Brosa model concepts

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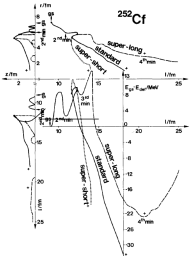

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Nuclear Scission by Ulrich Brosa (`Brosa model`)

- **Scission versus fission**
    - "Scission" denotes the "instant of rupture"
    - do not discuss the complete process of fission

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  - several exit channels in fission
  - Leaving the compound state, the nucleus may choose between various paths to disintegrate.

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  - several exit channels in fission
  - Leaving the compound state, the nucleus may choose between various paths to disintegrate.
- **Random neck rupture**
  - "rupture": the neck breaks suddenly "when" the nucleus stretches beyond the prescission shape
  - "Random": not decided "where" the neck breaks

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Nuclear shapes suitable for fission

(i) Shape representation $\Rightarrow$ 3 essential degrees of freedom :

- stretching of the nucleus
- thinning of the neck
- deformation to asymmetry

(ii) "A single sphere and two fragments"

- among allowed configurations

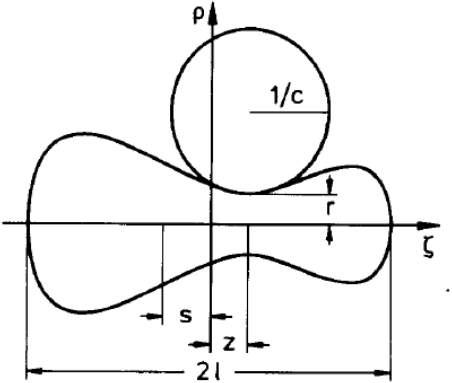
(iii) "Flatness of the neck"

- an independent variable

Shape parameters

- Suitable set of shape parameters (degrees of freedom):

$$(\, l \,,\, r \,,\, z \,,\, c \,,\, s \,)$$

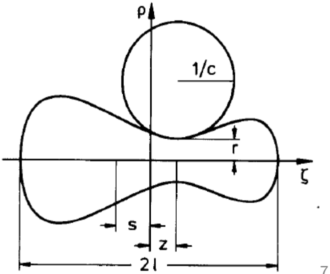


Shape parameters

- Semilength l : elongation of the nucleus
- r : radius of neck
- z : position on neck where neck is thinnest /

  where shape is thickest if neck does not yet exist

- c : curvature of neck
- s : position of centroid

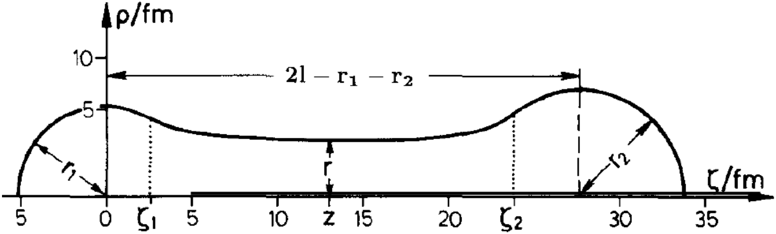


Real flat neck representation (prescission shape)

Shape function:

$$\rho(\zeta) = \begin{cases} (r_1^2 - \zeta^2)^{\frac{1}{2}} & ; -r_1 \leq \zeta \leq \zeta_1 \\ r + a^2 c[\cosh(\frac{\zeta - z + l - r_1}{a}) - 1] & ; \zeta_1 \leq \zeta \leq \zeta_2 \\ (r_2^2 - [2l - r_1 - r_2 - \zeta]^2)^{\frac{1}{2}} & ; \zeta_2 \leq \zeta \leq 2l - r_1 \end{cases}$$

`$r_1$ and $r_2$ : radii of the spherical heads`
`$\zeta_1$ and $\zeta_2$ : transitional positions (where 3 parts of equation join)`
`$a$ : extension of neck`

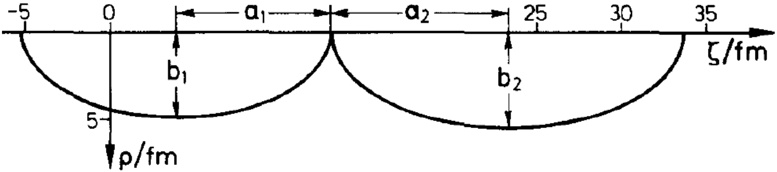


Embedded spheroids

- When a nucleus scissions ⇒ decays into fragments
- Newborn fragments are modelled as ⇒

  two spheroids in contact

`(The strong surface tension quickly smooths all centers and edges.)`

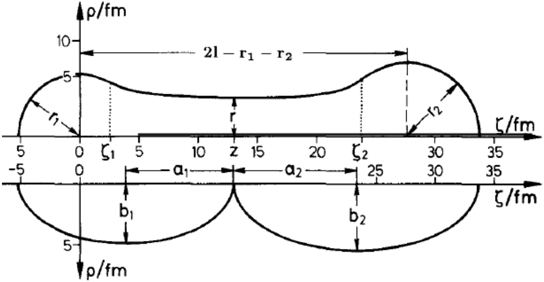


Embedded spheroids

- These spheroids:
- major axes a_1 and a_2 :

`fixed by the total length` $2l$ `and actual rupture point` z_r

$$a_1 = \frac{1}{2}(r_1 + z_r) \qquad , \qquad a_2 = l - \frac{1}{2}(r_1 + z_r)$$

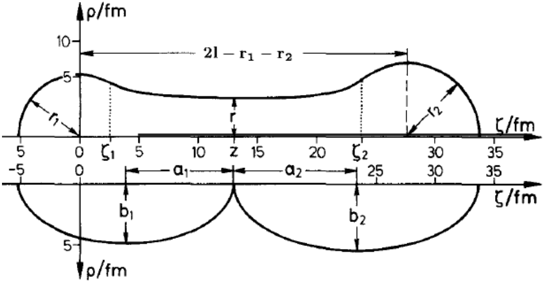


Embedded spheroids

- These spheroids :
- minor axes b_1 and b_2 :

`followed from the volume conservation`

$$b_1^2 = \frac{3}{4a_1}\int_{-r_1}^{z_r} \rho^2 \mathrm{d}\zeta \qquad , \qquad b_2^2 = \frac{3}{4a_2}\int_{z_r}^{2l-r_1} \rho^2 \mathrm{d}\zeta$$

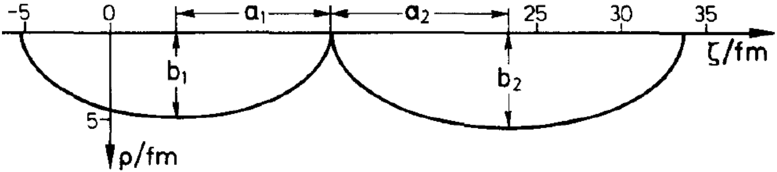


Embedded spheroids

`Applications`

- Embedded spheroids are to estimate:
  - repulsion between the fragments
  - energies of deformation

    fragments have immediately after formation



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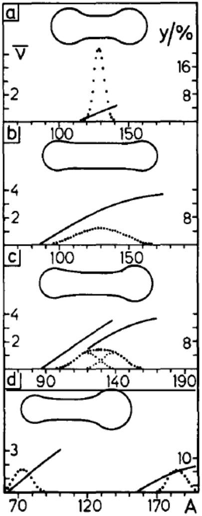
Fundamentals of random neck rupture

Slaves of prescission shape:

- mass yield $Y(A)$
- neutron multiplicity $\overline{\nu}$
- total kinetic energy $\overline{TKE}$



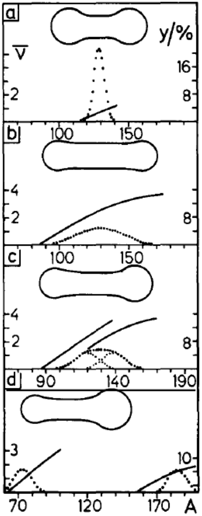
Fundamentals of random neck rupture

Figure:

- mass yields $Y(A)$ (dotted lines)

  and

- neutron multiplicities $\overline{\nu}$ (solid lines)
- as functions of fragment mass number A



Fundamentals of random neck rupture

`Part(a):`

supershort prescission shape and its products

`Part(b):`

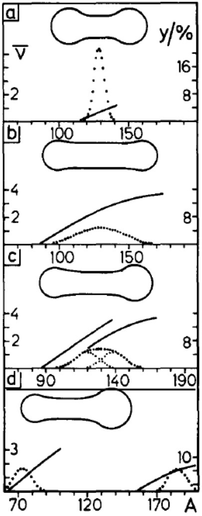
symmetric prescission shape

`Part(c):`

standard prescission shape

`Part(d):`

superasymmetrical prescission shape



Fundamentals of random neck rupture

- Total kinetic energy $\overline{TKE}$

  `inverse measure of prescission shape's length`

- `When rupture takes place` ⇒

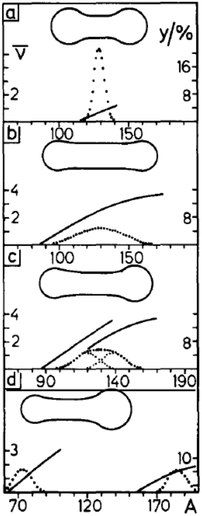
  `Coulomb repulsion accelerates fragments`

- Fragments high kinetic energy

  ⇒ `short prescission shape`

- Fragments low kinetic energy

  ⇒ `long prescission shape`



Fundamentals of random neck rupture

- Part (a, b)

Variance σ^2 of mass yield $Y(A)$ measures ⇒

prescission shape's length

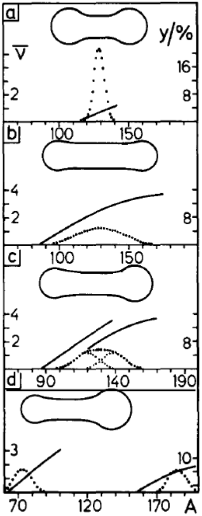
(Length of neck)

- Random neck rupture :

  produces different fragments

  by chopping neck

  at different positions



Fundamentals of random neck rupture

- The larger the neck

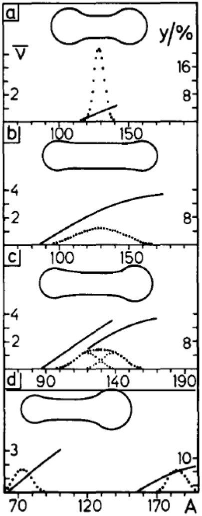
  ⇒ the more possibilities to chop it,

  and

  ⇒ the larger variety of fragments

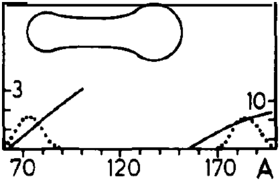
- the most frequent rupture

  ⇒ where the neck is thinnest



Fundamentals of random neck rupture

- `Part(d)`



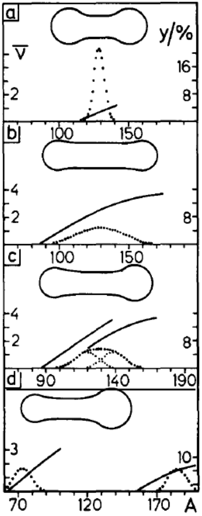
when prescission shape is asymmetric

$\Rightarrow$ neck is shifted away from center

- `Consequently:`

one light and one heavy fragment $\Rightarrow$

a double-humped yield $Y(A)$



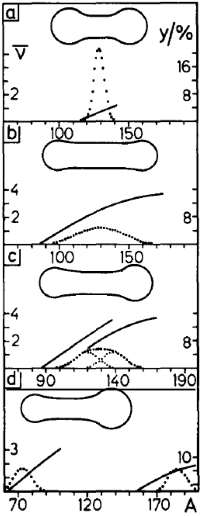
Fundamentals of random neck rupture

- `Part(d, c, b)`

With decreasing asymmetry :

the two humps merge

⇒ until a single bump remains



Fundamentals of random neck rupture

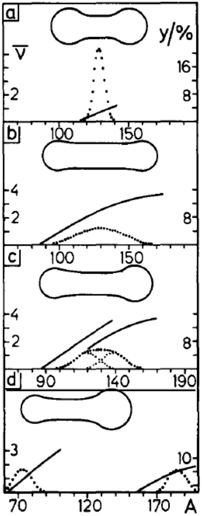
- `Part(a, b)`

  `(solid lines)`

A large average neutron multiplicity $\overline{\nu}$

is caused by :

" a long prescission shape "



Fundamentals of random neck rupture

- `Part(b)`

"symmetric" prescission shape :

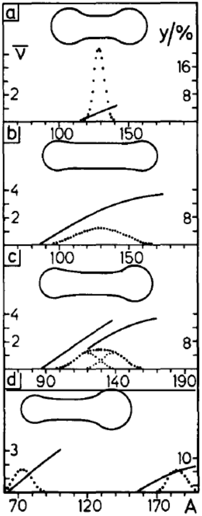
gives rise to neutron multiplicity $\overline{\nu}$

