Level density parameter
A correction on CRISP

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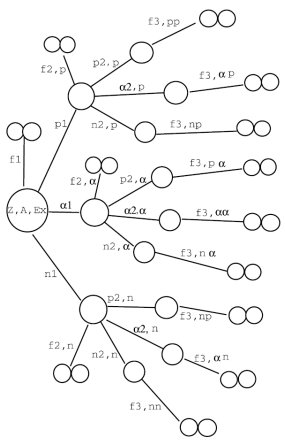

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Monte Carlo evaporation-fissions process

- **MCEF process**
  - a competition between evaporation and fission

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Monte Carlo evaporation-fissions process

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  - a competition between evaporation and fission
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  - calculation of level density parameters
- **Bohr-Wheeler model**
  - fission

Weisskopf model

- Relative probability of emission of two particles i and j:

$$\frac{\Gamma_i}{\Gamma_j} = \frac{\gamma_i}{\gamma_j}\left(\frac{E_i}{E_j}\right)\frac{a_j}{a_i} exp\{2[(a_i E_i)^{\frac{1}{2}} - (a_j E_j)^{\frac{1}{2}}]\}$$

a_i and a_j : level density parameters

Weisskopf model

$$\frac{\Gamma_i}{\Gamma_j} = \frac{\gamma_i}{\gamma_j}\left(\frac{E_i}{E_j}\right)\frac{a_j}{a_i} exp\{2[(a_i E_i)^{\frac{1}{2}} - (a_j E_j)^{\frac{1}{2}}]\}$$

$$\gamma_i = \frac{gm}{\pi^2\hbar^3}$$

- g : number of states for the spin of particle
- $g = 2$: for neutrons and protons
- $g = 1$: for α-particles
- m : mass of particle i
- $m_p \approx m_n$
- $m_\alpha \approx 4m_n$

Weisskopf model

$$\gamma_i = \frac{gm}{\pi^2\hbar^3}$$

- $g = 2$: `for neutrons and protons`
- $g = 1$: `for` α`-particles`
- $m_p \approx m_n$
- $m_\alpha \approx 4m_n$

$$\frac{\gamma_p}{\gamma_n} = \frac{g_p}{g_n} \times \frac{m_p}{m_n} = \frac{2}{2} \times \frac{1}{1} = 1$$

$$\frac{\gamma_\alpha}{\gamma_n} = \frac{g_\alpha}{g_n} \times \frac{m_\alpha}{m_n} = \frac{1}{2} \times \frac{4}{1} = 2$$

Weisskopf model

$$\frac{\Gamma_i}{\Gamma_j} = \frac{\gamma_i}{\gamma_j}\left(\frac{E_i}{E_j}\right)\frac{a_j}{a_i} exp\{2[(a_i E_i)^{\frac{1}{2}} - (a_j E_j)^{\frac{1}{2}}]\}$$

$$\frac{\gamma_p}{\gamma_n} = 1 \qquad and \qquad \frac{\gamma_\alpha}{\gamma_n} = 2$$

$$\Downarrow$$

- For proton emission:

$$\frac{\Gamma_p}{\Gamma_n} = \left(\frac{1E_p}{E_n}\right)\left(\frac{a_n}{a_p}\right)exp\{2[(a_p E_p)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

- For α-particle emission:

$$\frac{\Gamma_\alpha}{\Gamma_n} = \left(\frac{2E_\alpha}{E_n}\right)\left(\frac{a_n}{a_\alpha}\right)exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

Dostrovsky model

$$\frac{\Gamma_p}{\Gamma_n} = (\frac{E_p}{E_n})(\frac{a_n}{a_p})exp\{2[(a_p E_p)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n})(\frac{a_n}{a_\alpha})exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

- a_n, a_p, a_α : level density parameters for neutrons, protons, alpha particles

$$a_n = \frac{A}{a_1}(1 - a_2\frac{A-2Z}{A^2})^2$$

$$a_p = \frac{A}{a_3}(1 + a_4\frac{A-2Z}{A^2})^2$$

$$a_\alpha = \frac{A}{a_5}(1 - \frac{a_6}{A})^2$$

- a_1 to a_6 : fitted parameters

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Level density parameters in the old formula

- Level density parameter for neutron emission:

$$a_n = (0.134A - 1.21) \times 10^{-4} A^2 MeV^{-1}$$

- Level density parameter for proton and α-particle emission:

$$a_j = r_j a_n$$

- where:

$$r_p = r_\alpha = 1 \qquad \Longrightarrow \qquad \boxed{a_p = a_\alpha = a_n}$$

Level density parameters in the old formula

$$\frac{\Gamma_p}{\Gamma_n} = (\frac{E_p}{E_n})(\frac{a_n}{a_p})exp\{2[(a_p E_p)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n})(\frac{a_n}{a_\alpha})exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

.

$$a_p = a_\alpha = a_n$$

$$\Downarrow$$

$$\frac{a_n}{a_p} = \frac{a_n}{a_\alpha} = 1$$

Level density parameters in the old formula

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$$\Downarrow$$

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$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n})exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_nE_n)^{\frac{1}{2}}]\}$$

Level density parameters in the new formula

$$a_n = \frac{A}{a_1}(1 - a_2\frac{A-2Z}{A^2})^2$$

$$a_p = \frac{A}{a_3}(1 + a_4\frac{A-2Z}{A^2})^2$$

$$a_\alpha = \frac{A}{a_5}(1 - \frac{a_6}{A})^2$$

$$a_n \neq a_p \neq a_\alpha$$

$$\Downarrow$$

$$\frac{a_n}{a_p} \ , \ \frac{a_n}{a_\alpha} \quad \texttt{should not be omitted!}$$

