

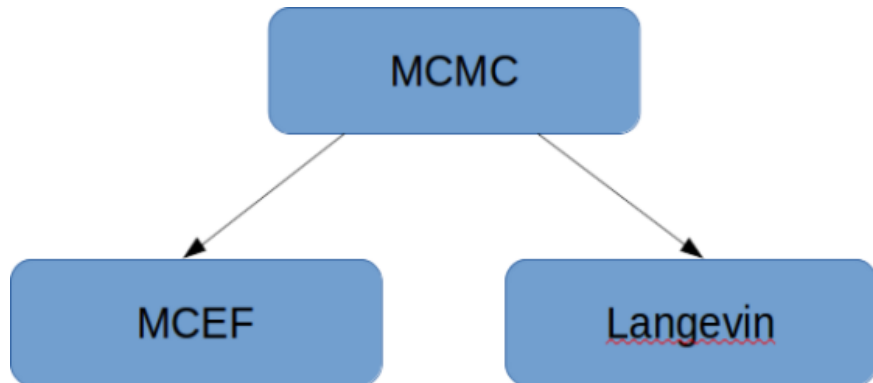
# Langevin description for evaporation-fission processes in CRISP code

Ramón Pérez

Universidade de São Paulo

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Langevin dynamic:

- The time evolution of the nucleus is considered and friction  $\beta$  parameter that has a direct influence in the fission probability and evaporation multiplicity.
- This study is more interesting for ion reaction, where a considerable angular momentum is transferred to compound nucleus.

# Parametrization (c,h)

- The following simple polynomial expression is in rather good agreement with the known results of exact LDM calculation:

$$\pi(u, v) = v^2 - (1 - u)^2(A + Bu^2 + \alpha u) \quad (1)$$

- Volume conservation:

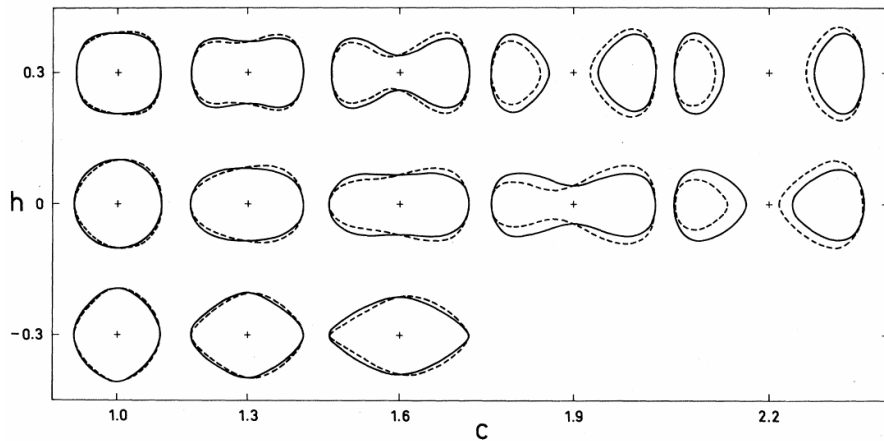
$$c = (A + \frac{1}{5}B)^{-1/3} \quad (2)$$

- Connection between the parameter sets A,B and c,h:

$$B = 2h + 1/2(c - 1) \quad (3)$$

$$A = \frac{1}{c^3} - \frac{1}{5}B \quad (4)$$

# Parametrization (c,h)



# Langevin equation

- Langevin equation for over-damped motion:

$$\dot{q} = -\frac{1}{M\beta} \frac{dV(q)}{dq} + \sqrt{\frac{T}{M\beta}} \Gamma(t) \quad (5)$$

- Langevin discretized form:

$$q_{n+1} = q_n + \left[ \sqrt{\frac{T(q)}{\beta(q)M} \frac{dS(q)}{dq}} \right]_n \tau + \sqrt{\left[ \frac{T(q)}{\beta(q)M} \right]_n} \tau \omega_n \quad (6)$$

$$q(c, h) = \frac{3}{8}c \left( 1 + \frac{2}{15}c^3 \right)$$

# Entropy and temperature

- Fermi's gas expression for the entropy:

$$S(q, E_{tot}^*, A, Z, L) = 2\sqrt{a(q, A)[E_{tot}^* - V(q, A, Z, L)]} \quad (7)$$