$\Rightarrow$ which increases steadily with fragment mass number A

- `Part(d)`

"asymmetric" prescission shape :

causes a sawtooth

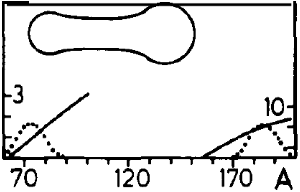


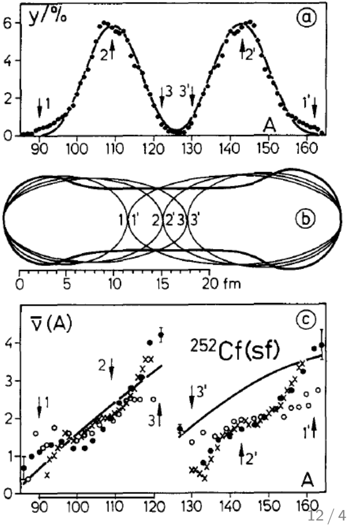
Fundamentals of random neck rupture

- `Random neck rupture`

  `and`

- `sawtooth shape of neutron multiplicity` $\overline{\nu}$





Fundamentals of random neck rupture

- `Part(b)`

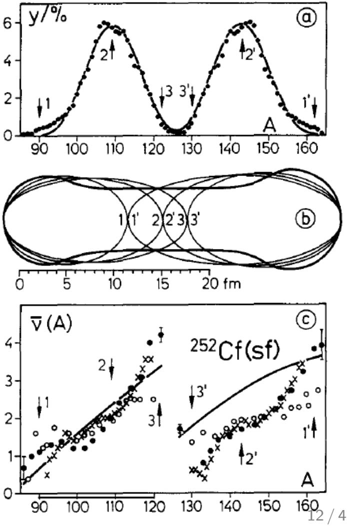
some embedded spheroids

- 2 and $2'$ `fragments`

$\Rightarrow$ neck is the thinnest

$\Rightarrow$ `Part(a):`

maxima of yield $Y(A)$ of fragments production



Fundamentals of random neck rupture

- `3 and 3' fragments`

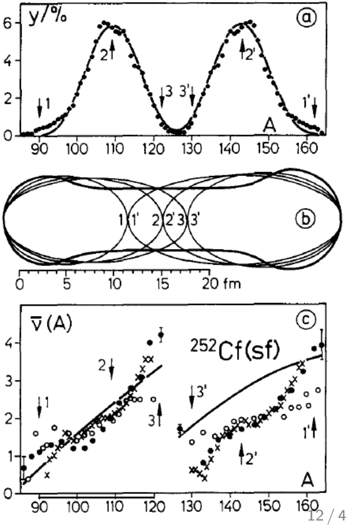
$\Rightarrow$ `Part(a):`

- rupture rarely happens

  (increased thickness of neck)
- fragments are equal by mass

  but

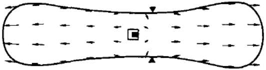
  very different by deformation



Scission as a sequence of instabilities

- `Fission`

  3 instabilities for evolution:



1. `Passing the barriers`
   - the first step of fission
2. `Shift instability`
   - Shortly behind the last barrier, the neck starts to appear.
   - Shift instability arises, because for fission : nucleus has to change from a spheroidal to a necked-in configuration
3. `Capillarity (Rayleigh) instability`
   - accomplishes what the shift instability prepares
   - takes the dent where it is and deepens it until two fragments appear

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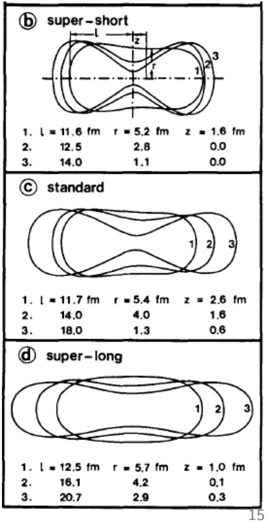
Standard, superlong and supershort

`Prescission shapes:`



ⓑ super – short

1. l = 11.6 fm r = 5.2 fm z = 1.6 fm
2. 12.5 2.8 0.0
3. 14.0 1.1 0.0

ⓒ standard

1. l = 11.7 fm r = 5.4 fm z = 2.6 fm
2. 14.0 4.0 1.6
3. 18.0 1.3 0.6

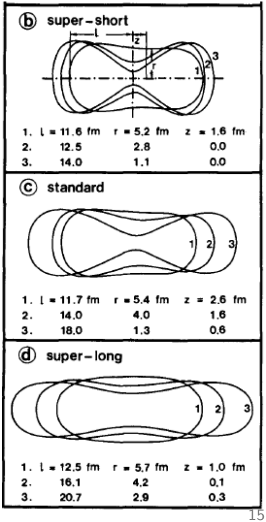
ⓓ super – long

1. l = 12.5 fm r = 5.7 fm z = 1.0 fm
2. 16.1 4.2 0.1
3. 20.7 2.9 0.3

Standard, superlong and supershort

Prescission shapes:

- Standard (Part c)
  - slightly asymmetric
  - of "normal" length



ⓑ super-short

1. l = 11.6 fm r = 5.2 fm z = 1.6 fm
2. 12.5 2.8 0.0
3. 14.0 1.1 0.0

ⓒ standard

1. l = 11.7 fm r = 5.4 fm z = 2.6 fm
2. 14.0 4.0 1.6
3. 18.0 1.3 0.6

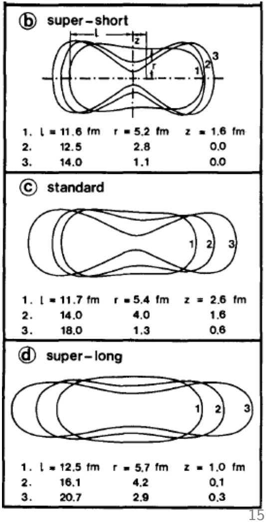
ⓓ super-long

1. l = 12.5 fm r = 5.7 fm z = 1.0 fm
2. 16.1 4.2 0.1
3. 20.7 2.9 0.3

Standard, superlong and supershort

`Prescission shapes:`

- `Standard` `(Part c)`
  - slightly asymmetric
  - of "normal" length
- `Superlong` `(Part d)`
  - almost symmetric
  - longer than standard



ⓑ super – short

	l	r	z
1.	l = 11.6 fm	r = 5.2 fm	z = 1.6 fm
2.	12.5	2.8	0.0
3.	14.0	1.1	0.0

ⓒ standard

	l	r	z
1.	l = 11.7 fm	r = 5.4 fm	z = 2.6 fm
2.	14.0	4.0	1.6
3.	18.0	1.3	0.6

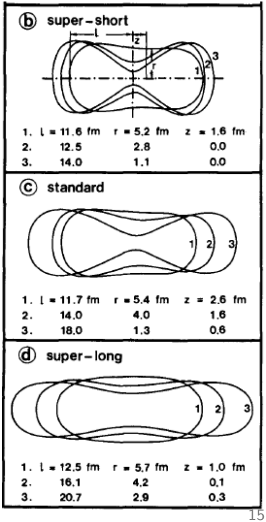
ⓓ super – long

	l	r	z
1.	l = 12.5 fm	r = 5.7 fm	z = 1.0 fm
2.	16.1	4.2	0.1
3.	20.7	2.9	0.3

Standard, superlong and supershort

Prescission shapes:

- Standard (Part c)
  - slightly asymmetric
  - of "normal" length
- Superlong (Part d)
  - almost symmetric
  - longer than standard
- Supershort (Part b)
  - almost symmetric
  - shorter than standard



1. l = 11.6 fm r = 5,2 fm z = 1,6 fm
2. 12.5 2.8 0,0
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1. l = 12,5 fm r = 5,7 fm z = 1,0 fm
2. 16,1 4,2 0,1
3. 20.7 2,9 0,3

Standard, superlong and supershort

- These differences in "mean" length are larger than those caused by fluctuations,

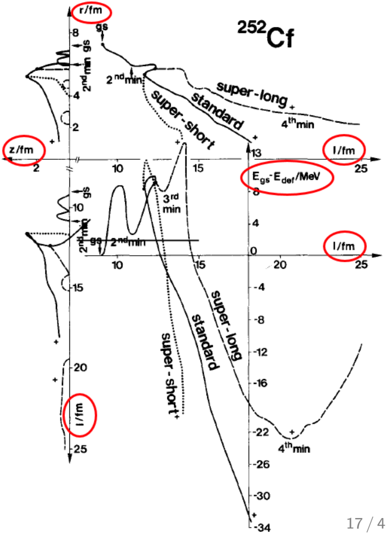
`Hence, we expect`

- separable components in exit-channel observables

Channel graph

`Channels traverse :`

"space of deformation"



The simplest set of coordinates:

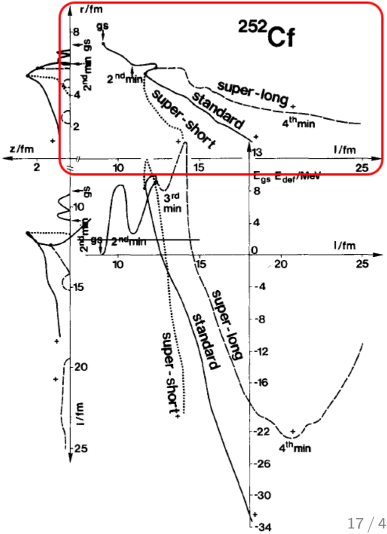
(l, r, z)

- semilength l
- neck radius r
- location of the dent z

Channel graph

(l, r) `projection :`

- Top right
- standard channel (full line)
- superlong channel (dashed)
- supershort channel (dotted)
- rooted in ground state gs

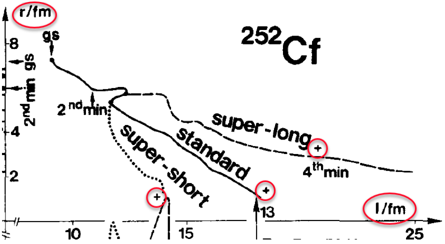


Channel graph

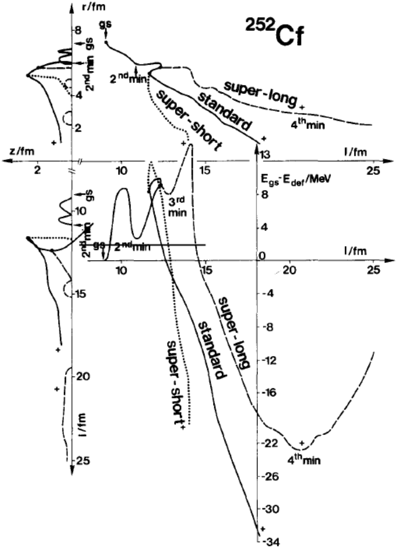
(l, r) `projection`

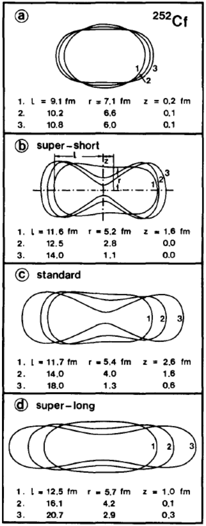
`To initiate fission :`

- the nucleus lengthens (l increases)
- its radius r decreases
- shortly after 2^{nd} min :
- "the big loop"
- necleus stretches
- thins its neck
- until just behind the prescission shape at $+$
- two fragments appear



Channel graph





Channel graph

Part(a)

- evolution from gs to 2^{nd} min

Part(b)

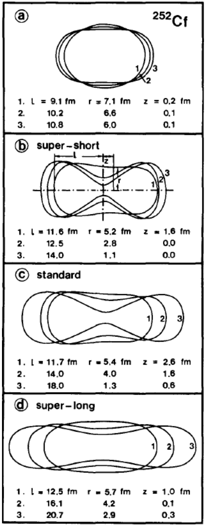
- deformation in supershort fission channel
- starting from bifurcation
- ending at prescission $(+)$

Part(c)

- similar changes along standard channel

Part(d)

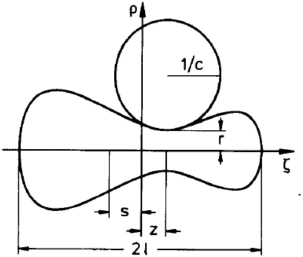
- similar change along superlong channel

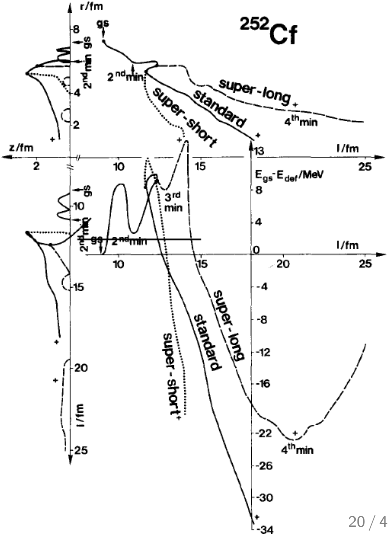


Channel graph

`Symmetry and Asymmetry`

- z : position on the neck where neck is thinnest

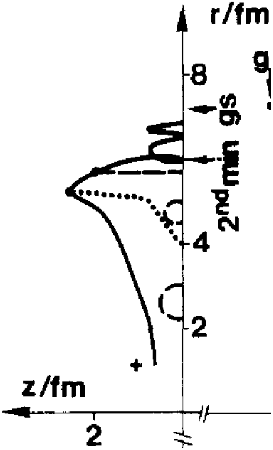


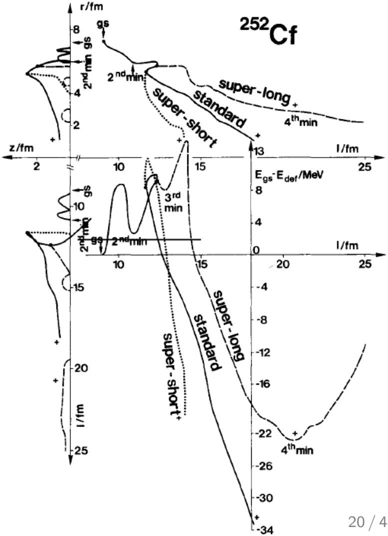


- Symmetry : $z = 0$

Channel graph

- (r, z) `projection`





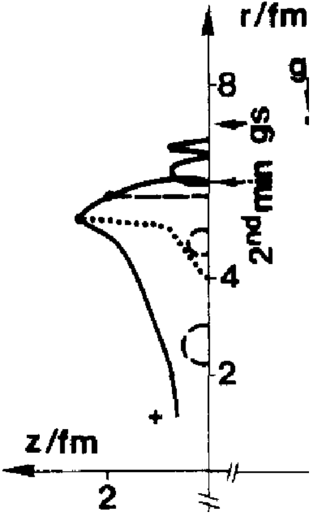
Channel graph

(r, z) `projection :`

- gs : nearly symmetric

  (minor deviations)

- nucleus stays symmetric until big loop
- $z \approx 2.5$: asymmetry becomes sizable
- approach to scission : asymmetry decreases again

