PREY-PREDATOR MODELING OF CO2 ATMOSPHERIC CONCENTRATION

Luis A. Trevisan[1], Fabiano Meira De Moura Luz[1],Moises de Souza Santos[1]

Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84010-790, Ponta Grossa, PR, Brazil

Keywords:CO2 atmospheric concentration,Photosynthesis rate ,Prey Predator model, vegetal death rate *abreviação do título: PPMCa.*

Secao academica: Ciencias da Terra e Matematica.

autor correspondente: Luis Augusto Trevisan, luisaugustotrevisan@yahoo.com.br, 042-32203050

ABSTRACT

In this work we propose a mathematical model, based in a modified version of the Lotka-Volterra prey-predator equations, to predict the increasing in CO2 atmospheric concentration. We consider how the photosynthesis rate has changed with the increase of CO2 and how this affects plant reproduction and CO2 absorptions rates. Total CO2 emissions (natural and manmade) and biomass numerical parameter changes are considered. It is shown that the atmospheric system can be in equilibrium under some specific conditions, and also some comparisons with historical data and predictions are done. A striking feature of the model is to adjust data with a small number of parameters.

I. INTRODUCTION

Several billion of years ago, special terrestrial conditions made the formation of life possible. Since its formation, the biosphere has played an active role in controlling environmental conditions. An interaction has developed between the evolution of living species and the environment. Changes in environmental conditions modified the biosphere and vice-versa. When life began, the Earths atmosphere was not similar to the present air. The main peculiarity of our atmosphere - the presence of oxygen is the result of the biospheres evolution (Meszaros 2000). Natural changes in atmospheric composition and climate are slow processes when compared with typical human time scales. During the last 8.000 to 10.000 years, the climate has been stable. Such stability has been favorable for humans and made social and economic development possible. In the present industrial era, this development has reached such a level that human activities have become able to modify environmental conditions on a time scale ($\gg$ 100years) that is quite shorter than periods of natural changes. It is well known that the CO2 emission due to human activities has contributed to cause the greenhouse effect, a warning in the Earth mean temperature. Another important effect of the increase of the CO2 concentration is acceleration in the photosynthesis rate, a subject widely studied (Kirsshbaum 1994)(Laisk and Edwards 2000). With a higher photosynthesis rate, plants can absorb more CO2, and have a faster growing and reproduction rate, again consuming more CO2. In this work we present a model to predict the increase of CO2 atmospheric concentration, considering the plant-atmospheric carbon interaction. Basically, we consider the increase of photosynthesis rates, that is a consequence of the increase of Ca (CO2 atmospheric concentration), and apply a modified version of Lotka-Volterra (Lotka 1931) predator-prey model to describe a possible time evolution of Ca (the concentration of CO_2 in the atmosphere, given in part per million,ppm) and a function of plant biomass P. This function includes all life beings that use photosynthesis, even if they are not vegetables. In the section method we show the adapted Lotka-Volterra model, in the section results there are comparisons with the data. In discussion some points to be improved are considered. The main goal of the present paper is to show that the adapted prey-predator model works.

II. METHODS

The original model by Lotka-Volterra (Lotka 1931) was proposed in the twenty's, and have been widely used in ecological studies and also in another fields of knowledge. In our model, adapted to the problem of plants CO2 interaction, we will consider the CO2 as the inorganic prey (not reproducing) and plants are predator. The CO2 molecule does not reproduce by itself, therefore its concentration evolution, denoted by Ca (in ppm), depends only on emissions (i.e. combustion reactions) denoted by Q(t), and absorptions (i.e photosynthesis), assumed to be proportional to vegetal biomass surface, denoted by P(t) and also to the relative photosynthesis rate A(Ca , T), where T is the temperature. The relative (in comparison with the maximum possible) photosynthesis rate is given, at some temperature is given by (Kirshbaum 1994)

$$A(Ca,T) = \frac{V_j(C_a - 1.5\Gamma_*)}{C_a + 3\Gamma_*}$$

where A is the relative photosyntheses rate (in comparison to the possible maxima value, at a given temperature.We choose $V_j = 1$. The Γ factor depends on the foliar temperature T, in the following way,

$$\Gamma_* = 42e^{9,46(T_l-25)/(T_l+273.2)}$$

where 42μmol/mol is the value of Γ_* at 25^oC. In this way, we have the set of equations to be solved:

$$\frac{dC_a}{dt} = -A(C_a,T) * P + kQ(t) \tag{1}$$

$$\frac{dP}{dt} = -e * P + f * P * A(C_a,T) \tag{2}$$

To estimate k, we consider the work by P.Tans, F.Y.Inez and T.Takahashi (Tans et al. 1990) , that studied emissions and concentration during the 80s, the value is obtained considering the annual average increase on concentration of 1.4ppm and difference between emissions and absorption, which the mean value is 3GtC/year. The ratio (1.4/3.0) reads k= 0.47. This means that for each 1GtC emitted more than consumed , the Ca increases 0.47 ppm. We also consider that the oceans are a balanced system, that absorbs all the CO2 they emit and is also able to absorb around 30% of human emission (Sabine et.al 2004).

