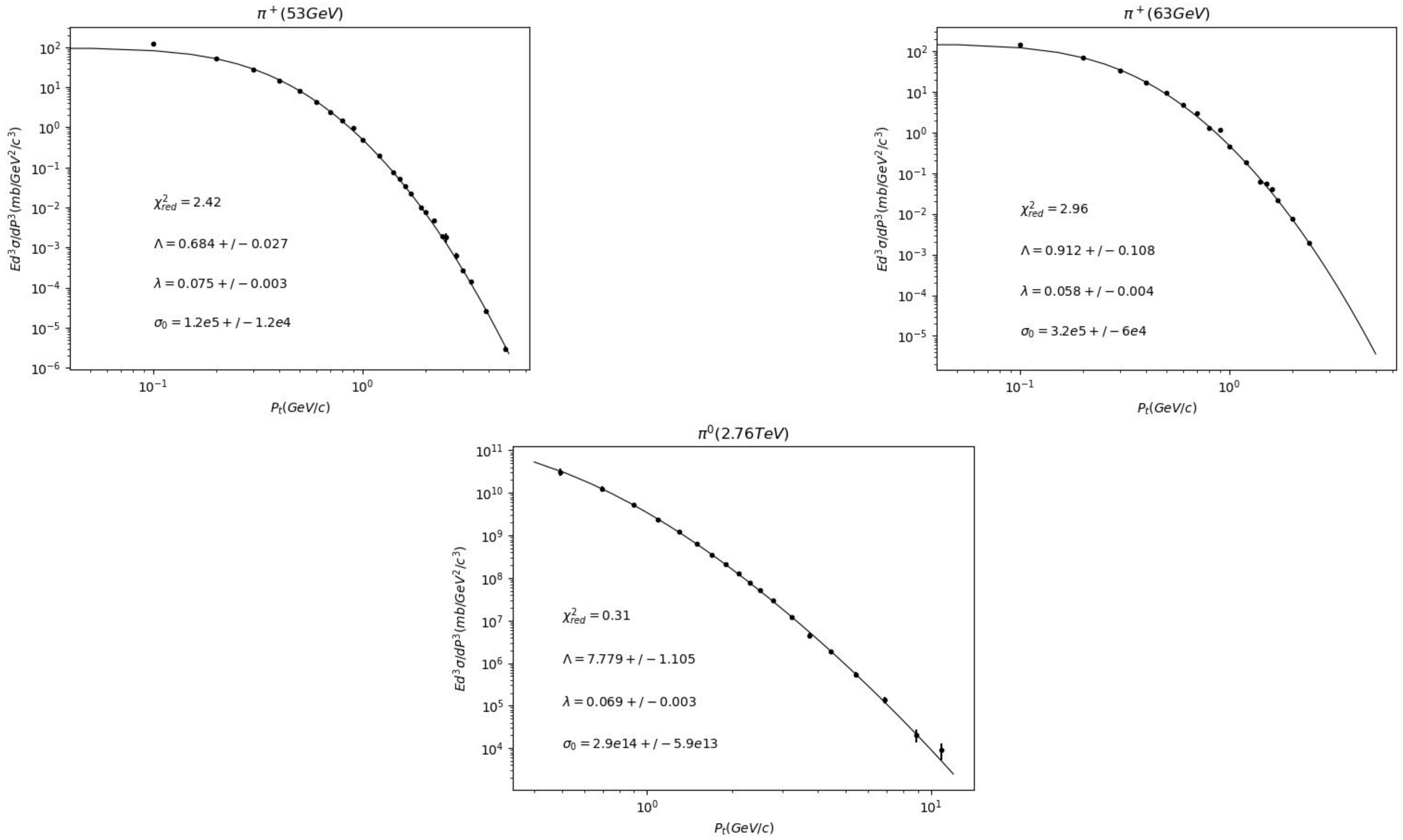
Conexão com termofractal

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2) \quad \Longrightarrow \quad E\frac{d^3\sigma}{dp^3} = \int dx_1 dx_2 e_{\bar{q}}\left(\frac{x_1 E_1}{\Lambda}\right) e_{\bar{q}}\left(\frac{x_2 E_2}{\Lambda}\right) e_q\left(\frac{\varepsilon_m}{\lambda}\right)$$