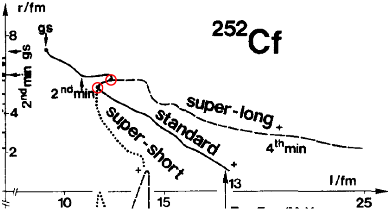


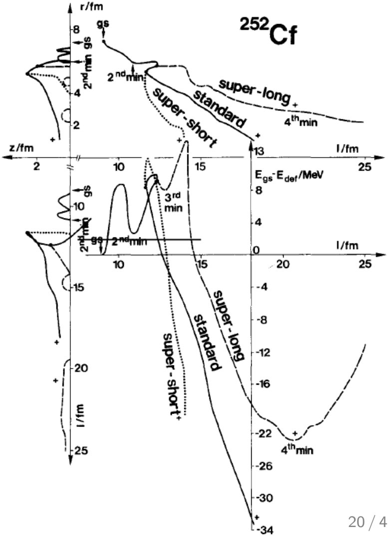
Channel graph

(r, l) `projection :`

superlong & supershort channels

- branching from standard channel
- at "bifurcation points " (full circles)





Channel graph

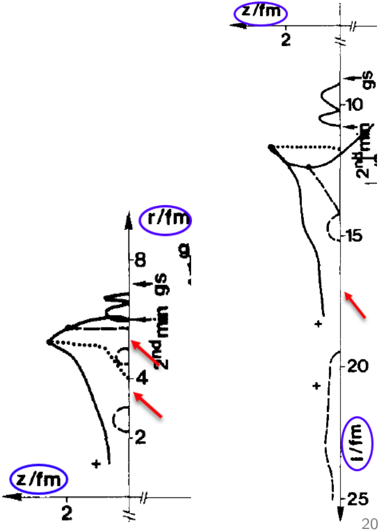
(r, z) & (z, l) `projections`

- trails of superlong & supershort channels

Due to

little deviation from symmetry $(z = 0)$:

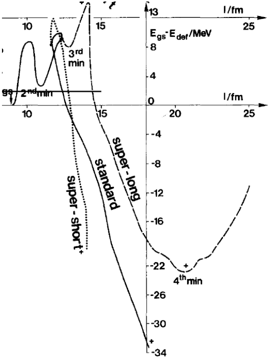
- their lines coincide with the axes

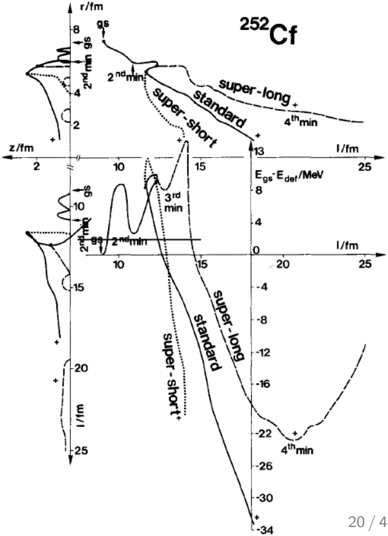


Channel graph

$(E_{gs} - E_{def}, l)$ `projection :`

- Potential energy contained in the nucleus as it floats through one of channels

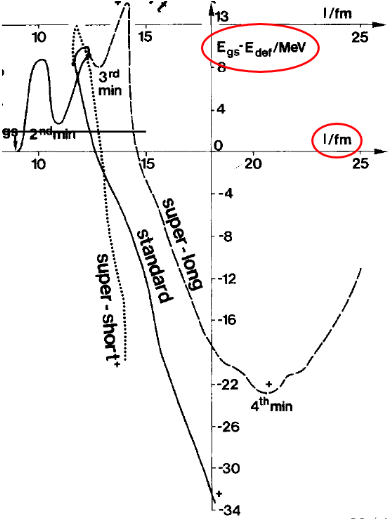




Channel graph

$(E_{gs} - E_{def}, l)$ `projection :`

- nucleus starts at gs with energy 0 (normalized)
- climbs the first barrier at $l \approx 10(fm)$
- falls into the 2^{nd} min
- rises to the second barrier
- descends to scission

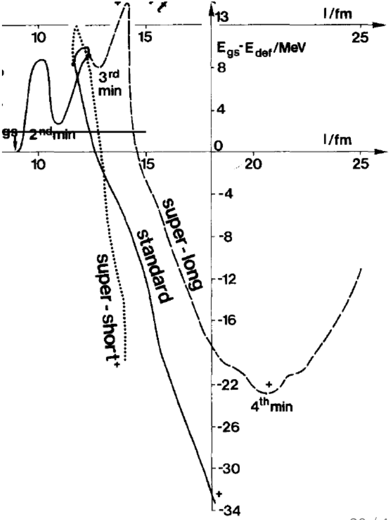


Channel graph

$(E_{gs} - E_{def}, l)$ `projection :`

`Standard channel`

- double-humped barrier
- second minimum

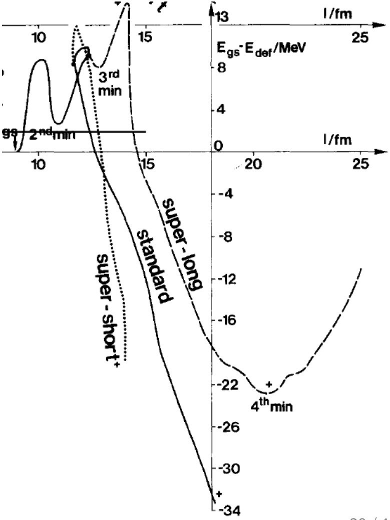


Channel graph

$(E_{gs} - E_{def}, l)$ `projection :`

`Superlong channel` (^{252}Cf)

- barrier at $l \approx 14(fm)$
- higher than any of the standard barriers

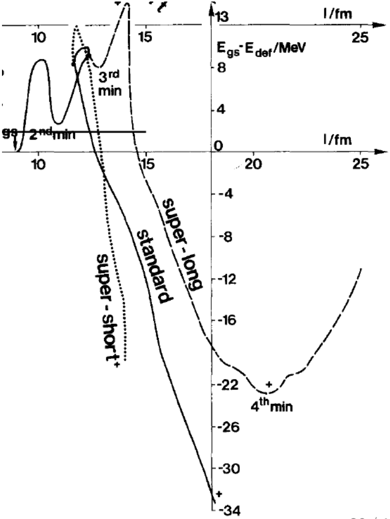


Channel graph

$(E_{gs} - E_{def}, l)$ `projection`

`Supershort channel`

- barrier at $l \approx 12(fm)$
- higher than any of the standard barriers



Channel graph

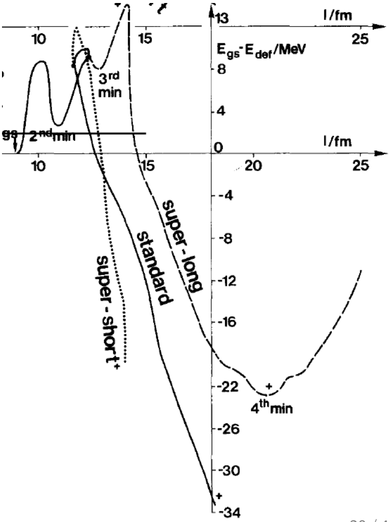
$(E_{gs} - E_{def}, l)$ `projection`

`That's why :`

- standard channel

  is much more used than

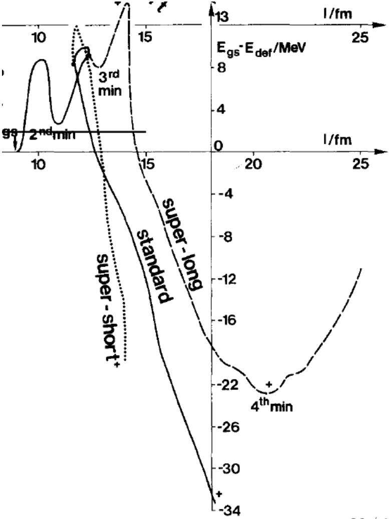
- superlong channel



Channel graph

Role of
bifurcation points

- divide the flux to various prescission shapes
- in corporation with barriers decide the distribution of exit-channel observables



Application of such diagrams

- measure semilength l
- measure asymmetry z

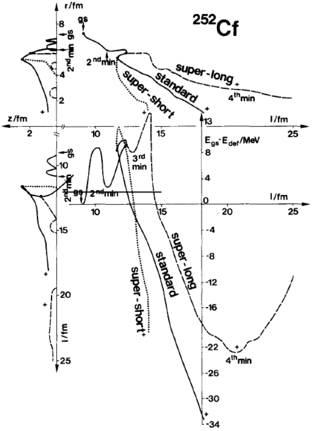
  of various prescission shapes

`With these data :`

- construct shapes with flat necks
- find the yield $Y(A)$
- other exit-channel variables

`Finally :`

- compare with experiments



Application of such diagrams

- Potential-energy calculations $\Rightarrow$
  - prescission shapes
- Prescission shapes $\Rightarrow$
  - individual yields $Y_c(A)$
  - individual total kinetic energies $\overline{TKE}_c(A)$
  - individual neutron multiplicities $\overline{\nu}_c(A)$
- `subscript` c `labels various channels.`

Application of such diagrams

- To compare with measurements, we form superpositions:

$$Y(A) = \sum_c p_c Y_c(A) \qquad (\mathtt{p_c : channel\ probability})$$

$$\overline{TKE}(A) = \sum_c p_c \, \overline{TKE}_c(A)\, Y_c(A)/Y(A)$$

$$\overline{\nu}(A) = \sum_c p_c \, \overline{\nu}_c(A)\, Y_c(A)/Y(A)$$

Application of such diagrams

- To compare with measurements, we form superpositions:

$$Y(A) = \sum_c p_c Y_c(A) \qquad (\mathrm{p_c} : \texttt{channel probability})$$

$$\overline{TKE}(A) = \sum_c p_c \, \overline{TKE}_c(A) \, Y_c(A)/Y(A)$$

$$\overline{\nu}(A) = \sum_c p_c \, \overline{\nu}_c(A) \, Y_c(A)/Y(A)$$

`For information reduction, without loss of accuracy :`

$$Y_c(A) = \frac{1}{(2\pi\sigma^2_{A,c})^{1/2}}[exp(-\frac{(A-\overline{A}_c)^2}{2\sigma^2_{A,c}}) + exp(-\frac{(A-A_{cn}+\overline{A}_c)^2}{2\sigma^2_{A,c}})]$$

$$\overline{TKE}_c(A) = \frac{A(A_{cn}-A)}{\overline{A}_c(A_{cn}-\overline{A}_c)-\sigma^2_{A,c}}\,\overline{TKE}_c$$

$\overline{A}_c$: `average mass` , $\sigma^2_{A,c}$: `mass variance` , $\overline{TKE}_c$: `average total kinetic energy`

$\overline{A}_c$, $\sigma^2_{A,c}$, $\overline{TKE}_c$: `byproducts of random neck rupture`

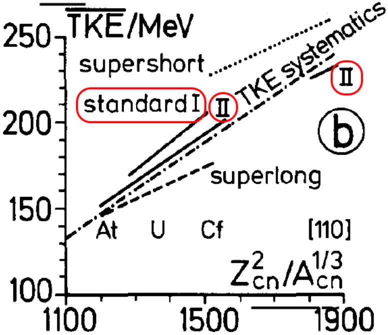
Mean total kinetic energy ($\overline{TKE}$)

`Standard channel`

- splits into I and II
- stays close to overall $\overline{TKE}$ systematics `(dash-dotted line)`

`Standard` II

- closeness to overall $\overline{TKE}$ systematics
- the most abundant channel in most actinides



Mean total kinetic energy ($\overline{TKE}$)

`Supershort channel`

- gives naturally

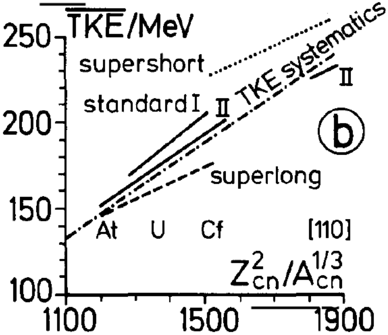
  too high kinetic energies

`Superlong channel`

- $\overline{TKE}s$ are too low

`For light systems`

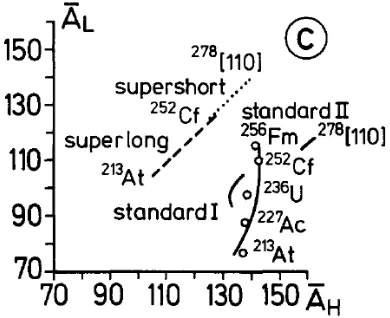
- convergence of superlong $\overline{TKE}$ to overall $\overline{TKE}$ systematics



Mean mass number $\overline{A}_H$ of heavy fragments

`Figure :`

- light fragment mean mass number $\overline{A}_L$ as a function of $\overline{A}_H$



`System size` A_{cn}

- $\overline{A}_H + \overline{A}_L$

Mean mass number $\overline{A}_H$ of heavy fragments

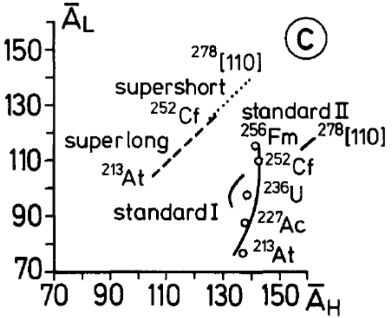
Standard channel

particularly standard II

- remains nearly constant at $\overline{A}_H \approx 140$

Superlong & supershort channels

- symmetrical fission $(\overline{A}_H \approx \overline{A}_L)$



Channel probabilities P_c

`Supershort channel`

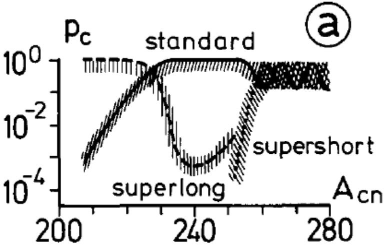
- disappears for systems smaller than $A_{cn} \approx 250$