- The temperature is:

$$T(q, E_{tot}^*, A, Z, L) = \frac{S(q, E_{tot}^*, A, Z, L)}{2a(q, A)} \quad (8)$$



- The potential energy is given by the liquid-drop model:

$$V_s(q, A, Z) = a_2 \left[ 1 - k \left( \frac{N - Z}{A} \right)^2 \right] A^{2/3} [B_s(q) - 1] \quad (9)$$

$$B_s(q) = \begin{cases} 1 + 0.4(64/9)(q - 0.375)^2 & \text{if } q < 0.452 \\ 0.983 + 0.439(q - 0.375) & \text{if } q \geq 0.452 \end{cases} \quad (10)$$

$$a_2 = 17.9439 \text{ MeV} \quad k = 1.7826 \quad (11)$$

- The potential energy is given by the liquid-drop model:

$$V_c(q, A, Z) = c_3 \frac{Z^2}{A^{1/3}} [B_s(q) - 1] \quad (12)$$

$$B_c(x) = \begin{cases} 1 + \left(1 - B_s + \frac{B_f}{E_{ssp}}\right) / 2X & \text{if } q < 0.452 \\ 1 - 0.2 \frac{64}{9} (q - 0.375)^2 & \text{if } q \geq 0.452 \end{cases} \quad (13)$$

$$\frac{B_f}{E_{ssp}}(q) = \begin{cases} 0.2599 - 0.2151X - 0.1643X^2 \\ \quad - 0.0673X^3 & \text{if } q < 0.6 \\ 0.7259Y^3 - 0.3302Y^4 + 0.6387Y^5 \\ \quad + 7.827Y^6 - 12.0061Y^7 & \text{if } q \geq 0.452 \end{cases}$$

$$X = 0.05 \ln[0.875(q_{sd} - 0.375)^{-1} - 1] + 0.74 \quad (14)$$

# Deformation potential

- The potential energy is given by the liquid-drop model:

$$B_r = J_{\parallel}^{-1} \text{ if } J_{\perp} < J_{\parallel} \text{ and } q > 0.375$$

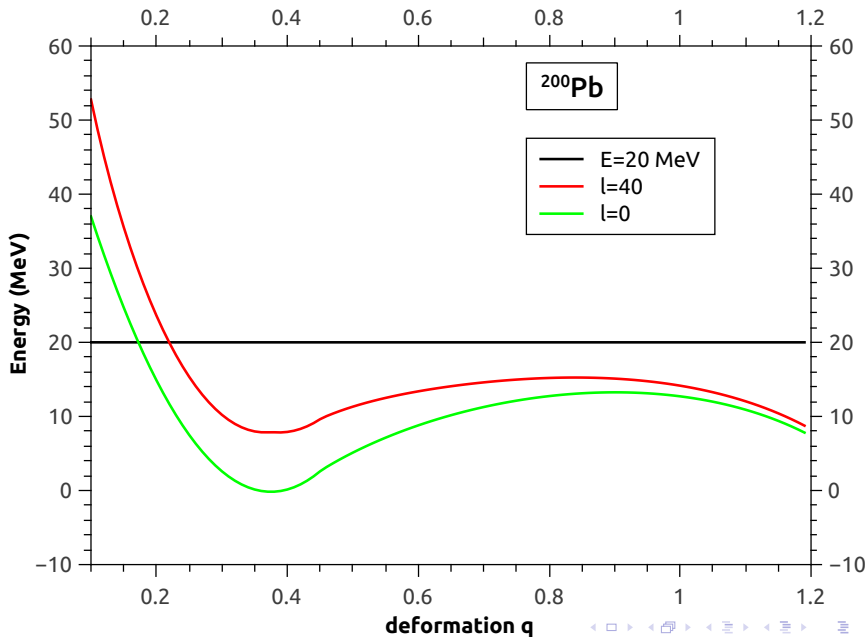
$$B_r = J_{\perp}^{-1} \text{ in all other cases}$$