Level density parameters in the new formula

$$a_n \neq a_p \neq a_\alpha$$

$$\Downarrow$$

- For proton emission:

$$\frac{\Gamma_p}{\Gamma_n} = (\frac{E_p}{E_n})(\frac{a_n}{a_p})exp\{2[(a_pE_p)^{\frac{1}{2}} - (a_nE_n)^{\frac{1}{2}}]\}$$

- For α-particle emission:

$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n})(\frac{a_n}{a_\alpha})exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_nE_n)^{\frac{1}{2}}]\}$$

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Test done on CRISP

- Comparison between:

$$\frac{\Gamma_p}{\Gamma_n} = (\frac{E_p}{E_n}) exp\{2[(a_p E_p)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n}) exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

- and the corrected formulae:

$$\frac{\Gamma_p}{\Gamma_n} = (\frac{E_p}{E_n})(\frac{a_n}{a_p}) exp\{2[(a_p E_p)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

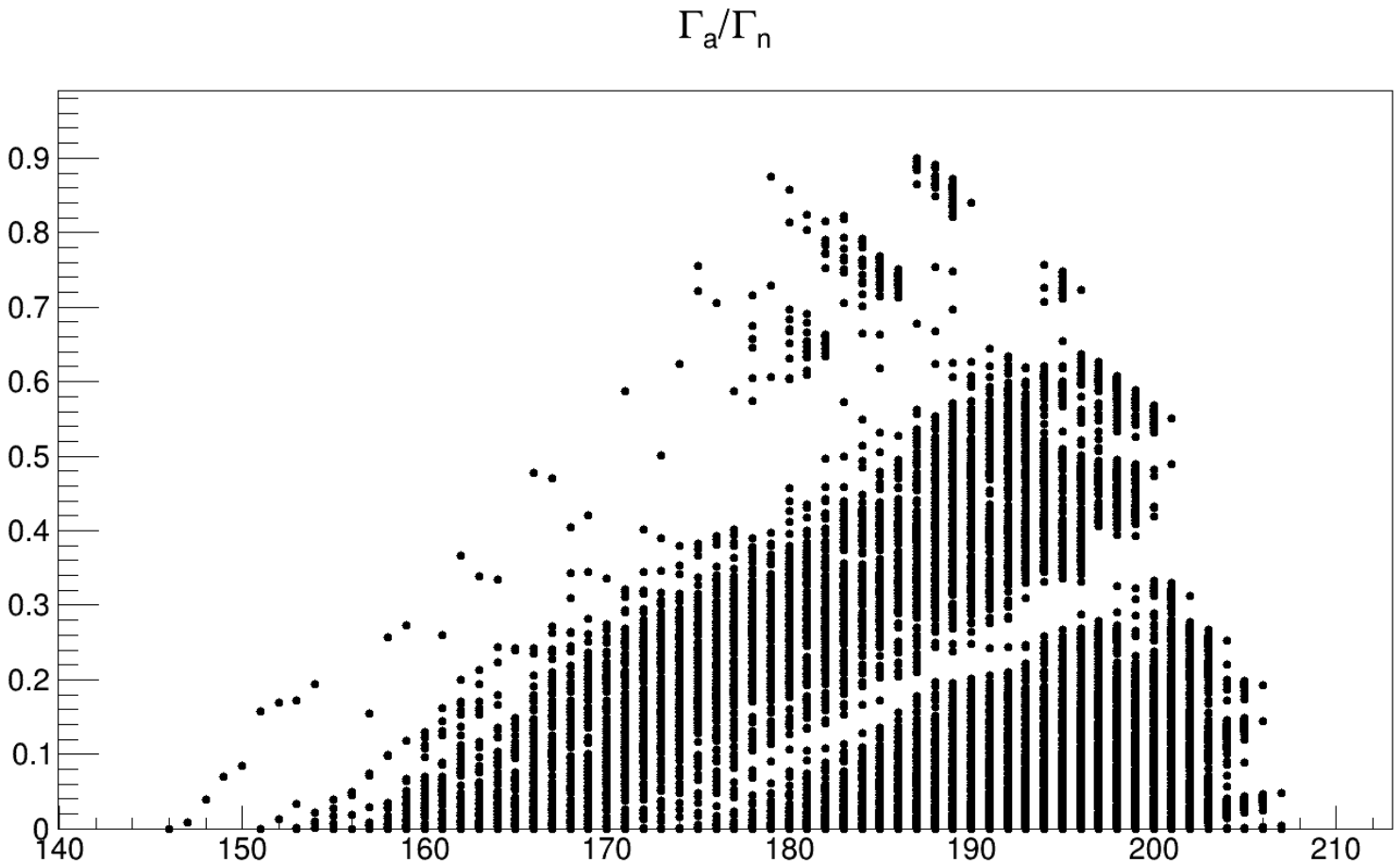
$$\frac{\Gamma_\alpha}{\Gamma_n} = (\frac{2E_\alpha}{E_n})(\frac{a_n}{a_\alpha}) exp\{2[(a_\alpha E_\alpha)^{\frac{1}{2}} - (a_n E_n)^{\frac{1}{2}}]\}$$