`Superlong channel`

- breaks up for systems larger than $A_{cn} \approx 260$

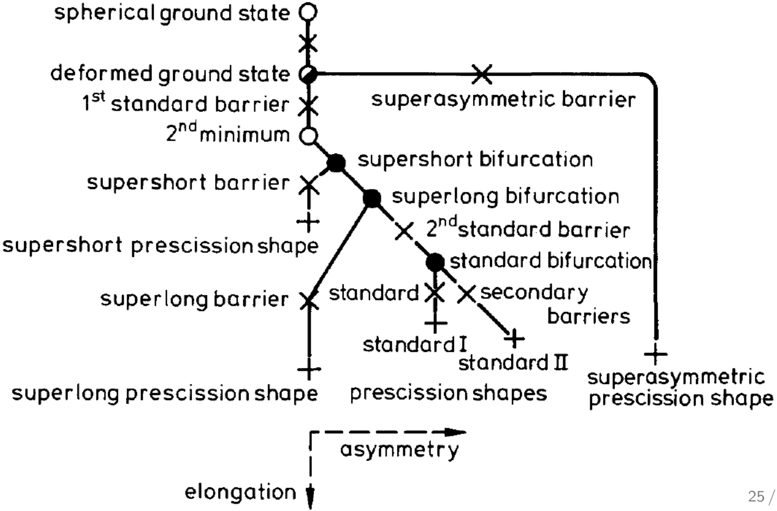
`Standard channel`

- exists everywhere



Tree of nuclear fission

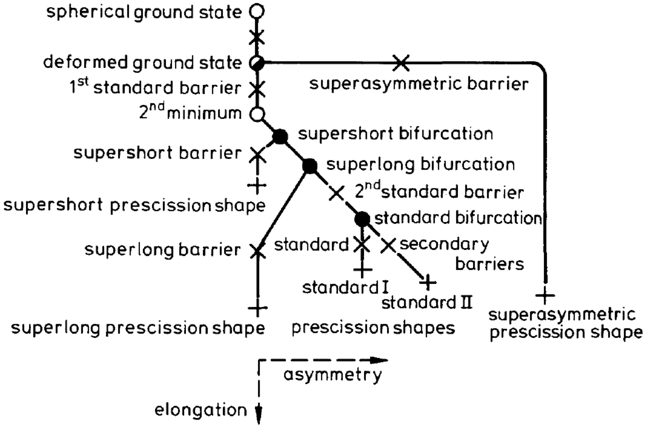
- `Cayley tree of nuclear fission`



Tree of nuclear fission

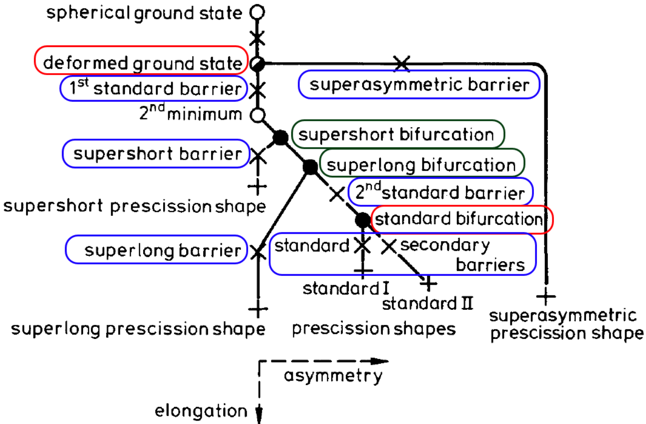
○ : potential energy as minima
● : bifurcation points

\+ : prescission shapes
× : barriers
– : their connections



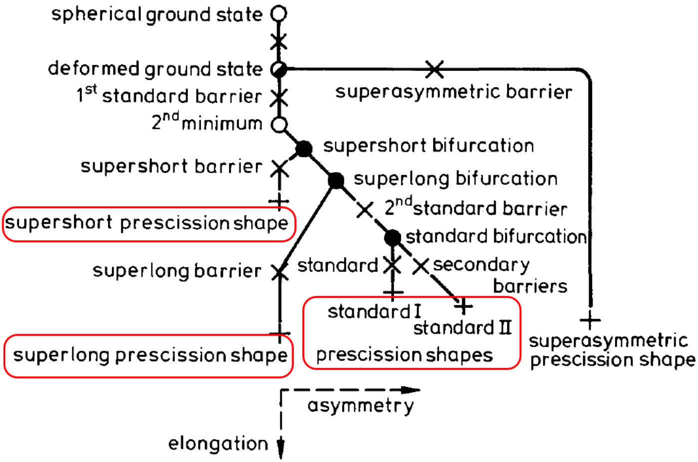
Tree of nuclear fission

- nucleus starts to fission from the deformed ground state
- overcome barriers
- pass bifurcation points



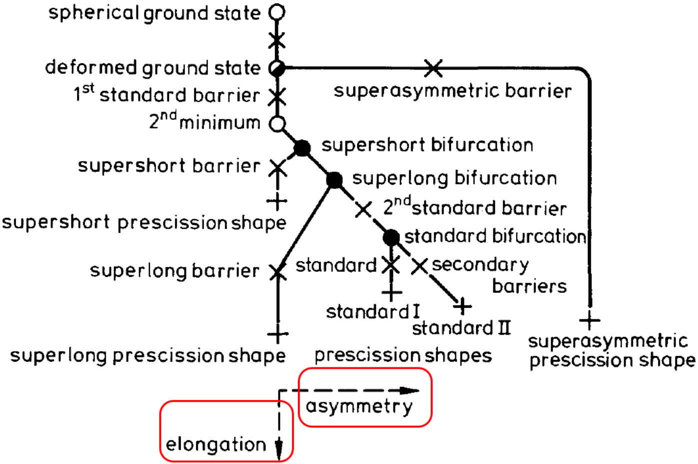
Tree of nuclear fission

- choose different channels
- rupture at different prescssion shapes



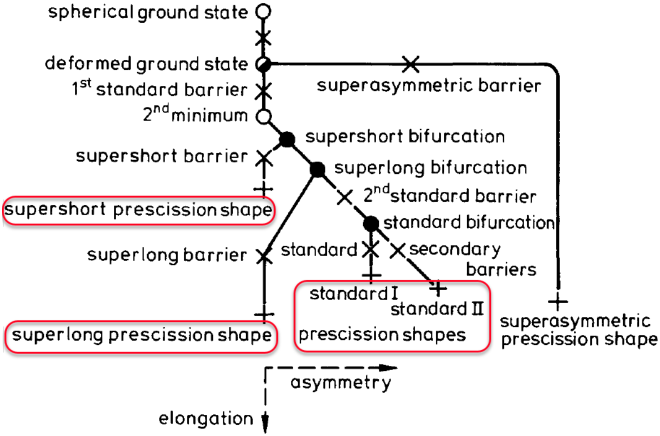
Tree of nuclear fission

- `downward motion` : increase of semilength l
- `motion to right` : growth of asymmetry z



Tree of nuclear fission

- standard channel : `slightly asymmetric`
- superlong & supershort channels : `almost symmetric`



Computed ground states

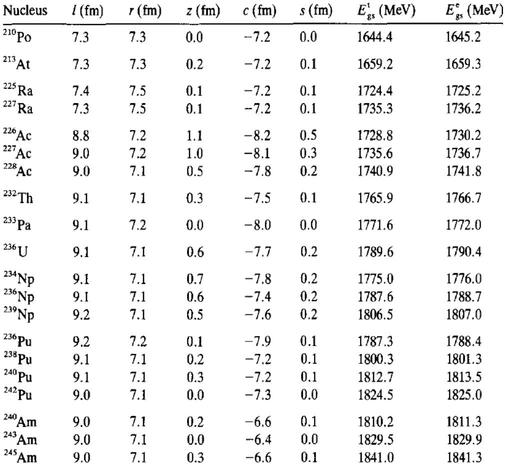
- Shape parameters

  (l, r, z, c, s)

- Ground state energies

  E^t_{gs} & E^e_{gs}

- accuracy of Strutinsky's method

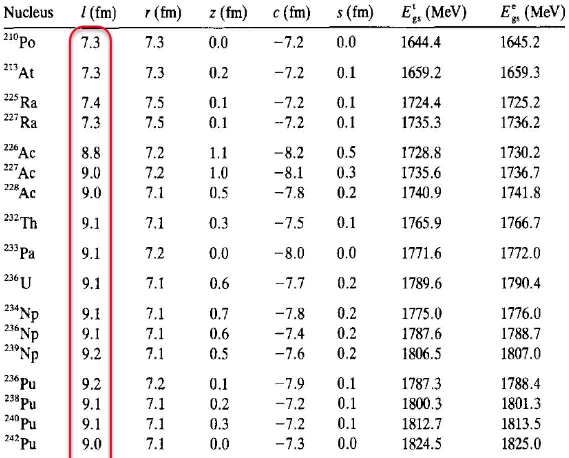


Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	E^t_{gs} (MeV)	E^e_{gs} (MeV)
^{210}Po	7.3	7.3	0.0	−7.2	0.0	1644.4	1645.2
^{213}At	7.3	7.3	0.2	−7.2	0.1	1659.2	1659.3
^{225}Ra	7.4	7.5	0.1	−7.2	0.1	1724.4	1725.2
^{227}Ra	7.3	7.5	0.1	−7.2	0.1	1735.3	1736.2
^{226}Ac	8.8	7.2	1.1	−8.2	0.5	1728.8	1730.2
^{227}Ac	9.0	7.2	1.0	−8.1	0.3	1735.6	1736.7
^{228}Ac	9.0	7.1	0.5	−7.8	0.2	1740.9	1741.8
^{232}Th	9.1	7.1	0.3	−7.5	0.1	1765.9	1766.7
^{233}Pa	9.1	7.2	0.0	−8.0	0.0	1771.6	1772.0
^{236}U	9.1	7.1	0.6	−7.7	0.2	1789.6	1790.4
^{234}Np	9.1	7.1	0.7	−7.8	0.2	1775.0	1776.0
^{236}Np	9.1	7.1	0.6	−7.4	0.2	1787.6	1788.7
^{239}Np	9.2	7.1	0.5	−7.6	0.2	1806.5	1807.0
^{236}Pu	9.2	7.2	0.1	−7.9	0.1	1787.3	1788.4
^{238}Pu	9.1	7.1	0.2	−7.2	0.1	1800.3	1801.3
^{240}Pu	9.1	7.1	0.3	−7.2	0.1	1812.7	1813.5
^{242}Pu	9.0	7.1	0.0	−7.3	0.0	1824.5	1825.0
^{240}Am	9.0	7.1	0.2	−6.6	0.1	1810.2	1811.3
^{243}Am	9.0	7.1	0.0	−6.4	0.0	1829.5	1829.9
^{245}Am	9.0	7.1	0.3	−6.6	0.1	1841.0	1841.3

Computed ground states

- transition from spherical to deformed gs :

  not too far from $^{208}_{82}Pb_{126}$: $l \approx 7.3$ to $l \approx 9.0$



Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	E^{t}_{gs} (MeV)	E^{c}_{gs} (MeV)
^{210}Po	7.3	7.3	0.0	−7.2	0.0	1644.4	1645.2
^{213}At	7.3	7.3	0.2	−7.2	0.1	1659.2	1659.3
^{225}Ra	7.4	7.5	0.1	−7.2	0.1	1724.4	1725.2
^{227}Ra	7.3	7.5	0.1	−7.2	0.1	1735.3	1736.2
^{226}Ac	8.8	7.2	1.1	−8.2	0.5	1728.8	1730.2
^{227}Ac	9.0	7.2	1.0	−8.1	0.3	1735.6	1736.7
^{228}Ac	9.0	7.1	0.5	−7.8	0.2	1740.9	1741.8
^{232}Th	9.1	7.1	0.3	−7.5	0.1	1765.9	1766.7
^{233}Pa	9.1	7.2	0.0	−8.0	0.0	1771.6	1772.0
^{236}U	9.1	7.1	0.6	−7.7	0.2	1789.6	1790.4
^{234}Np	9.1	7.1	0.7	−7.8	0.2	1775.0	1776.0
^{236}Np	9.1	7.1	0.6	−7.4	0.2	1787.6	1788.7
^{239}Np	9.2	7.1	0.5	−7.6	0.2	1806.5	1807.0
^{236}Pu	9.2	7.2	0.1	−7.9	0.1	1787.3	1788.4
^{238}Pu	9.1	7.1	0.2	−7.2	0.1	1800.3	1801.3
^{240}Pu	9.1	7.1	0.3	−7.2	0.1	1812.7	1813.5
^{242}Pu	9.0	7.1	0.0	−7.3	0.0	1824.5	1825.0

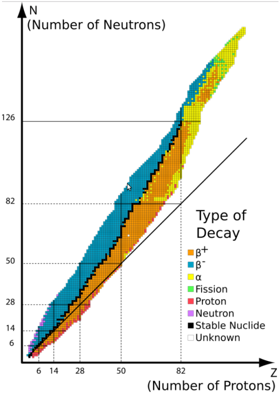
Computed ground states

- transition from spherical to deformed gs :

  not too far from $^{208}_{82}Pb_{126}$: $l \approx 7.3$ to $l \approx 9.0$

`Pb :`

- doubly magic spherical nucleus
- the heaviest stable element
- no other isotopes with $Z > 82$ are stable