$$J_{\parallel} = c^2 \left\{ c^{-3} + 4B_{sh}[(4/15)B_{sh}c^3 - 1]/35 \right\}$$

$$J_{\perp} = c^2 \left\{ 1 - c^{-3} + 4B_{sh}[(4/15)B_{sh}c^3 - 1]/35 \right\} / 2$$

$$B_{sh}(c, h) = 2h + (c - 1)/2$$

$$q(c, h) = \frac{3}{8}c \left( 1 + \frac{2}{15}c^3 \right)$$



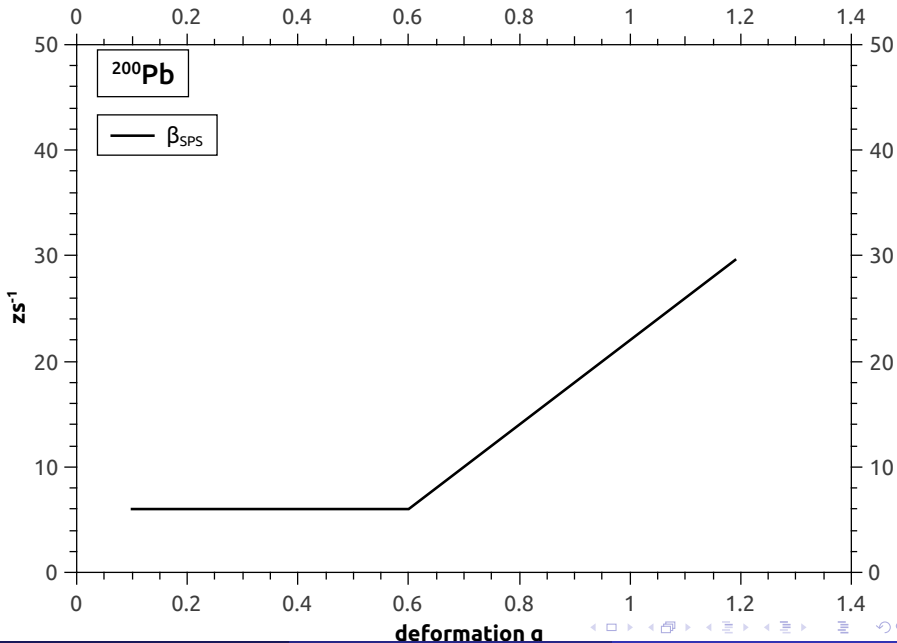
- Woods-Saxon single particle potential:

$$a(q, A) = \alpha_1 A + \alpha_2 A^{2/3} B_s(q)$$

$$\alpha_1 = 0.073 \text{ MeV}^{-1} \quad \alpha_2 = 0.095 \text{ MeV}^{-1}$$

- The Standard Parameter Set friction

$$\beta_{SPS}(q) = \begin{cases} \beta_0 & \text{if } q < q_{neck} \\ \beta_0 + \frac{\beta_{sc} - \beta_0}{q_{sc} - q_{neck}} (q - q_{neck}) & \text{if } q_{neck} \leq q \leq q_{sc} \end{cases}$$



- The emission width of a particle of kind  $\nu$  ( $\nu = n, p, \alpha$ ):

$$\Gamma_\nu = (2s_\nu + 1) \frac{m_\nu}{\pi^2 \hbar^2 \rho_c(E^*)} \int_0^{E^* - B_\nu} d\varepsilon_\nu \rho_R(E^* - \varepsilon_\nu) \varepsilon_\nu \sigma_{inv}(\varepsilon_\nu)$$

- Level densities of the compound and residual nuclei are:

$$\rho \sim (2L + 1) \exp(S)$$

- Inverse cross sections are:

$$\sigma_{inv}(\varepsilon_\nu) = \begin{cases} \pi R_\nu^2 (1 - V_\nu/\varepsilon_\nu) & \text{for } \varepsilon_\nu > V_\nu \\ 0 & \text{for } \varepsilon_\nu < V_\nu \end{cases}$$