Test done on CRISP

- Target Nucleus : Pb208
- Projectile : proton
- Initial Energy : 1000 MeV

Results

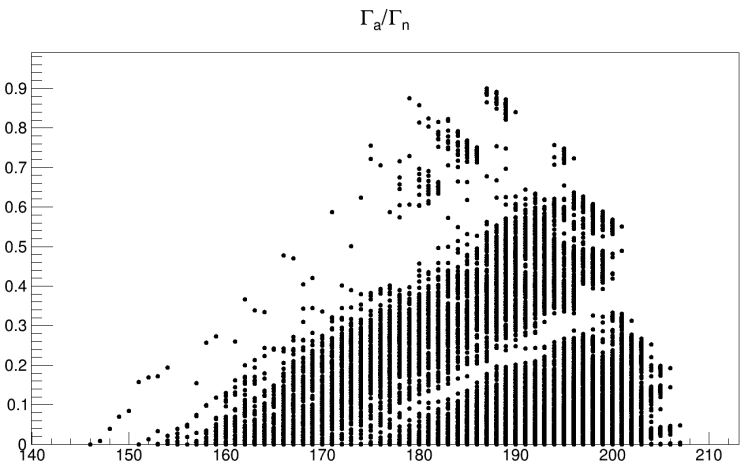
- `Results of CRISP:`



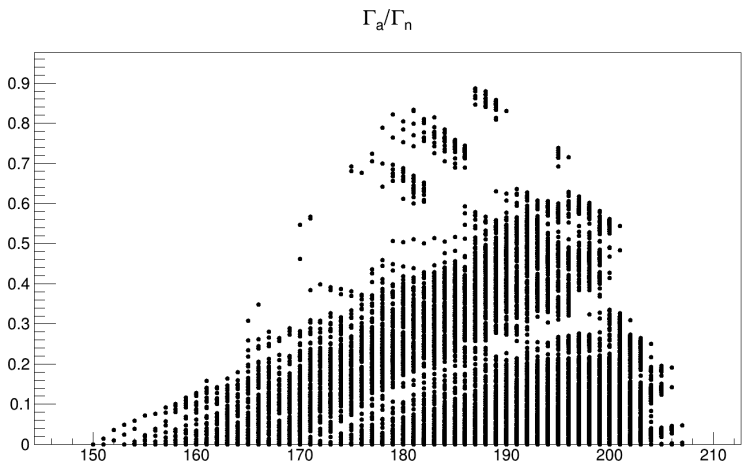
Results

- Comparison of CRISP results:

- new formula:



- old formula:



Results

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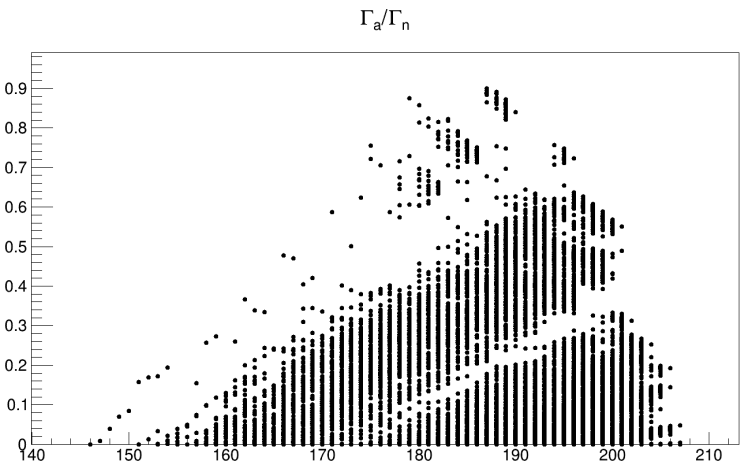
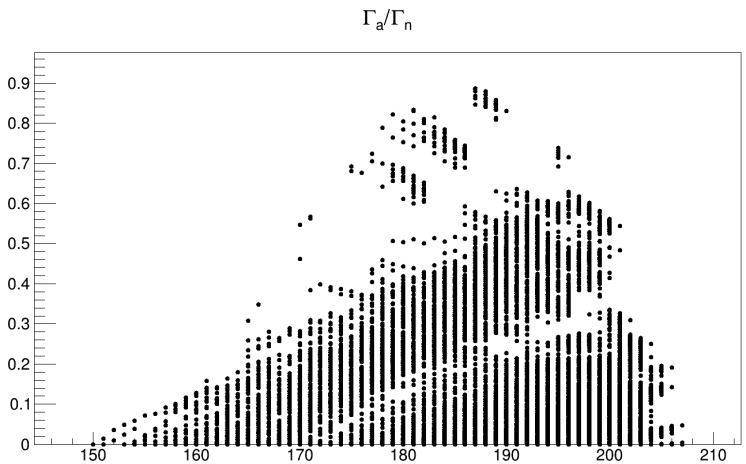


- old formula:



- Solution:

Averaging over the whole data for each mass number

Averaging Code

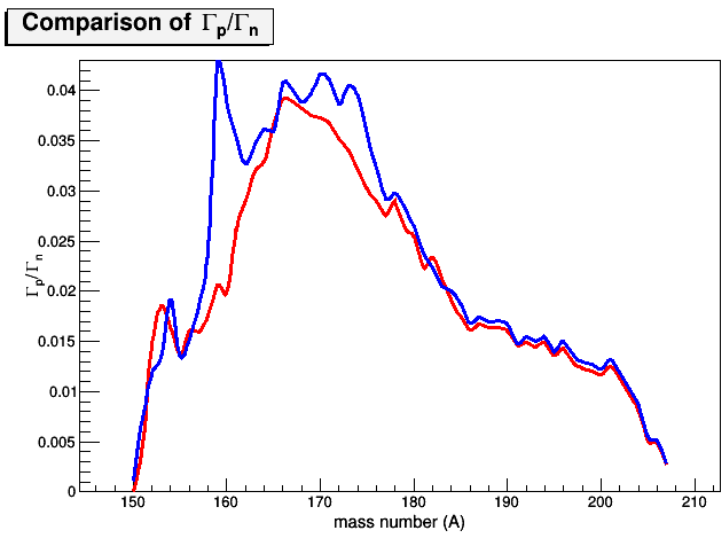

```cpp
#include <iostream>
#include <fstream>
using namespace std;

int main(){
    ifstream obj1("./A.txt");
    ifstream obj2("./Gp.txt");

    int A;
    double Gp;
    int B = 70774;  // number of rows (data)
    int k = 0;
    int l = 0;
    int j = 0;
    double b[70][4];  // 70 = last A - first A + 1
    int a[B];
    double c[B];

    ofstream writing("./output-Gp.xls");

    while(obj1 >> A){
        a[k] = A;
        k++;
    }

    while(obj2 >> Gp){
        c[l] = Gp;
        l++;
    }

    for (int i = 0; i < B; i++)
    {
        if (i==0)
        {
            b[j][0] = a[i];
            b[j][1] = 1;
            b[j][2] = c[i];
            b[j][3] = b[j][2] / b[j][1];
        }else{
            if(a[i]==a[i-1])
            {
                b[j][0] = a[i];
                b[j][1]++;
                b[j][2]+=c[i];
                b[j][3] = b[j][2] / b[j][1];
            }else{
                j++;
                b[j][0] = a[i];
                b[j][1] = 1;
                b[j][2] = c[i];
                b[j][3] = b[j][2] / b[j][1];
            }
        }
    }
    for (j=0; j < 70; j++)
    {
        writing << b[j][0] << " " << b[j][3] << endl;
    }
    return 0;
}
```