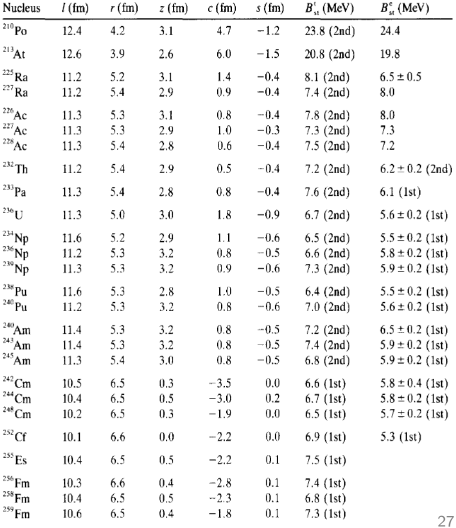
Standard barriers B_{st}

- All nuclei listed:

  a double-humped standard barrier

- higher barrier

  is presented



Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B^t_{st} (MeV)	B^c_{st} (MeV)
^{210}Po	12.4	4.2	3.1	4.7	−1.2	23.8 (2nd)	24.4
^{213}At	12.6	3.9	2.6	6.0	−1.5	20.8 (2nd)	19.8
^{225}Ra	11.2	5.2	3.1	1.4	−0.4	8.1 (2nd)	6.5 ± 0.5
^{227}Ra	11.2	5.4	2.9	0.9	−0.4	7.4 (2nd)	8.0
^{226}Ac	11.3	5.3	3.1	0.8	−0.4	7.8 (2nd)	8.0
^{227}Ac	11.3	5.3	2.9	1.0	−0.3	7.3 (2nd)	7.3
^{228}Ac	11.3	5.4	2.8	0.6	−0.4	7.5 (2nd)	7.2
^{232}Th	11.2	5.4	2.9	0.5	−0.4	7.2 (2nd)	6.2 ± 0.2 (2nd)
^{233}Pa	11.3	5.4	2.8	0.8	−0.4	7.6 (2nd)	6.1 (1st)
^{236}U	11.3	5.0	3.0	1.8	−0.9	6.7 (2nd)	5.6 ± 0.2 (1st)
^{234}Np	11.6	5.2	2.9	1.1	−0.6	6.5 (2nd)	5.5 ± 0.2 (1st)
^{236}Np	11.2	5.3	3.2	0.8	−0.5	6.6 (2nd)	5.8 ± 0.2 (1st)
^{239}Np	11.3	5.3	3.2	0.9	−0.6	7.3 (2nd)	5.9 ± 0.2 (1st)
^{238}Pu	11.6	5.3	2.8	1.0	−0.5	6.4 (2nd)	5.5 ± 0.2 (1st)
^{240}Pu	11.2	5.3	3.2	0.8	−0.6	7.0 (2nd)	5.6 ± 0.2 (1st)
^{240}Am	11.4	5.3	3.2	0.8	−0.5	7.2 (2nd)	6.5 ± 0.2 (1st)
^{243}Am	11.4	5.3	3.2	0.8	−0.5	7.4 (2nd)	5.9 ± 0.2 (1st)
^{245}Am	11.3	5.4	3.0	0.8	−0.5	6.8 (2nd)	5.9 ± 0.2 (1st)
^{242}Cm	10.5	6.5	0.3	−3.5	0.0	6.6 (1st)	5.8 ± 0.4 (1st)
^{244}Cm	10.4	6.5	0.5	−3.0	0.2	6.7 (1st)	5.8 ± 0.2 (1st)
^{248}Cm	10.2	6.5	0.3	−1.9	0.0	6.5 (1st)	5.7 ± 0.2 (1st)
^{252}Cf	10.1	6.6	0.0	−2.2	0.0	6.9 (1st)	5.3 (1st)
^{255}Es	10.4	6.5	0.5	−2.2	0.1	7.5 (1st)	
^{256}Fm	10.3	6.6	0.4	−2.8	0.1	7.4 (1st)	
^{258}Fm	10.4	6.5	0.5	−2.3	0.1	6.8 (1st)	
^{259}Fm	10.6	6.5	0.4	−1.8	0.1	7.3 (1st)	

Stanndard barriers B_{st}

- In lighter nuclei:

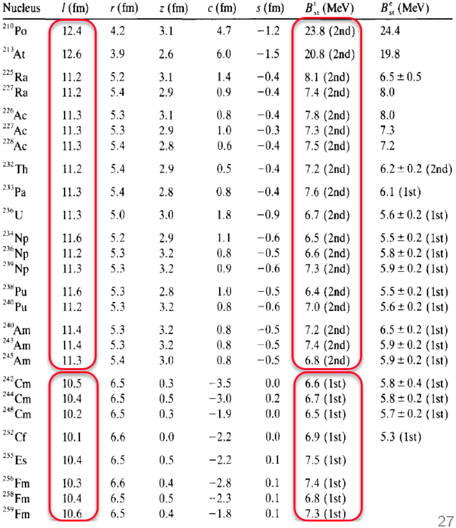
  second hump dominates

  `located at larger values of` l

- In heavier nuclei:

  first hump dominates

  `located at smaller values of` l

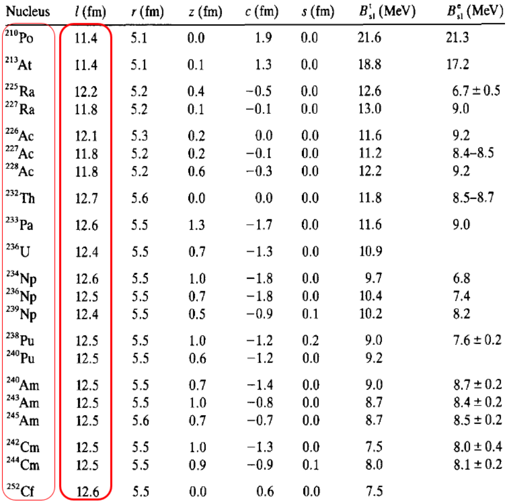


Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B^{t}_{st} (MeV)	B^{c}_{st} (MeV)
^{210}Po	12.4	4.2	3.1	4.7	−1.2	23.8 (2nd)	24.4
^{213}At	12.6	3.9	2.6	6.0	−1.5	20.8 (2nd)	19.8
^{225}Ra	11.2	5.2	3.1	1.4	−0.4	8.1 (2nd)	6.5 ± 0.5
^{227}Ra	11.2	5.4	2.9	0.9	−0.4	7.4 (2nd)	8.0
^{226}Ac	11.3	5.3	3.1	0.8	−0.4	7.8 (2nd)	8.0
^{227}Ac	11.3	5.3	2.9	1.0	−0.3	7.3 (2nd)	7.3
^{228}Ac	11.3	5.4	2.8	0.6	−0.4	7.5 (2nd)	7.2
^{232}Th	11.2	5.4	2.9	0.5	−0.4	7.2 (2nd)	6.2 ± 0.2 (2nd)
^{233}Pa	11.3	5.4	2.8	0.8	−0.4	7.6 (2nd)	6.1 (1st)
^{236}U	11.3	5.0	3.0	1.8	−0.9	6.7 (2nd)	5.6 ± 0.2 (1st)
^{234}Np	11.6	5.2	2.9	1.1	−0.6	6.5 (2nd)	5.5 ± 0.2 (1st)
^{236}Np	11.2	5.3	3.2	0.8	−0.5	6.6 (2nd)	5.8 ± 0.2 (1st)
^{239}Np	11.3	5.3	3.2	0.9	−0.6	7.3 (2nd)	5.9 ± 0.2 (1st)
^{238}Pu	11.6	5.3	2.8	1.0	−0.5	6.4 (2nd)	5.5 ± 0.2 (1st)
^{240}Pu	11.2	5.3	3.2	0.8	−0.6	7.0 (2nd)	5.6 ± 0.2 (1st)
^{240}Am	11.4	5.3	3.2	0.8	−0.5	7.2 (2nd)	6.5 ± 0.2 (1st)
^{243}Am	11.4	5.3	3.2	0.8	−0.5	7.4 (2nd)	5.9 ± 0.2 (1st)
^{245}Am	11.3	5.4	3.0	0.8	−0.5	6.8 (2nd)	5.9 ± 0.2 (1st)
^{242}Cm	10.5	6.5	0.3	−3.5	0.0	6.6 (1st)	5.8 ± 0.4 (1st)
^{244}Cm	10.4	6.5	0.5	−3.0	0.2	6.7 (1st)	5.8 ± 0.2 (1st)
^{248}Cm	10.2	6.5	0.3	−1.9	0.0	6.5 (1st)	5.7 ± 0.2 (1st)
^{252}Cf	10.1	6.6	0.0	−2.2	0.0	6.9 (1st)	5.3 (1st)
^{255}Es	10.4	6.5	0.5	−2.2	0.1	7.5 (1st)	
^{256}Fm	10.3	6.6	0.4	−2.8	0.1	7.4 (1st)	
^{258}Fm	10.4	6.5	0.5	−2.3	0.1	6.8 (1st)	
^{259}Fm	10.6	6.5	0.4	−1.8	0.1	7.3 (1st)	

Superlong barriers B_{sl}

with mass number increase:

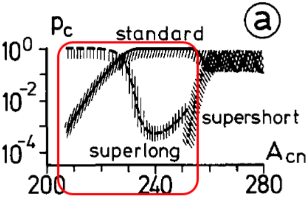
- superlong barrier shifts to larger values of l
- (opposite to standard barrier)

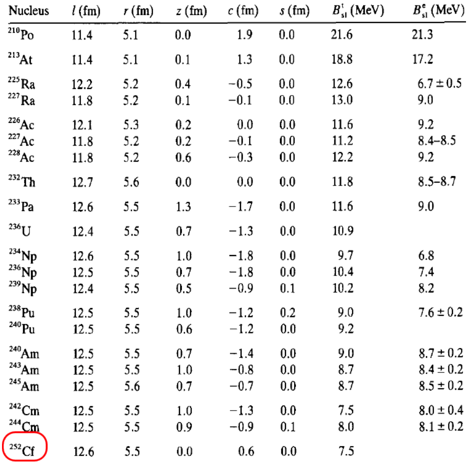


Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B^{t}_{sl} (MeV)	B^{e}_{sl} (MeV)
^{210}Po	11.4	5.1	0.0	1.9	0.0	21.6	21.3
^{213}At	11.4	5.1	0.1	1.3	0.0	18.8	17.2
^{225}Ra	12.2	5.2	0.4	−0.5	0.0	12.6	6.7 ± 0.5
^{227}Ra	11.8	5.2	0.1	−0.1	0.0	13.0	9.0
^{226}Ac	12.1	5.3	0.2	0.0	0.0	11.6	9.2
^{227}Ac	11.8	5.2	0.2	−0.1	0.0	11.2	8.4–8.5
^{228}Ac	11.8	5.2	0.6	−0.3	0.0	12.2	9.2
^{232}Th	12.7	5.6	0.0	0.0	0.0	11.8	8.5–8.7
^{233}Pa	12.6	5.5	1.3	−1.7	0.0	11.6	9.0
^{236}U	12.4	5.5	0.7	−1.3	0.0	10.9	
^{234}Np	12.6	5.5	1.0	−1.8	0.0	9.7	6.8
^{236}Np	12.5	5.5	0.7	−1.8	0.0	10.4	7.4
^{239}Np	12.4	5.5	0.5	−0.9	0.1	10.2	8.2
^{238}Pu	12.5	5.5	1.0	−1.2	0.2	9.0	7.6 ± 0.2
^{240}Pu	12.5	5.5	0.6	−1.2	0.0	9.2	
^{240}Am	12.5	5.5	0.7	−1.4	0.0	9.0	8.7 ± 0.2
^{243}Am	12.5	5.5	1.0	−0.8	0.0	8.7	8.4 ± 0.2
^{245}Am	12.5	5.6	0.7	−0.7	0.0	8.7	8.5 ± 0.2
^{242}Cm	12.5	5.5	1.0	−1.3	0.0	7.5	8.0 ± 0.4
^{244}Cm	12.5	5.5	0.9	−0.9	0.1	8.0	8.1 ± 0.2
^{252}Cf	12.6	5.5	0.0	0.6	0.0	7.5	

Superlong barriers B_{sl}

- Superlong channel is "broken" in heavy nuclei (such as ^{256}Fm)

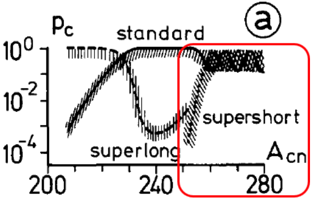




Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B^{t}_{sl} (MeV)	B^{e}_{sl} (MeV)
^{210}Po	11.4	5.1	0.0	1.9	0.0	21.6	21.3
^{213}At	11.4	5.1	0.1	1.3	0.0	18.8	17.2
^{225}Ra	12.2	5.2	0.4	−0.5	0.0	12.6	6.7 ± 0.5
^{227}Ra	11.8	5.2	0.1	−0.1	0.0	13.0	9.0
^{226}Ac	12.1	5.3	0.2	0.0	0.0	11.6	9.2
^{227}Ac	11.8	5.2	0.2	−0.1	0.0	11.2	8.4–8.5
^{228}Ac	11.8	5.2	0.6	−0.3	0.0	12.2	9.2
^{232}Th	12.7	5.6	0.0	0.0	0.0	11.8	8.5–8.7
^{233}Pa	12.6	5.5	1.3	−1.7	0.0	11.6	9.0
^{236}U	12.4	5.5	0.7	−1.3	0.0	10.9	
^{234}Np	12.6	5.5	1.0	−1.8	0.0	9.7	6.8
^{236}Np	12.5	5.5	0.7	−1.8	0.0	10.4	7.4
^{239}Np	12.4	5.5	0.5	−0.9	0.1	10.2	8.2
^{238}Pu	12.5	5.5	1.0	−1.2	0.2	9.0	7.6 ± 0.2
^{240}Pu	12.5	5.5	0.6	−1.2	0.0	9.2	
^{240}Am	12.5	5.5	0.7	−1.4	0.0	9.0	8.7 ± 0.2
^{243}Am	12.5	5.5	1.0	−0.8	0.0	8.7	8.4 ± 0.2
^{245}Am	12.5	5.6	0.7	−0.7	0.0	8.7	8.5 ± 0.2
^{242}Cm	12.5	5.5	1.0	−1.3	0.0	7.5	8.0 ± 0.4
^{244}Cm	12.5	5.5	0.9	−0.9	0.1	8.0	8.1 ± 0.2
^{252}Cf	12.6	5.5	0.0	0.6	0.0	7.5	

Supershort barriers B_{ss}

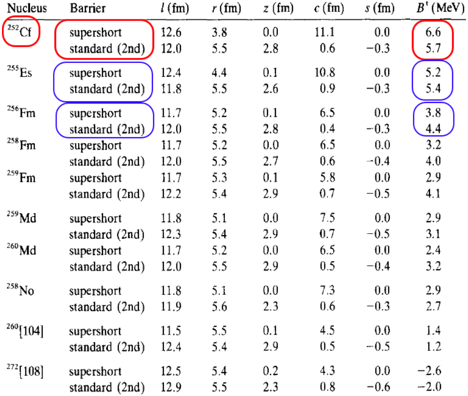
- Supershort channel does not exist in light nuclei