- The barriers for the charged particles are:

$$V_\nu = \frac{(Z - Z_\nu)Z_\nu K_\nu}{R_\nu + 1.6}$$

$$R_\nu = 1.22 \left[ (A - A_\nu)^{1/3} + A_\nu^{1/3} \right] + \frac{3.4}{\varepsilon_\nu^{1/2} \delta_{\nu,n}}$$

$$K_\nu = 1.32 \text{ for } \alpha \quad K_\nu = 1.15 \text{ for proton}$$



- For giant dipole  $\gamma$  – *quanta*:

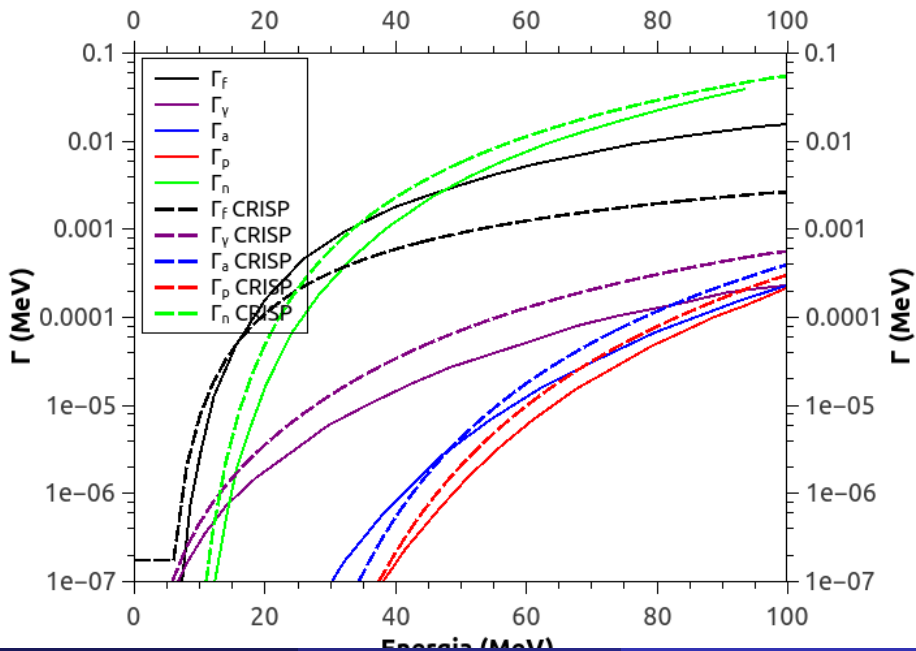
$$\Gamma_\gamma = \frac{3}{\rho_C(E^*)} \int_0^{E^*} d\varepsilon \rho_C(E^* - \varepsilon) f(\varepsilon)$$

$$f(\varepsilon) = \frac{4}{3\pi} \frac{1+k}{m_n c^2} \frac{e^2}{\hbar c} \frac{NZ}{A} \frac{\Gamma_G \varepsilon^4}{(\Gamma \varepsilon)^2 + (\varepsilon^2 - E_G^2)^2}$$

with  $k=0.75$ ,  $E_G=80/A^{1/3}$  MeV,  $\Gamma_G=5$  MeV

- For fission:

$$\Gamma_f = \hbar \frac{T_{gs}}{\beta_{gs}} \frac{\sqrt{|S''_{gs}| S''_{sd}}}{\pi M} \exp(S(q_{sd}) - S(q_{gs})) \times \\ \times \left\{ 1 + \operatorname{erf} \left[ (q_{sc} - q_{sd}) \sqrt{S''_{sd}/2} \right] \right\}^{-1}$$