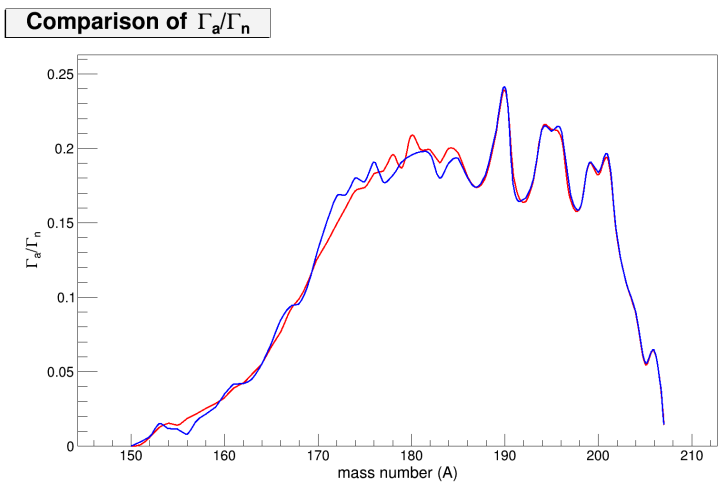
Comparison of Results before and after correction

- 39000 points:

— old formula

— new formula



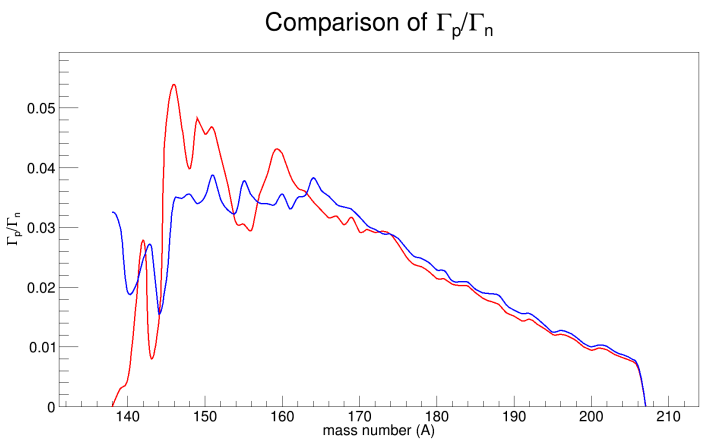


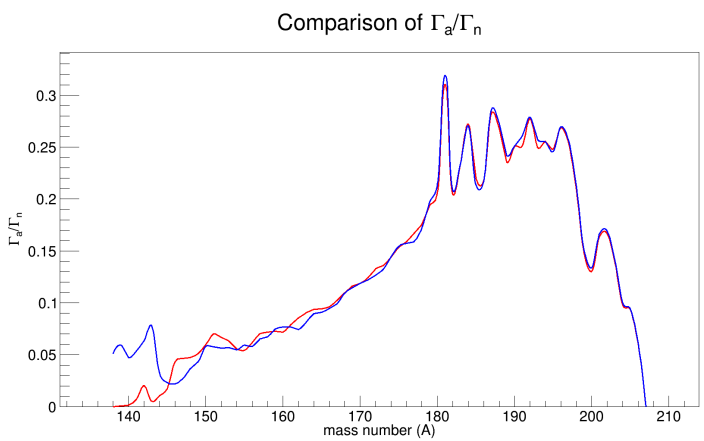
Comparison of Results before and after correction

- 71000 points:

— old formula

— new formula

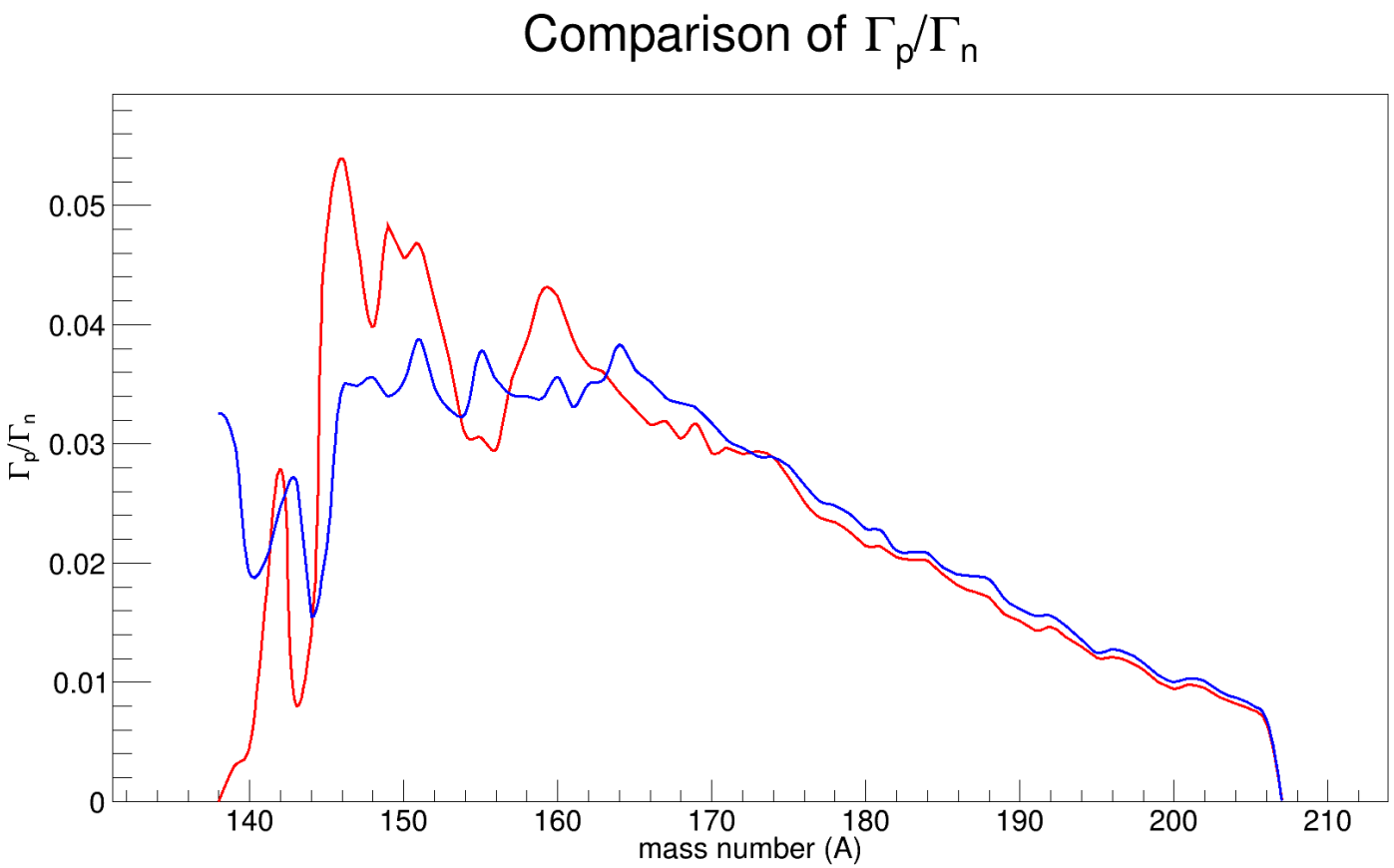




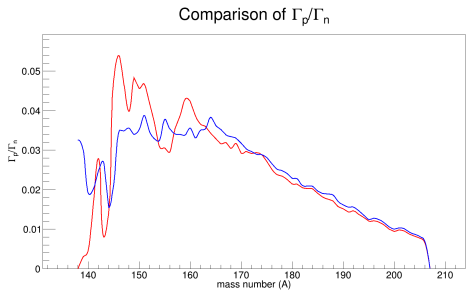
Γ_p/Γ_n

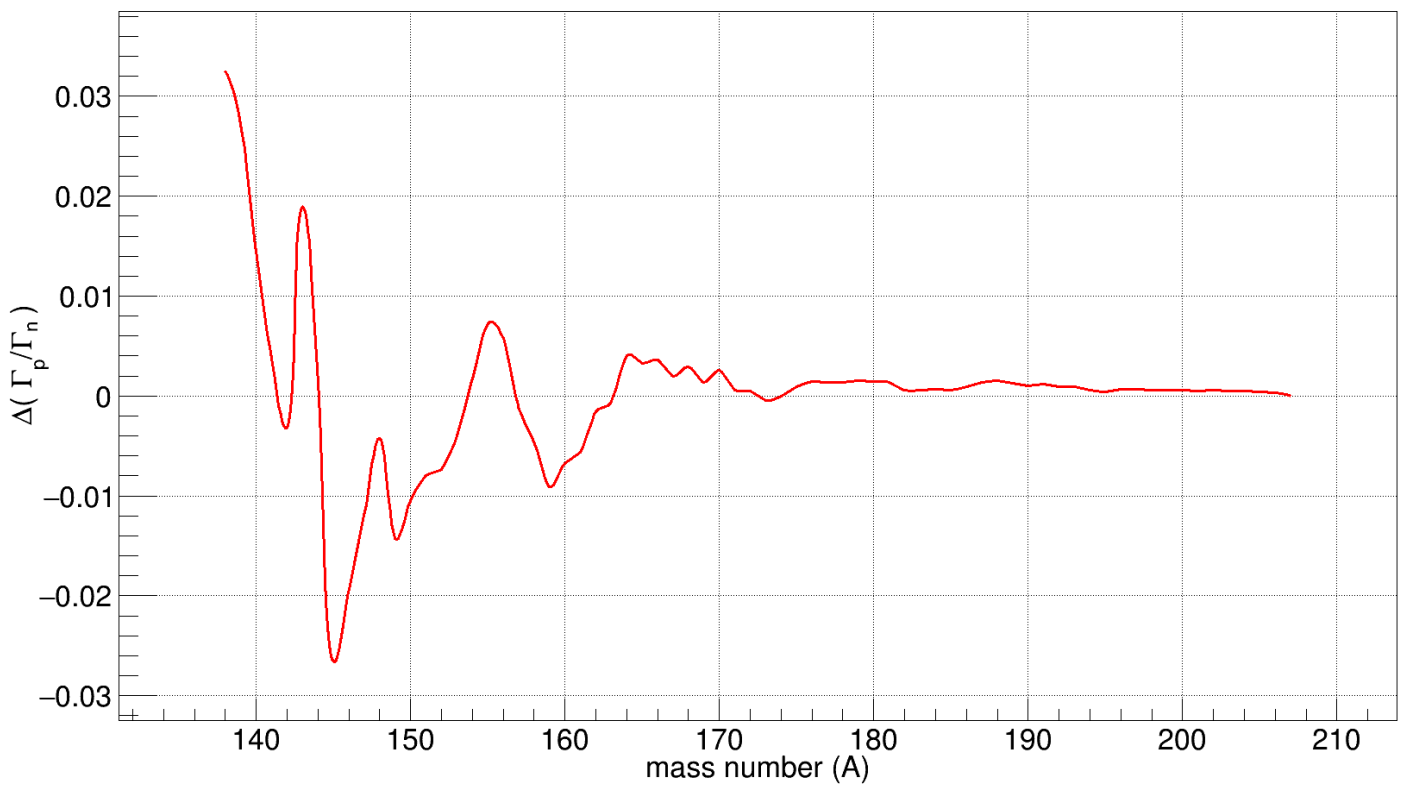