Nucleus	Barrier	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B' (MeV)
^{252}Cf	supershort	12.6	3.8	0.0	11.1	0.0	6.6
	standard (2nd)	12.0	5.5	2.8	0.6	−0.3	5.7
^{255}Es	supershort	12.4	4.4	0.1	10.8	0.0	5.2
	standard (2nd)	11.8	5.5	2.6	0.9	−0.3	5.4
^{256}Fm	supershort	11.7	5.2	0.1	6.5	0.0	3.8
	standard (2nd)	12.0	5.5	2.8	0.4	−0.3	4.4
^{258}Fm	supershort	11.7	5.2	0.0	6.5	0.0	3.2
	standard (2nd)	12.0	5.5	2.7	0.6	−0.4	4.0
^{259}Fm	supershort	11.7	5.3	0.1	5.8	0.0	2.9
	standard (2nd)	12.2	5.4	2.9	0.7	−0.5	4.1
^{259}Md	supershort	11.8	5.1	0.0	7.5	0.0	2.9
	standard (2nd)	12.3	5.4	2.9	0.7	−0.5	3.1
^{260}Md	supershort	11.7	5.2	0.0	6.5	0.0	2.4
	standard (2nd)	12.0	5.5	2.9	0.5	−0.4	3.2
^{258}No	supershort	11.8	5.1	0.0	7.3	0.0	2.9
	standard (2nd)	11.9	5.6	2.3	0.6	−0.3	2.7
^{260}[104]	supershort	11.5	5.5	0.1	4.5	0.0	1.4
	standard (2nd)	12.4	5.4	2.9	0.5	−0.5	1.2
^{272}[108]	supershort	12.5	5.4	0.2	4.3	0.0	−2.6
	standard (2nd)	12.9	5.5	2.3	0.8	−0.6	−2.0

Supershort barriers B_{ss}

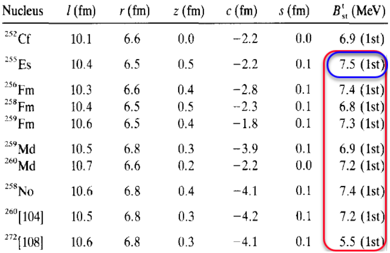
- Supershort barriers are "lower" than Standard barriers (except for ^{252}Cf)



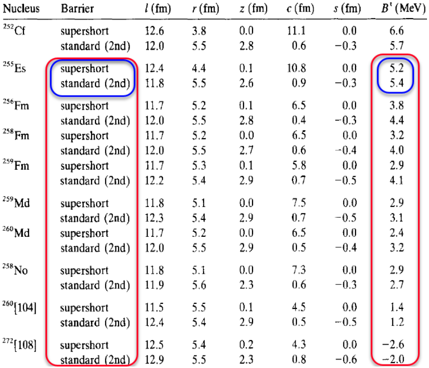
Nucleus	Barrier	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B' (MeV)
^{252}Cf	supershort	12.6	3.8	0.0	11.1	0.0	6.6
	standard (2nd)	12.0	5.5	2.8	0.6	−0.3	5.7
^{255}Es	supershort	12.4	4.4	0.1	10.8	0.0	5.2
	standard (2nd)	11.8	5.5	2.6	0.9	−0.3	5.4
^{256}Fm	supershort	11.7	5.2	0.1	6.5	0.0	3.8
	standard (2nd)	12.0	5.5	2.8	0.4	−0.3	4.4
^{258}Fm	supershort	11.7	5.2	0.0	6.5	0.0	3.2
	standard (2nd)	12.0	5.5	2.7	0.6	−0.4	4.0
^{259}Fm	supershort	11.7	5.3	0.1	5.8	0.0	2.9
	standard (2nd)	12.2	5.4	2.9	0.7	−0.5	4.1
^{259}Md	supershort	11.8	5.1	0.0	7.5	0.0	2.9
	standard (2nd)	12.3	5.4	2.9	0.7	−0.5	3.1
^{260}Md	supershort	11.7	5.2	0.0	6.5	0.0	2.4
	standard (2nd)	12.0	5.5	2.9	0.5	−0.4	3.2
^{258}No	supershort	11.8	5.1	0.0	7.3	0.0	2.9
	standard (2nd)	11.9	5.6	2.3	0.6	−0.3	2.7
^{260}[104]	supershort	11.5	5.5	0.1	4.5	0.0	1.4
	standard (2nd)	12.4	5.4	2.9	0.5	−0.5	1.2
^{272}[108]	supershort	12.5	5.4	0.2	4.3	0.0	−2.6
	standard (2nd)	12.9	5.5	2.3	0.8	−0.6	−2.0

Supershort barriers B_{ss}

- Lower than both 1^{st} and 2^{nd} standard barriers



Nucleus	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B'_{st} (MeV)
^{252}Cf	10.1	6.6	0.0	−2.2	0.0	6.9 (1st)
^{255}Es	10.4	6.5	0.5	−2.2	0.1	7.5 (1st)
^{256}Fm	10.3	6.6	0.4	−2.8	0.1	7.4 (1st)
^{258}Fm	10.4	6.5	0.5	−2.3	0.1	6.8 (1st)
^{259}Fm	10.6	6.5	0.4	−1.8	0.1	7.3 (1st)
^{259}Md	10.5	6.8	0.3	−3.9	0.1	6.9 (1st)
^{260}Md	10.7	6.6	0.2	−2.2	0.0	7.2 (1st)
^{258}No	10.6	6.8	0.4	−4.1	0.1	7.4 (1st)
^{260}[104]	10.5	6.8	0.3	−4.2	0.1	7.2 (1st)
^{272}[108]	10.6	6.8	0.3	−4.1	0.1	5.5 (1st)



Nucleus	Barrier	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	B' (MeV)
^{252}Cf	supershort	12.6	3.8	0.0	11.1	0.0	6.6
	standard (2nd)	12.0	5.5	2.8	0.6	−0.3	5.7
^{255}Es	supershort	12.4	4.4	0.1	10.8	0.0	5.2
	standard (2nd)	11.8	5.5	2.6	0.9	−0.3	5.4
^{256}Fm	supershort	11.7	5.2	0.1	6.5	0.0	3.8
	standard (2nd)	12.0	5.5	2.8	0.4	−0.3	4.4
^{258}Fm	supershort	11.7	5.2	0.0	6.5	0.0	3.2
	standard (2nd)	12.0	5.5	2.7	0.6	−0.4	4.0
^{259}Fm	supershort	11.7	5.3	0.1	5.8	0.0	2.9
	standard (2nd)	12.2	5.4	2.9	0.7	−0.5	4.1
^{259}Md	supershort	11.8	5.1	0.0	7.5	0.0	2.9
	standard (2nd)	12.3	5.4	2.9	0.7	−0.5	3.1
^{260}Md	supershort	11.7	5.2	0.0	6.5	0.0	2.4
	standard (2nd)	12.0	5.5	2.9	0.5	−0.4	3.2
^{258}No	supershort	11.8	5.1	0.0	7.3	0.0	2.9
	standard (2nd)	11.9	5.6	2.3	0.6	−0.3	2.7
^{260}[104]	supershort	11.5	5.5	0.1	4.5	0.0	1.4
	standard (2nd)	12.4	5.4	2.9	0.5	−0.5	1.2
^{272}[108]	supershort	12.5	5.4	0.2	4.3	0.0	−2.6
	standard (2nd)	12.9	5.5	2.3	0.8	−0.6	−2.0

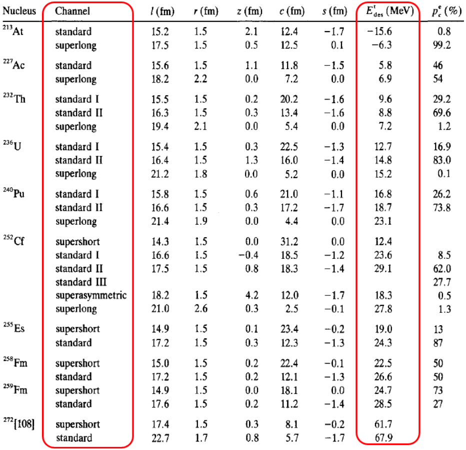
Prescission shapes

Energy of descent E_{des} :

- difference between potential energies of "ground state" and "prescission shape"

Table :

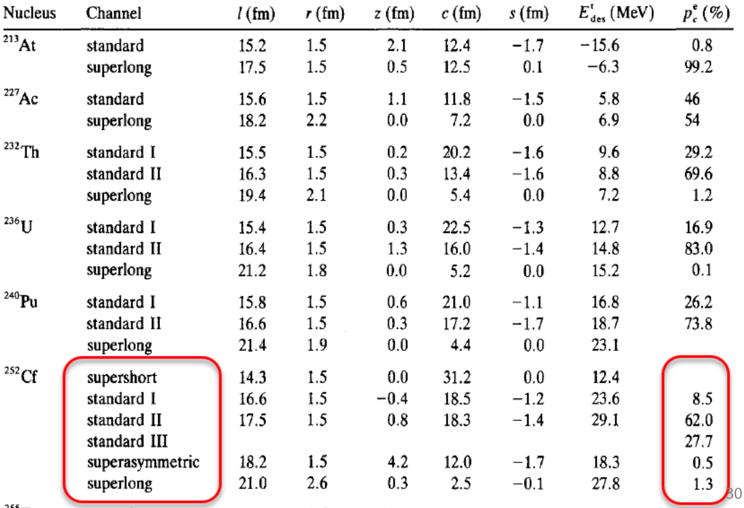
- different nuclei with different channels



Nucleus	Channel	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	E^{t}_{des} (MeV)	p^{e}_{c} (%)
^{213}At	standard	15.2	1.5	2.1	12.4	−1.7	−15.6	0.8
	superlong	17.5	1.5	0.5	12.5	0.1	−6.3	99.2
^{227}Ac	standard	15.6	1.5	1.1	11.8	−1.5	5.8	46
	superlong	18.2	2.2	0.0	7.2	0.0	6.9	54
^{232}Th	standard I	15.5	1.5	0.2	20.2	−1.6	9.6	29.2
	standard II	16.3	1.5	0.3	13.4	−1.6	8.8	69.6
	superlong	19.4	2.1	0.0	5.4	0.0	7.2	1.2
^{236}U	standard I	15.4	1.5	0.3	22.5	−1.3	12.7	16.9
	standard II	16.4	1.5	1.3	16.0	−1.4	14.8	83.0
	superlong	21.2	1.8	0.0	5.2	0.0	15.2	0.1
^{240}Pu	standard I	15.8	1.5	0.6	21.0	−1.1	16.8	26.2
	standard II	16.6	1.5	0.3	17.2	−1.7	18.7	73.8
	superlong	21.4	1.9	0.0	4.4	0.0	23.1	
^{252}Cf	supershort	14.3	1.5	0.0	31.2	0.0	12.4	
	standard I	16.6	1.5	−0.4	18.5	−1.2	23.6	8.5
	standard II	17.5	1.5	0.8	18.3	−1.4	29.1	62.0
	standard III							27.7
	superasymmetric	18.2	1.5	4.2	12.0	−1.7	18.3	0.5
	superlong	21.0	2.6	0.3	2.5	−0.1	27.8	1.3
^{255}Es	supershort	14.9	1.5	0.1	23.4	−0.2	19.0	13
	standard	17.2	1.5	0.3	12.3	−1.3	24.3	87
^{258}Fm	supershort	15.0	1.5	0.2	22.4	−0.1	22.5	50
	standard	17.2	1.5	0.2	12.1	−1.3	26.6	50
^{259}Fm	supershort	14.9	1.5	0.0	18.1	0.0	24.7	73
	standard	17.6	1.5	0.2	11.2	−1.4	28.5	27
^{272}[108]	supershort	17.4	1.5	0.3	8.1	−0.2	61.7	
	standard	22.7	1.7	0.8	5.7	−1.7	67.9	

Prescission shapes

- Channel probabilities P_c^e



Nucleus	Channel	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	E^t_{des} (MeV)	p^e_c (%)
^{213}At	standard	15.2	1.5	2.1	12.4	−1.7	−15.6	0.8
	superlong	17.5	1.5	0.5	12.5	0.1	−6.3	99.2
^{227}Ac	standard	15.6	1.5	1.1	11.8	−1.5	5.8	46
	superlong	18.2	2.2	0.0	7.2	0.0	6.9	54
^{232}Th	standard I	15.5	1.5	0.2	20.2	−1.6	9.6	29.2
	standard II	16.3	1.5	0.3	13.4	−1.6	8.8	69.6
	superlong	19.4	2.1	0.0	5.4	0.0	7.2	1.2
^{236}U	standard I	15.4	1.5	0.3	22.5	−1.3	12.7	16.9
	standard II	16.4	1.5	1.3	16.0	−1.4	14.8	83.0
	superlong	21.2	1.8	0.0	5.2	0.0	15.2	0.1
^{240}Pu	standard I	15.8	1.5	0.6	21.0	−1.1	16.8	26.2
	standard II	16.6	1.5	0.3	17.2	−1.7	18.7	73.8
	superlong	21.4	1.9	0.0	4.4	0.0	23.1	
^{252}Cf	supershort	14.3	1.5	0.0	31.2	0.0	12.4	
	standard I	16.6	1.5	−0.4	18.5	−1.2	23.6	8.5
	standard II	17.5	1.5	0.8	18.3	−1.4	29.1	62.0
	standard III							27.7
	superasymmetric	18.2	1.5	4.2	12.0	−1.7	18.3	0.5
	superlong	21.0	2.6	0.3	2.5	−0.1	27.8	1.3

superlong & supershort barriers

`Why superlong's probability` (P_{sl})

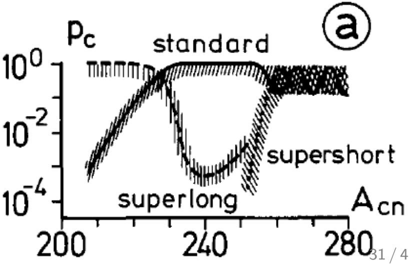
- dominates for light nuclei
- diminishes with increasing A_{cn}

`Why supershort's probability` (P_{ss})

- never able to push standard into forgetfulness

`Answer`

- height of barriers



superlong & supershort barriers

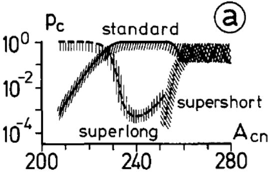
`For simplicity imagine :`

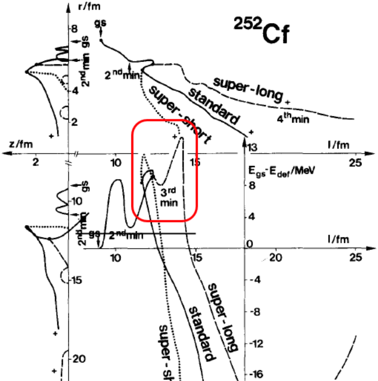
- only two fission channels
- one bifurcation point
- several barriers

superlong & supershort barriers

`Situation (i)`

- The highest barriers of channels lie behind bifurcation
- ⇒ channels have separate highest barriers