$^{226}\text{U} (l=0)$ 

- The total initial excitation energy is:

$$E_{tot}^* = E_{cm} + Q$$

where:

$$E_{cm} = E_{lab}A_T/(A_T + A_P)$$

- The angular momentum  $L = \hbar l$  is sampled from the spin distribution:

$$\sigma(l) = \frac{2\pi}{k^2} \frac{2l + 1}{1 + \exp[(l - l_c)/\delta l]}$$

- The quantity  $l_c$  scales as

$$l_c = \sqrt{A_P A_T / A} \left( A_P^{1/3} + A_T^{1/3} \right) \left( 0.33 + 0.205 \sqrt{E_{cm} - V_c} \right)$$

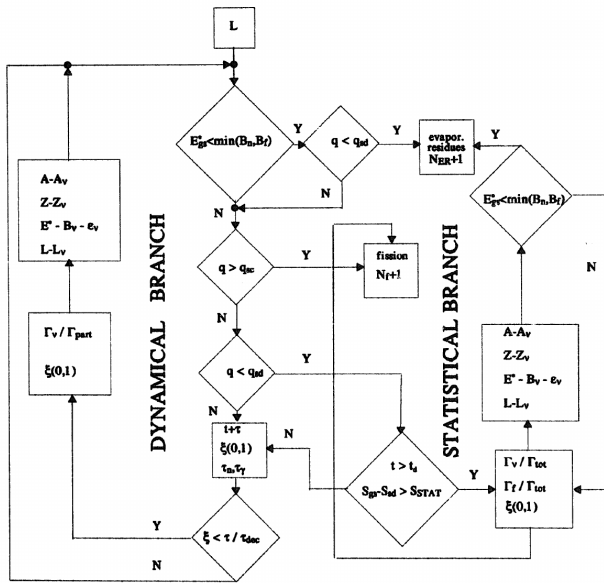
- The barrier  $V_c$  is:

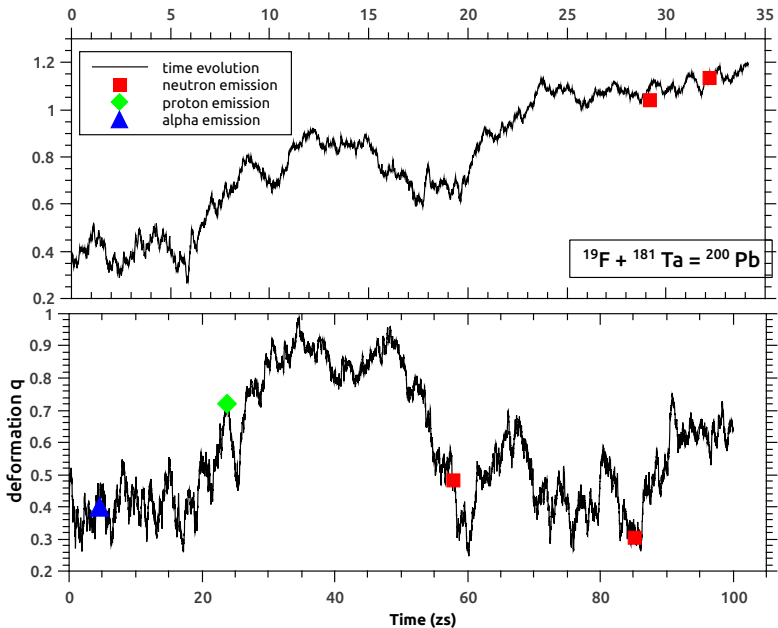
$$V_c = 5/3c_3Z_PZ_T / \left( A_P^{1/3} + A_T^{1/3} + 1.6 \right)$$