old formula

new formula



Γ_p/Γ_n

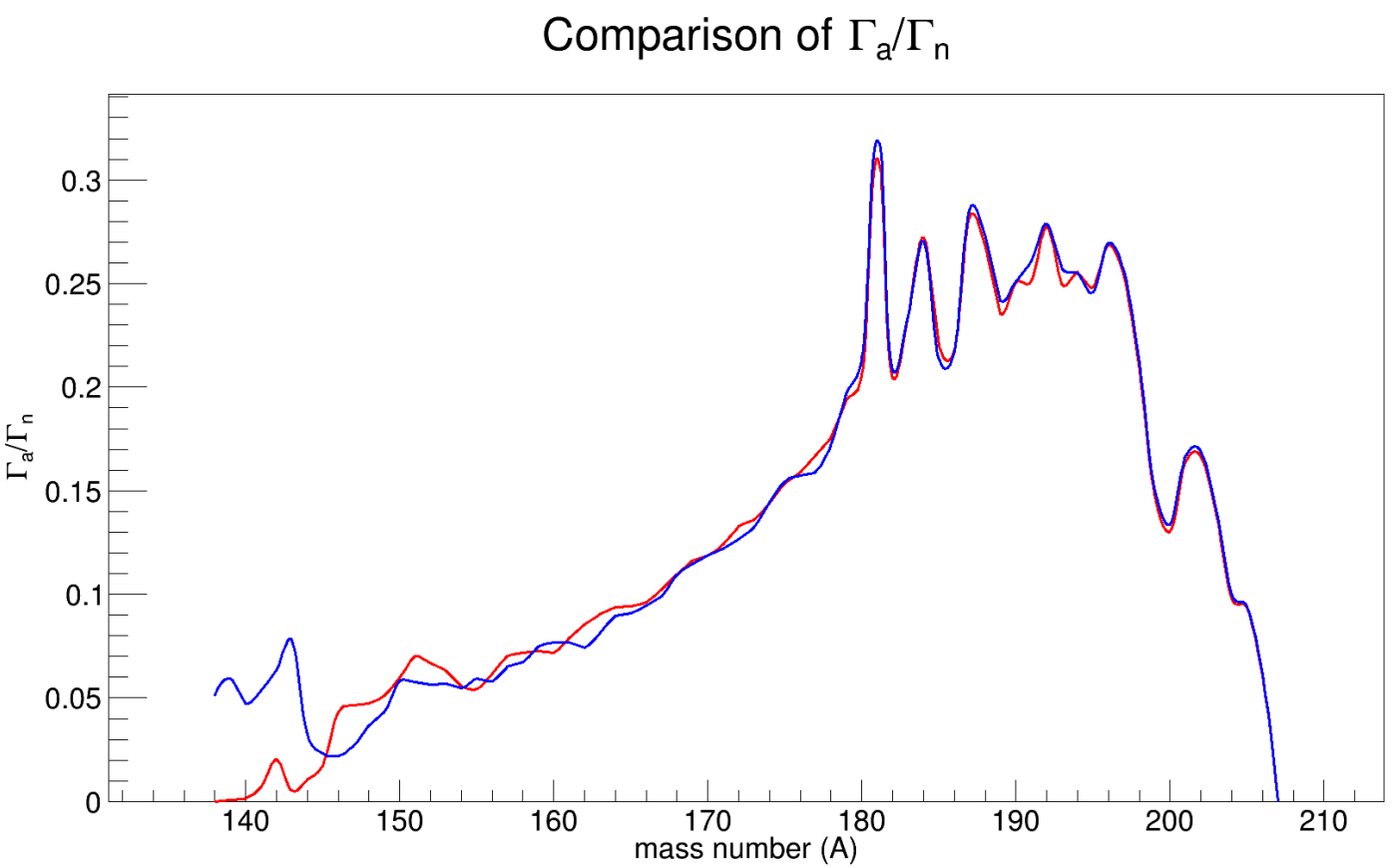




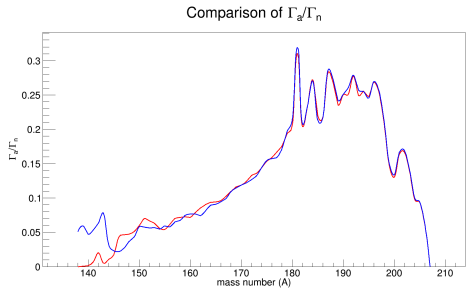
Γ_a/Γ_n