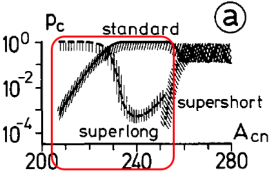


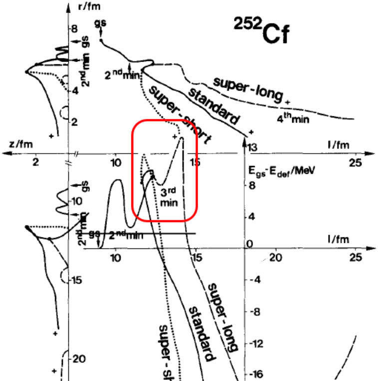


superlong & supershort barriers

`Situation (i)`

- superlong - standard bifurcation
- applies to nuclei lighter than ^{252}Cf

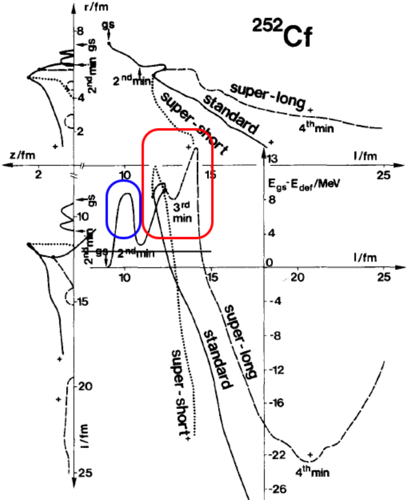




superlong & supershort barriers

`superlong and standard channels`

- first standard barrier is low
- after bifurcation (close to the 2^{nd} min)
- high second standard and superlong barriers



superlong & supershort barriers

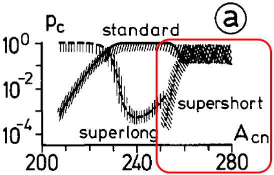
`Situation (ii)`

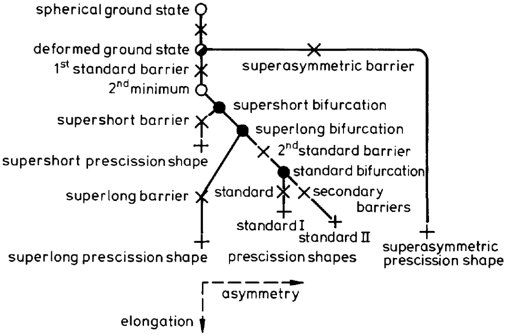
- before bifurcation, one highest barrier for both channels
- after bifurcation, lower secondary barriers

superlong & supershort barriers

Situation (ii)

- supershort - standard bifurcation
- applies to nuclei heavier than ^{252}Cf

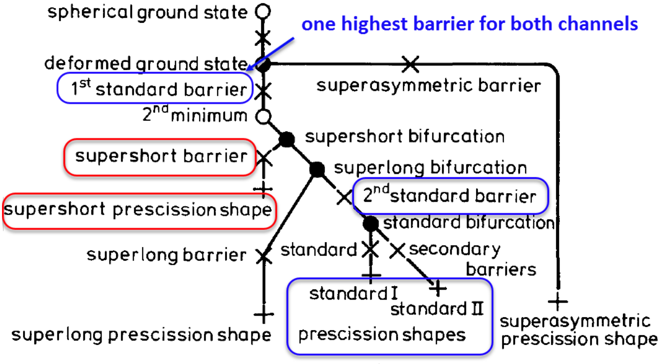




superlong & supershort barriers

`Situation (ii)`

- all nuclei fissioning via supershort channel
  `climb supershort barrier`
- all standard fissioners
  `overcome second standard barrier`



one highest barrier for both channels

superlong & supershort barriers

`Situation (i)`

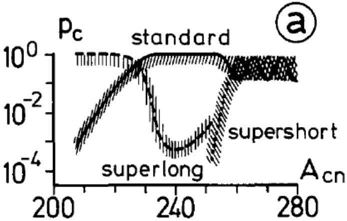
- behind bifurcation, the highest barriers of channels

`Situation (ii)`

- before bifurcation, one highest barrier for both channels

`That's why :`

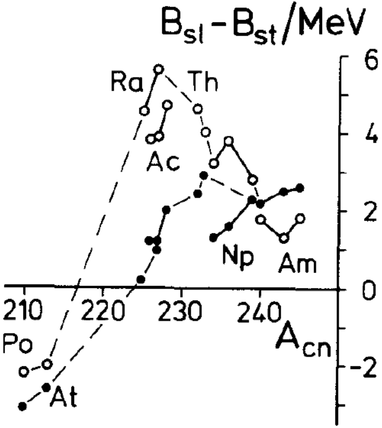
- superlong and standard channels can displace each other
- supershort and standard channels must coexist



superlong barrier B_{sl} over standard barrier B_{st}

`Barrier-height differences`

○ : computational results ● : measured data



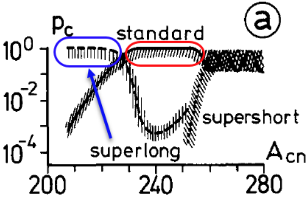
superlong barrier B_{sl} over standard barrier B_{st}

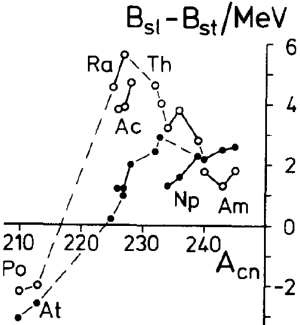
- `For lighter nuclei :`

  P_{sl} is higher $\Rightarrow$ B_{sl} is lower

- `by increasing mass :`

  P_{st} becomes larger $\Rightarrow$ B_{st} becomes smaller



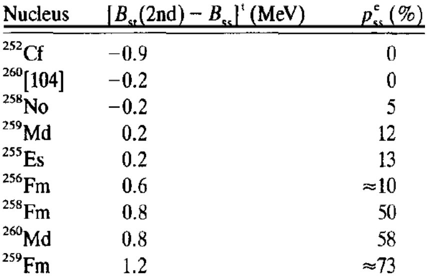


standard barrier B_{st} over supershort barrier B_{ss}

- `Table` : arranged according to increasing differences

- As the differences grow

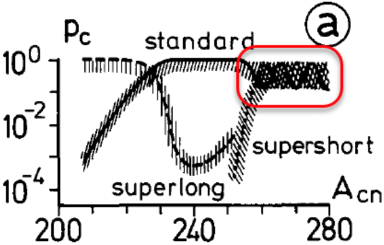
- supershort probabilities P_{ss} increase



Nucleus	$[B_{st}(2nd) - B_{ss}]^t$ (MeV)	p_{ss}^e (%)
^{252}Cf	−0.9	0
^{260}[104]	−0.2	0
^{258}No	−0.2	5
^{259}Md	0.2	12
^{255}Es	0.2	13
^{256}Fm	0.6	≈10
^{258}Fm	0.8	50
^{260}Md	0.8	58
^{259}Fm	1.2	≈73

standard barrier B_{st} over supershort barrier B_{ss}

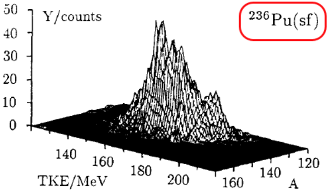
- `Table` : arranged according to increasing mass number A

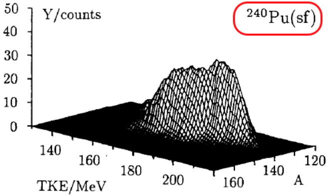


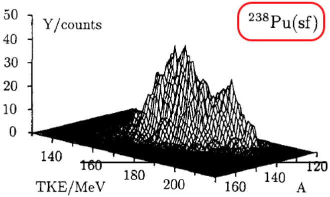
Nucleus	$(B_{st}(2nd) - B_{ss})$	$P_{ss}(\%)$
^{252}Cf	-0.9	0
^{255}Es	0.2	13
^{256}Fm	0.6	10
^{258}No	-0.2	5
^{258}Fm	0.8	50
^{259}Md	0.2	12
^{259}Fm	1.2	73
$^{260}[104]$	-0.2	0
^{260}Md	0.8	58

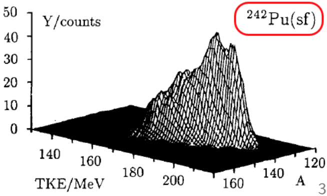
Standard splitting (Standard I and II)

- Fission yields of plutonium isotopes
  over the plane of fragment mass number A and TKE



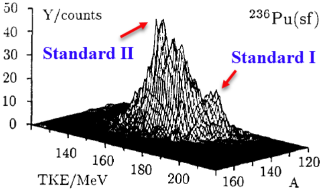


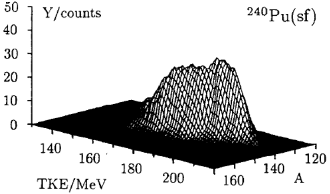


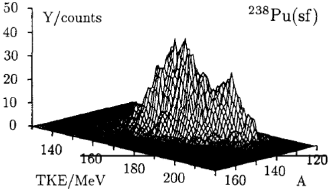


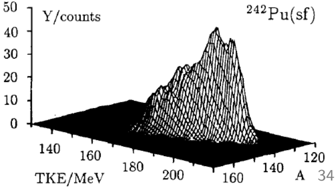
Standard splitting (Standard I and II)

- Standard I : around $A \approx 135$ and $TKE \approx 190MeV$
- Standard II : around $A \approx 142$ and $TKE \approx 175MeV$





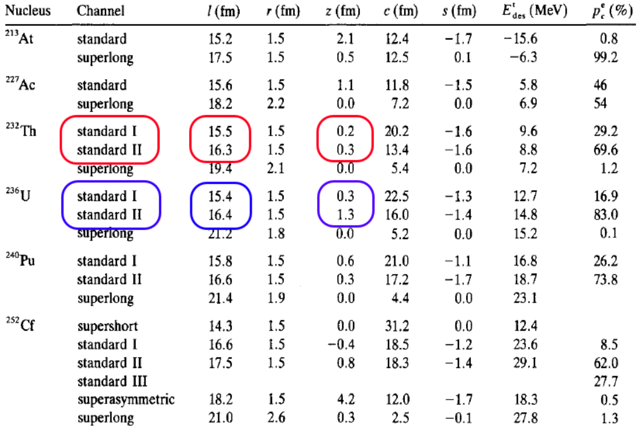




Standard splitting (Standard I and II)

`Standard I prescission shape:`

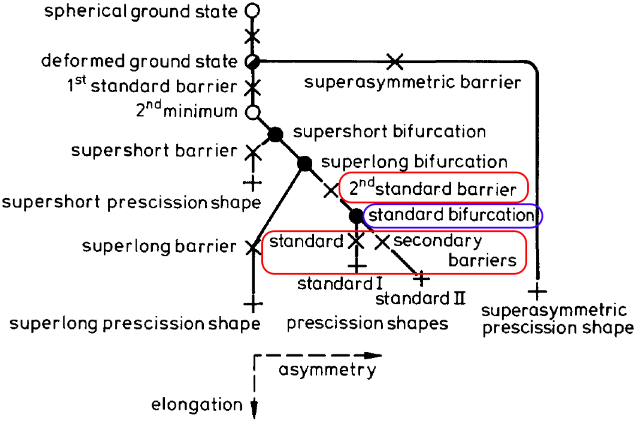
- less asymmetric and shorter than standard II
- make fragments with less deformation



Nucleus	Channel	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)	E^{t}_{des} (MeV)	p^{e}_{c} (%)
^{213}At	standard	15.2	1.5	2.1	12.4	−1.7	−15.6	0.8
	superlong	17.5	1.5	0.5	12.5	0.1	−6.3	99.2
^{227}Ac	standard	15.6	1.5	1.1	11.8	−1.5	5.8	46
	superlong	18.2	2.2	0.0	7.2	0.0	6.9	54
^{232}Th	standard I	15.5	1.5	0.2	20.2	−1.6	9.6	29.2
	standard II	16.3	1.5	0.3	13.4	−1.6	8.8	69.6
	superlong	19.4	2.1	0.0	5.4	0.0	7.2	1.2
^{236}U	standard I	15.4	1.5	0.3	22.5	−1.3	12.7	16.9
	standard II	16.4	1.5	1.3	16.0	−1.4	14.8	83.0
	superlong	21.2	1.8	0.0	5.2	0.0	15.2	0.1
^{240}Pu	standard I	15.8	1.5	0.6	21.0	−1.1	16.8	26.2
	standard II	16.6	1.5	0.3	17.2	−1.7	18.7	73.8
	superlong	21.4	1.9	0.0	4.4	0.0	23.1	
^{252}Cf	supershort	14.3	1.5	0.0	31.2	0.0	12.4	
	standard I	16.6	1.5	−0.4	18.5	−1.2	23.6	8.5
	standard II	17.5	1.5	0.8	18.3	−1.4	29.1	62.0
	standard III							27.7
	superasymmetric	18.2	1.5	4.2	12.0	−1.7	18.3	0.5
	superlong	21.0	2.6	0.3	2.5	−0.1	27.8	1.3

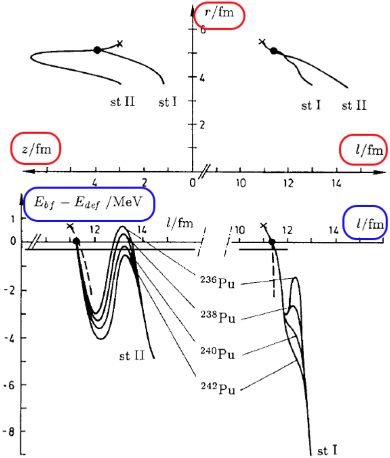
Standard splitting (Standard I and II)

- behind 2^{nd} standard barrier : standard bifurcation
- behind bifurcation : low "standard secondary barriers" - `situation (ii)`