$$x_1 E_1 + x_2 E_2 = \varepsilon_m \Rightarrow (x_1 + x_2)\frac{\sqrt{s}}{2} = \varepsilon_m \Rightarrow x_1 = \frac{2}{\sqrt{s}}\varepsilon_m - x_2$$

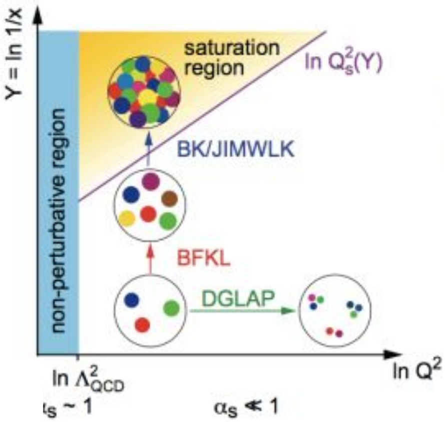






$\pi^+(53GeV)$
$Ed^3\sigma/dP^3(mb/GeV^2/c^3)$
$P_t(GeV/c)$
$\chi^2_{red} = 2.42$
$\Lambda = 0.684 +/- 0.027$
$\lambda = 0.075 +/- 0.003$
$\sigma_0 = 1.2e5 +/- 1.2e4$
$\pi^+(63GeV)$
$Ed^3\sigma/dP^3(mb/GeV^2/c^3)$
$P_t(GeV/c)$
$\chi^2_{red} = 2.96$
$\Lambda = 0.912 +/- 0.108$
$\lambda = 0.058 +/- 0.004$
$\sigma_0 = 3.2e5 +/- 6e4$
$\pi^0(2.76TeV)$
$Ed^3\sigma/dP^3(mb/GeV^2/c^3)$
$P_t(GeV/c)$
$\chi^2_{red} = 0.31$
$\Lambda = 7.779 +/- 1.105$
$\lambda = 0.069 +/- 0.003$
$\sigma_0 = 2.9e14 +/- 5.9e13$

Open questions and future EIC

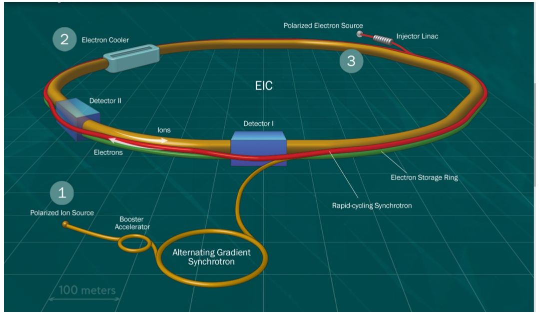


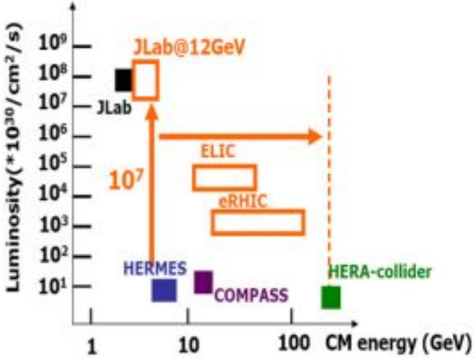
Color-glass condensate - as x go to very small values gluon distribution grows rapidly until saturation (gluon recombination balances gluon splitting) = new properties of hadronic matter

Confinement - why quark and gluons are confined in hadrons

Proton spin - how the spin of the proton emerges from quarks and gluon constituents

eRHIC
IHeC
ELIC





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