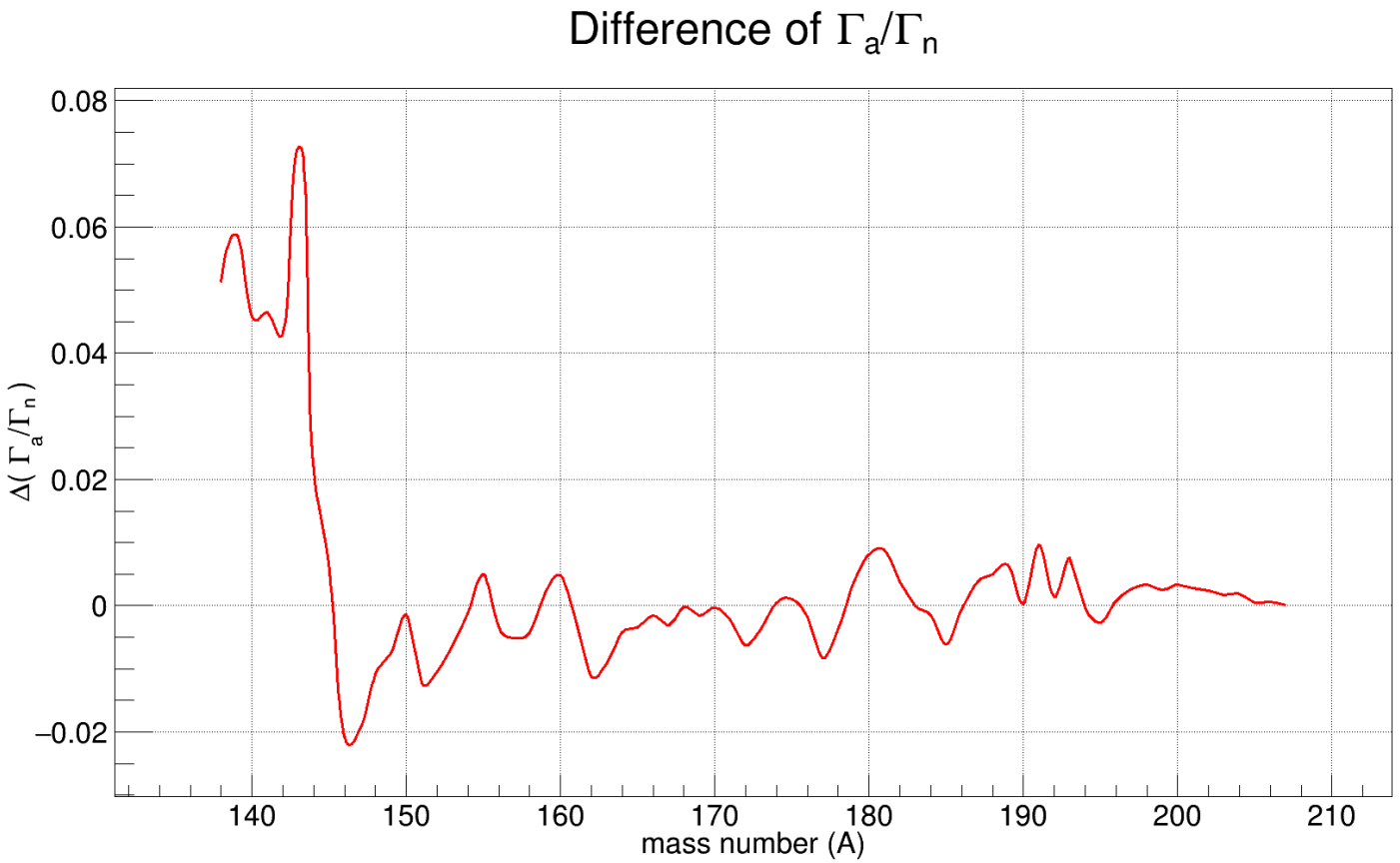
old formula

new formula



Γ_a/Γ_n

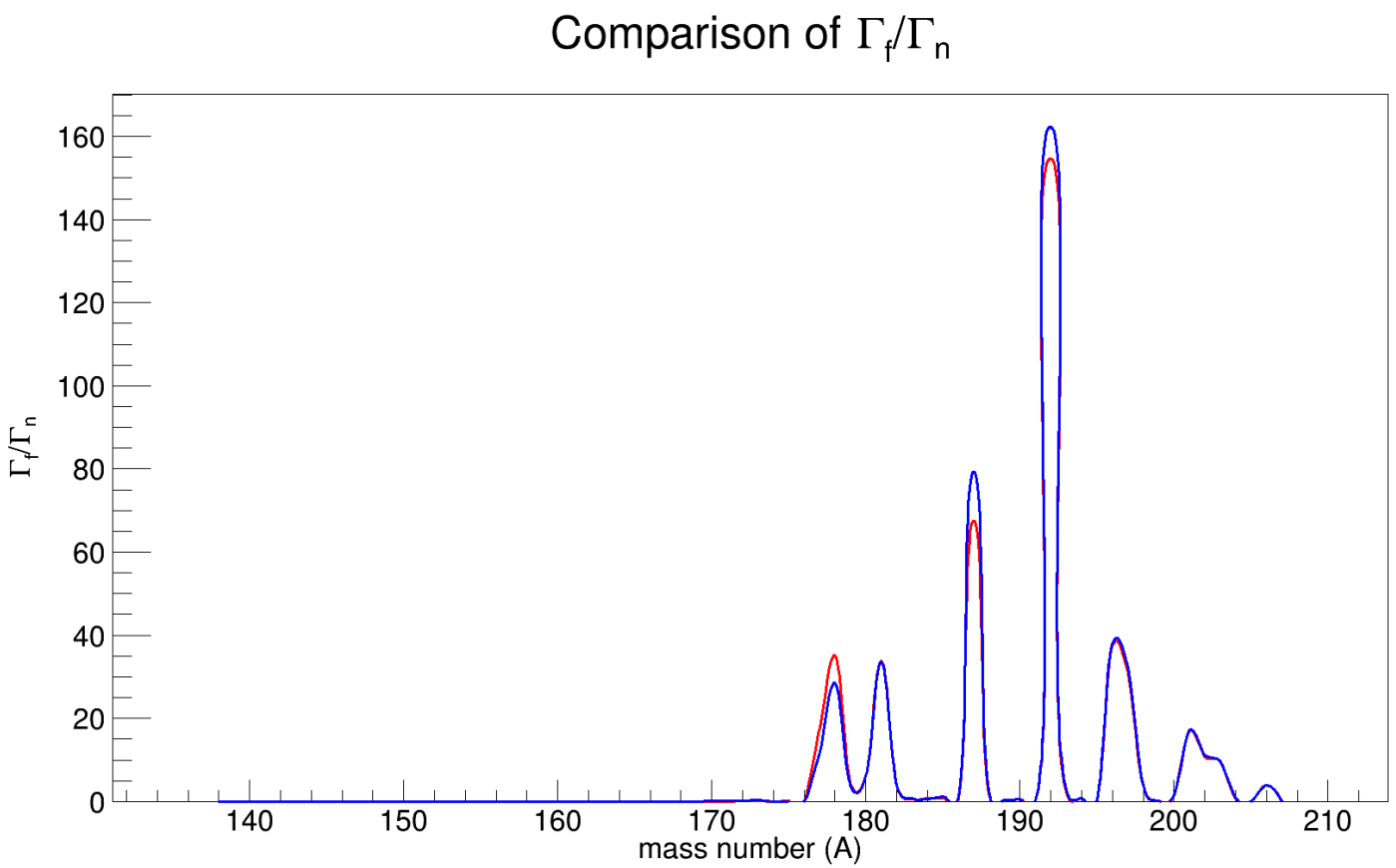




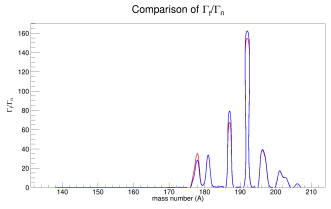