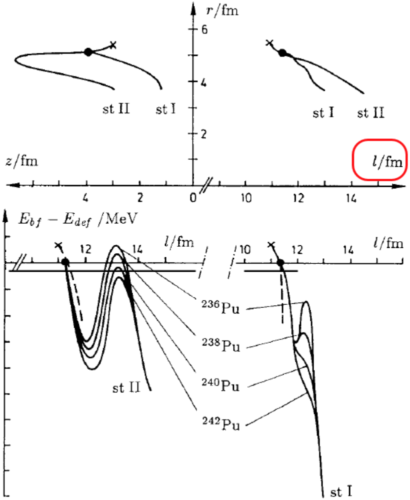
Standard splitting (Standard I and II)

- Geometric and energetic characteristics of standard I/II in Pu isotopes



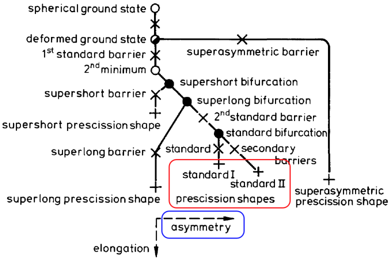
Standard splitting (Standard I and II)

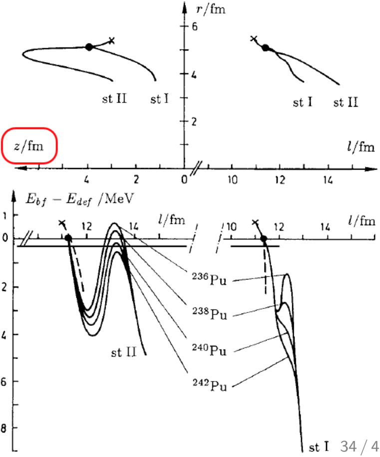
- Top-right: standard I ruptures at a shorter semilength l



Standard splitting (Standard I and II)

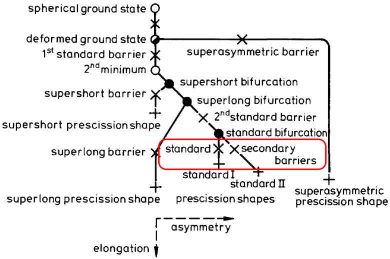
- Top-left: standard I ruptures with a smaller asymmetry z



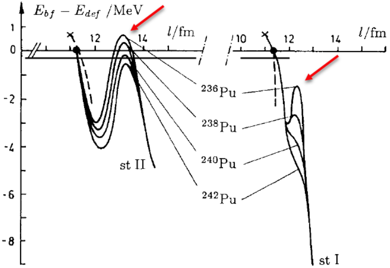


Standard splitting (Standard I and II)

- Bumps in potential energy : secondary standard barriers

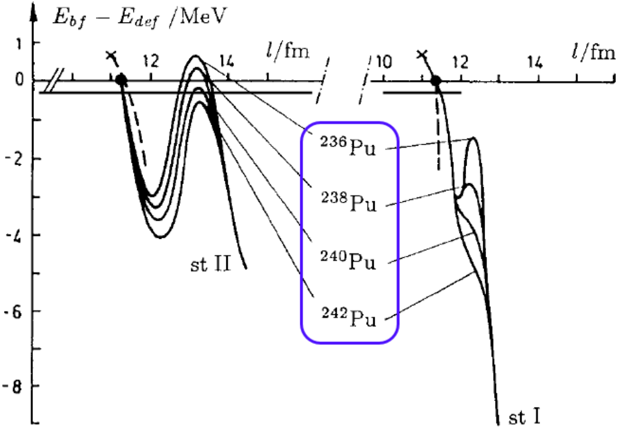


spherical ground state
deformed ground state
1st standard barrier
2nd minimum
superasymmetric barrier
supershort bifurcation
supershort barrier
superlong bifurcation
2nd standard barrier
supershort prescission shape
standard bifurcation
superlong barrier
standard
secondary barriers
standard I
standard II
superlong prescission shape
prescission shapes
superasymmetric prescission shape
asymmetry
elongation



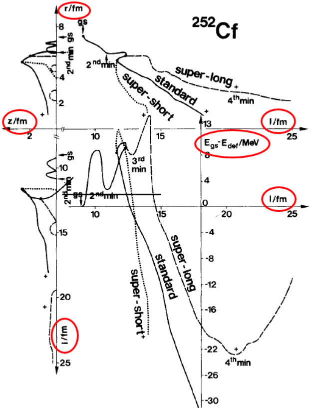
Standard splitting (Standard I and II)

- From light to heavy isotopes : barriers decrease
- Along standard I : barriers even disappear



Strutinsky's approach

- To reveal fission channels :
  - compute potential energy of deformed nuclei E_{def}
  - as a function of shape coordinates



Strutinsky's approach

- To reveal fission channels :
  - compute potential energy of deformed nuclei E_{def}
  - as a function of shape coordinates
- Strutinsky's approach :
  - the potential energy is composed of :
  - a "liquid-drop" part and a "shell" part
  - $E_{def} = E_{ld} + E_{shell}$
- `Myers-Swiatecki model :  liquid-drop part` E_{ld}

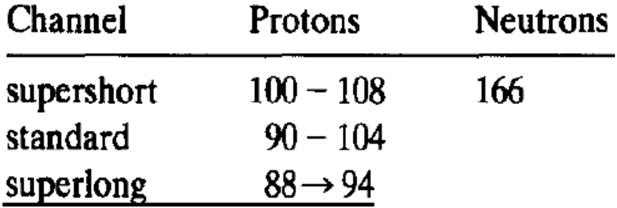
Magic numbers of fission

- Powerful shell effects are fundamental for :
  - `formation of exit channels in nuclear fission`
- Shell effects are caused by :
  - `significant gaps`
- gap existence ⇒ `magic numbers existence`

Magic numbers of fission

- Magic numbers of fission :

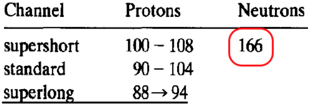
`properties of "fissioning nuclei"`



Channel	Protons	Neutrons
supershort	100 – 108	166
standard	90 – 104	
superlong	88 → 94	

- dash : a magic range
- arrow : transition
- neutron magic numbers or range :
  not clear in standard and superlong channels

Neutron magic number of supershort channel



Channel	Protons	Neutrons
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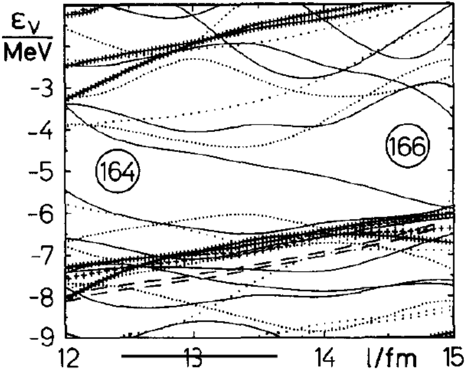
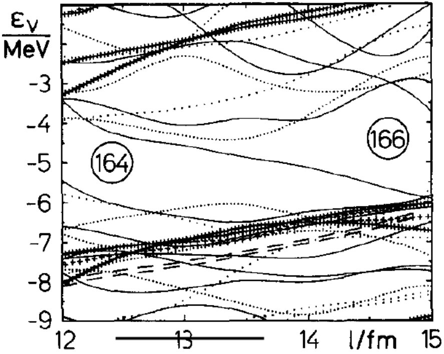


- Figure: neutron single-particle spectrum ϵ_ν as a function of elongation l

Neutron magic number of supershort channel

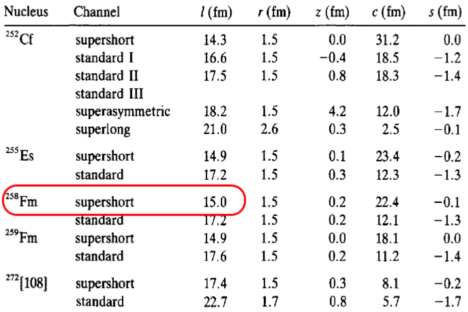
`Two large gaps :`

1. above the 82^{nd} level at moderate deformations (164 neutrons fit it)
2. above the 83^{rd} level at larger stretching (166 neutrons fit it)

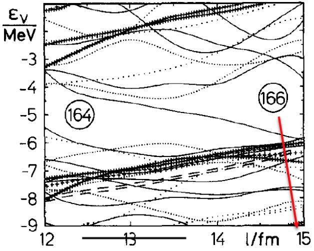


Neutron magic number of supershort channel

- at semilength $l \approx 15(fm)$: supershort channel ruptures



Nucleus	Channel	l (fm)	r (fm)	z (fm)	c (fm)	s (fm)
^{252}Cf	supershort	14.3	1.5	0.0	31.2	0.0
	standard I	16.6	1.5	−0.4	18.5	−1.2
	standard II	17.5	1.5	0.8	18.3	−1.4
	standard III					
	superasymmetric	18.2	1.5	4.2	12.0	−1.7
	superlong	21.0	2.6	0.3	2.5	−0.1
^{255}Es	supershort	14.9	1.5	0.1	23.4	−0.2
	standard	17.2	1.5	0.3	12.3	−1.3
^{258}Fm	supershort	15.0	1.5	0.2	22.4	−0.1
	standard	17.2	1.5	0.2	12.1	−1.3
^{259}Fm	supershort	14.9	1.5	0.0	18.1	0.0
	standard	17.6	1.5	0.2	11.2	−1.4
^{272}[108]	supershort	17.4	1.5	0.3	8.1	−0.2
	standard	22.7	1.7	0.8	5.7	−1.7



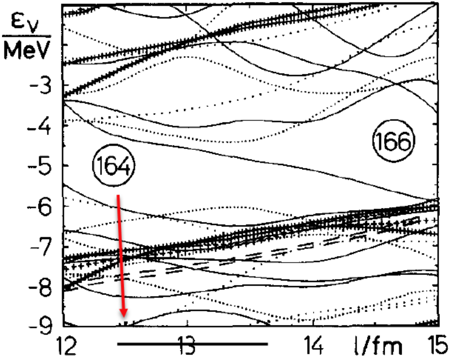
Neutron magic number of supershort channel

- at semilength $l \approx 12.5(fm)$:

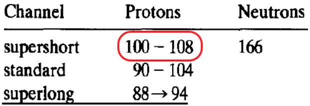
  liquid-drop energies are so adverse

$\Rightarrow$ shell cannot induce scission

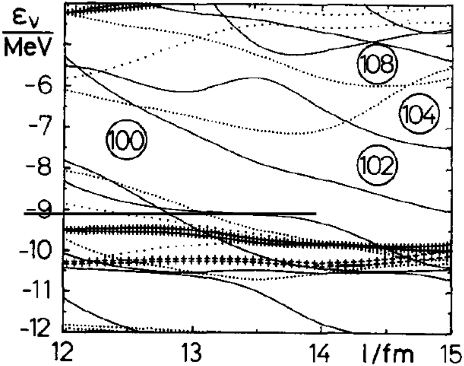
$\Rightarrow$ 166 : correct neutron magic number of supershort channel



Proton magic number of supershort channel



Channel	Protons	Neutrons
supershort	100 – 108	166
standard	90 – 104	
superlong	88→94	

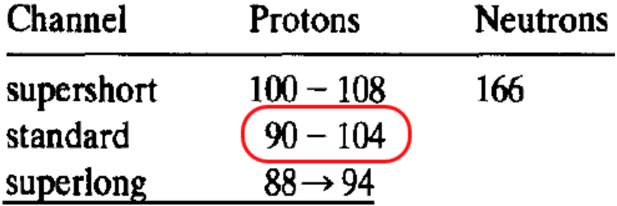


- not a definite gap
- a broad zone of level thining
- a magic range : $100 - 108$

Proton magic number of standard channel

- Proton magic number : wide range of $90 - 104$
- `Universality of standard channel`

  through many preactinides and all actinides

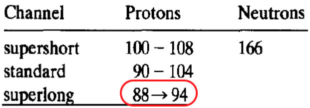


Channel	Protons	Neutrons
supershort	100 – 108	166
standard	90 – 104	
superlong	88 → 94	

Proton magic number of superlong channel

Two gaps :

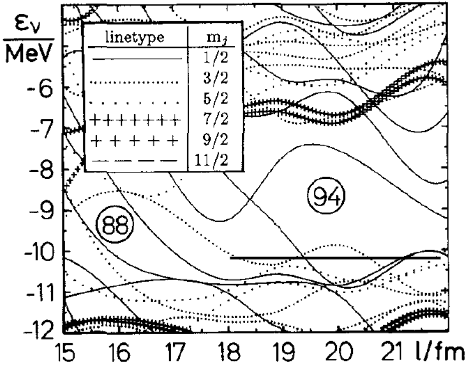
1. at 88 protons
2. at 94 protons



Channel	Protons	Neutrons
supershort	100 – 108	166
standard	90 – 104	
superlong	88 → 94	

Liquid-drop energies :

- not adverse to hinder nucleus from breaking



Magic numbers of fission

`Associate :`

- magic numbers of fissioning nucleus
- magic numbers of fragments `(2, 8, 20, 28, 50, 82, 126)`

– – – – – – – – – – – – – – – –

`Supershort neutron magic number 166 :`

- almost the sum $82 + 82$

– – – – – – – – – – – – – – – –

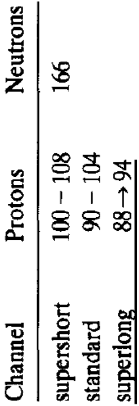
`Lower standard magic number 90 :`

- constructed as $40 + 50$

– – – – – – – – – – – – – – – –

`Superlong magic number :`

- no symmetric decomposition of 88 or 94



Channel	Protons	Neutrons
supershort	100 – 108	166
standard	90 – 104	
superlong	88 → 94	

Magic numbers of fission

`Magic numbers of fission :`

- sum of magic numbers of fragments

$+$

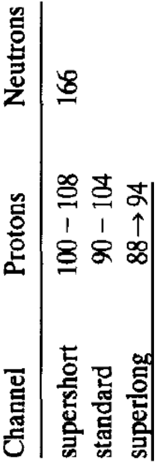
- some nucleons for the neck

`For superlong channel :`

- number of nucleons added

  so large that

- the argument looses its credibility



Channel	Protons	Neutrons
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Thank you for your attention!