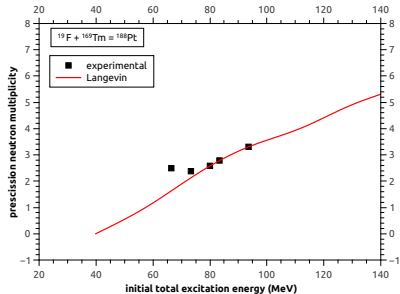
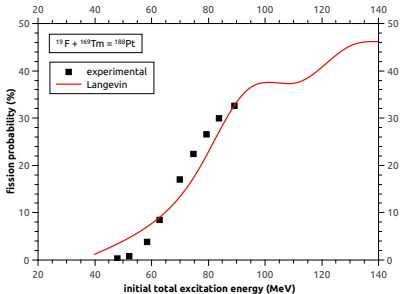
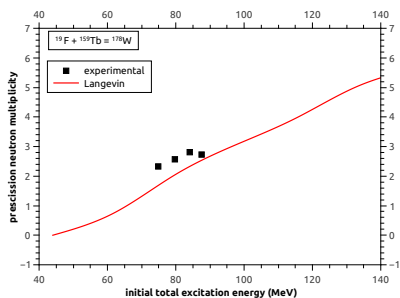
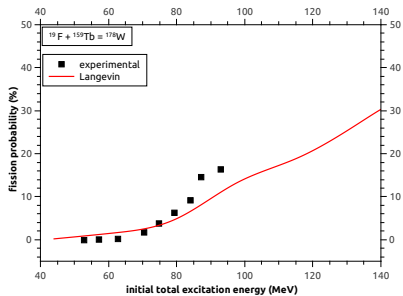
with  $c_3 = 0.7053$  MeV

- The diffuseness  $\delta l$  is:

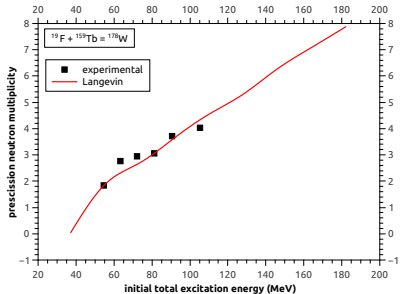
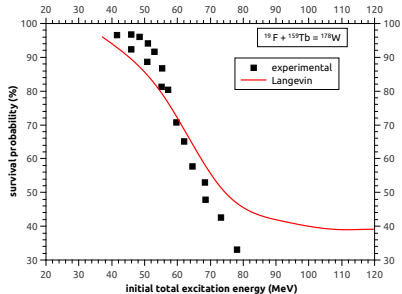
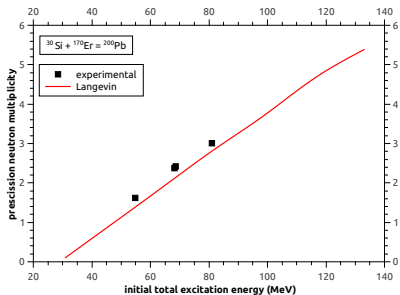
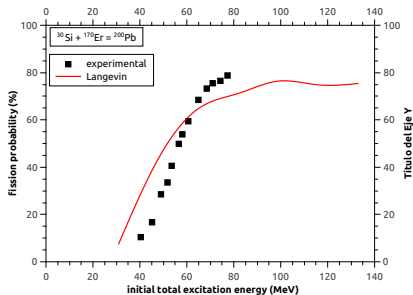
$$\begin{cases} (A_P A_T)^{3/2} * 10^{-5} [1.5 - 0.02 (E_{cm} - V_c - 10)] & \text{for } E_{cm} > V_c + 10 \\ (A_P A_T)^{3/2} * 10^{-5} [1.5 - 0.04 (E_{cm} - V_c - 10)] & \text{for } E_{cm} < V_c + 10 \end{cases}$$

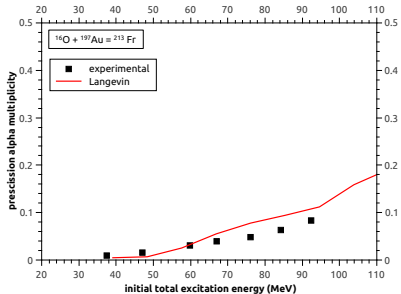
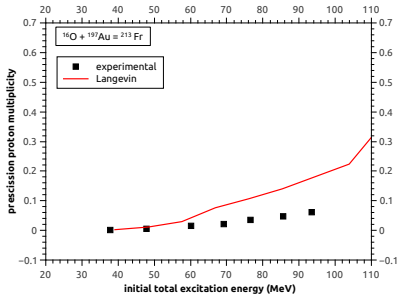
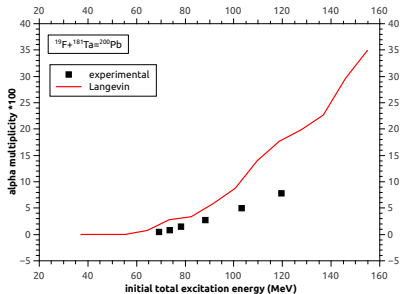
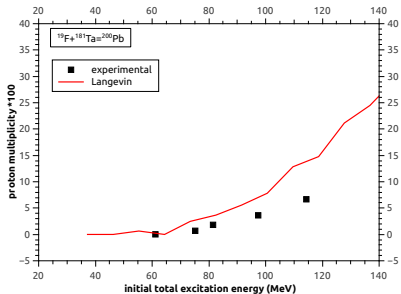