Γ_f/Γ_n

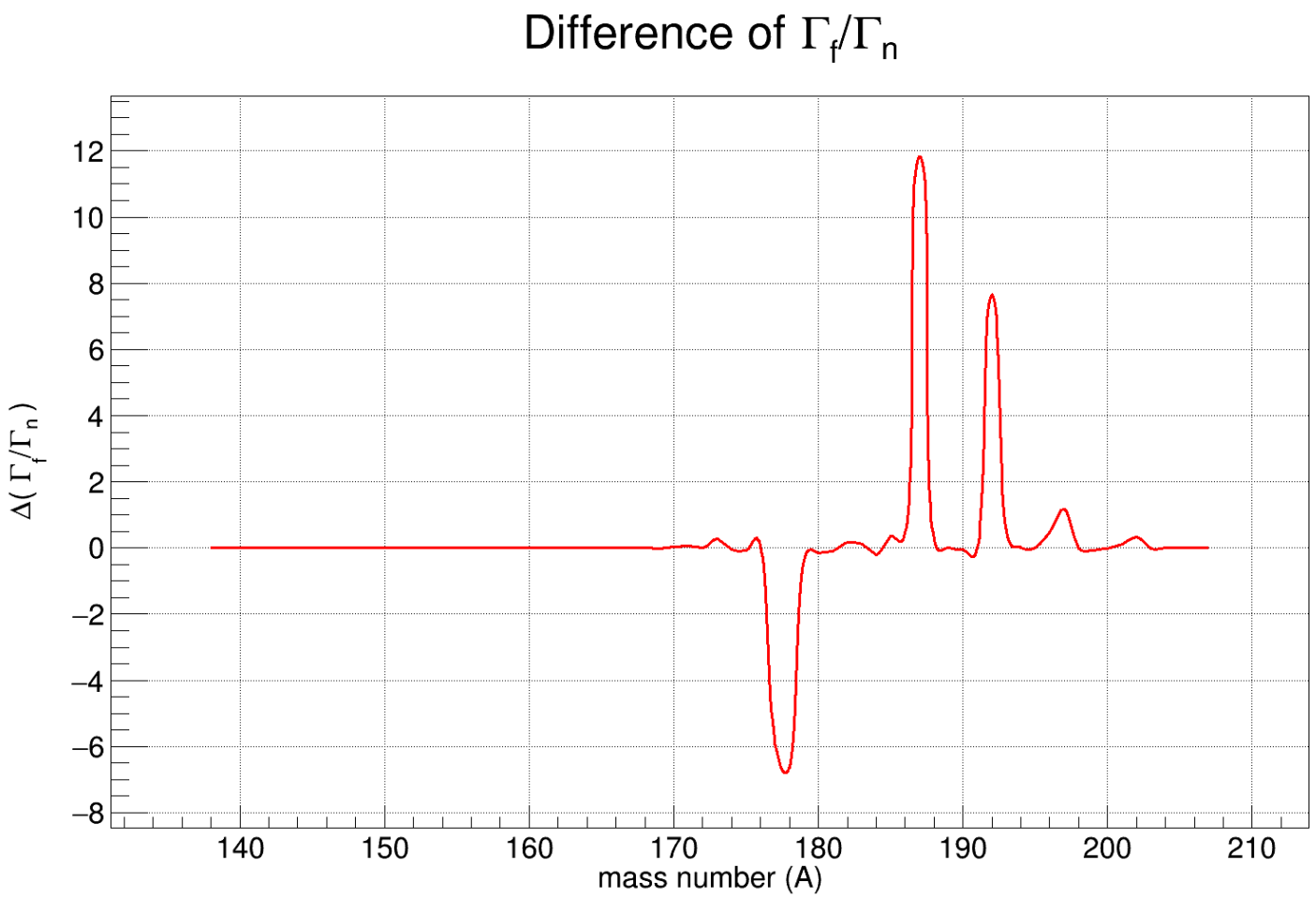
old formula

new formula



Γ_f/Γ_n





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