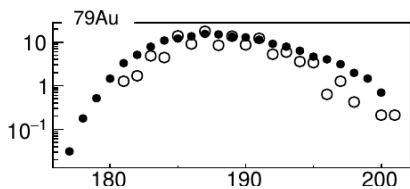
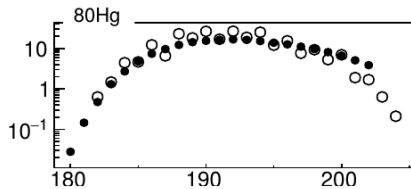
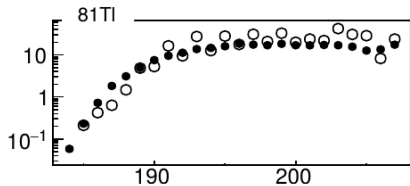
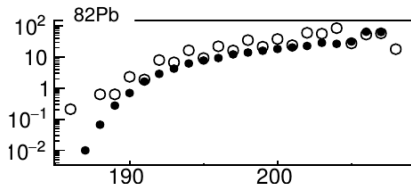


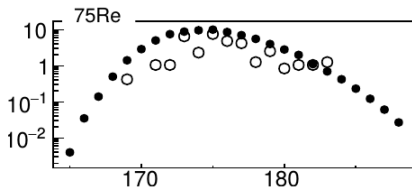
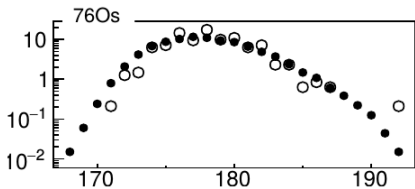
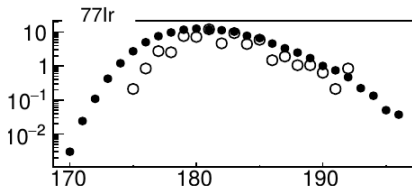
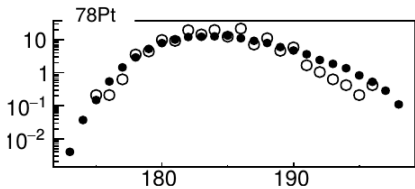


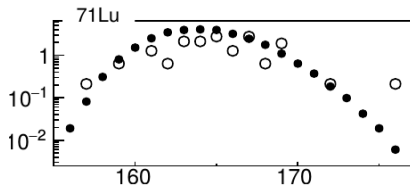
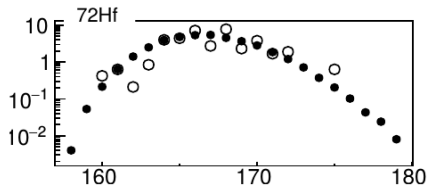
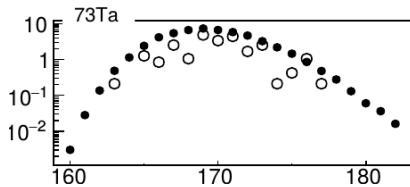
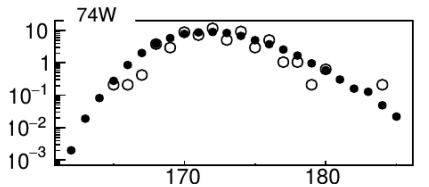












# Conclusions

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Thanks