The Global emission of CO_2 has 3 sources : the ocean- which contributes with 100GtC, but absorbs all; the terrestrial, divided in natural , with 90GtC and human, variable. We also consider that the oceans are also able to absorb around 30% of human emission (Sabine et.al 2004). Equation 1 says that the growing of the numerical value for P is proportional to itself and also to photosynthesis rate. The last equation is rewritten The last equation is rewrite:

$$\frac{dP}{dt} = f * P(-b + A(C_a, T))$$

where $b = e/f$, $0 \leq b \leq 1$. And separating the sources,

$$Q(t) = Q_n + Q_h(t)$$

where Q_n are the natural emissions of CO_2 and $Q_h(t)$ are the one made by man. Note that if

$$b = A(C_a, T)$$

the derivative is zero, so we get a stabilized system.

We have important remarks about the balance in the system. First, we note that if the system is balanced, $b = A(Ca, T)$, the photosynthesis rate is equal to vegetable death rate (b factor). Then, a condition to the system get balanced, after sometime is

$$b < 1$$

In fact, if $b > 1$ the system will never be stabilized, because the derivative of the vegetal population parameter always will be negative. This fact is explored in this work, to show how we can use the parameter b to study possible stabilization scenarios in the atmospheric CO2 concentration. The b factor is time dependent (more exactly, depends on the historical moment) and is also related to the human emissions, since some fraction of the vegetable death and human emission are due to the same fact: burning. Deforesting and urbanization also contribute to increase the b factor. In this way

$$b \rightarrow b(t)$$

Besides, the f value can be used to help to fit desirable value of the derivative. In this way, the model has only two free parameters, b(t) and f that could be estimate using environmental data.

To better understand the role of the b factor, we should observe the figures 2 below: In figure 2, we use the same human emissions (Qh = 7.5GtC/Y, which taking into account ocean absorption, should be read 0.7x7.5=5,25GtC/Y). In cases 1, 3 and 5 we have used the same f value (f =0.0185), the value of the b factor is supposed constant and fitted to given three different final CO2 concentrations; 500ppm; 600ppm and 700ppm, corresponding to b=0.70; b=0.74 and b= 0.77 respectively. The initial concentration is Ca = 345ppm. Therefore there is a relationship between the vegetal death rate (b) and the final carbon dioxide atmospheric concentration (Caf):

$$b = A(Caf, T)$$

Besides this, in curves 2 and 4 , we have used the same b value of curve 3, b=0.74, with different values for f (f= 0.03 in curve 2 and f=0.01 in curve 4, while f=0.0185 in curve 3). We observe that, using the values f=0.0185, and b=0.74 (which corresponds to final concentration of 600ppm) we may fit the data between 1985 (345ppm, jan) [8] and 1995 (360ppm, jan) .

We emphasize that one important difference of the present model to the IPCC (www.ipcc.ch) is the way the absorptions are considered The system can reach a stable concentration even with constant emissions when the b factor is less than unity (the greatest value the relative photosynthesis rate can reach)

III. RESULTS

The model parameters are adjusted to give results near the data of CO2 concentration in the 1800- 2000a.c. period, when the concentration increased from approximately 282 ppm (Neftel et al 1985) up to 370 ppm (2000), and with the recent annual average increase of 1.4 ppm (in the 80s)(Tans,Ynez,Takahashi 1990). We also suppose the human emissions (Qh) to have an exponential growing, from nearly zero up to the present level. The function to represent the emissions is obtained from the data of CDIAC (Citar ref), with 20 points for each century (we have used the BRoffice). The emission function is:

$$Qh = 0.01 * 1.04^t$$

where $t = year - 1800$.

The initial b value is: bi = A(282 ppm;25C) , that is, the initial b factor was equal to photosynthesis rate because, at that time, the system has been supposed to be balanced. And the final b value, combined with emissions, gives the increase of concentration that is close to 1.4 ppm/year, at 80s and a concentration near to the present one.

$$b = b_i * exp[t * ln(b_f/b_i)/195]$$

so, if t = 195, $b = b_f$; and, if $t = 0$,$b = b_i$. The value for b_f and f are also obtained by the statistical methods, minimizing the χ^2. As an important information, in the model, the natural emission (Qn) and absorption of CO2 are initially supposed equal, 90GtC/year (the same of nowadays), where we consider only terrestrial absorptions and emissions. The absorptions by the oceans may occur also due to physical-chemical process, not only by photosynthesis We consider that oceans take around 30% of human emissions, so we multiply the human emissions by 0.7. The initial value for the P parameter is fixed by:

$$P = k90/A(282, 25)$$

The value of $T = 25$ is an anzats, it may represents the mean temperature in which photosyntheses may occurs. This temperature must be different of the mean temperature of the planet. The results are $b_f = 0.579$, $f = 0.0822$ and $\chi^2 = 1.06603$.

In the figure 4 below, we fit a short time evolution for the concentration, considering the mean value of emissions (9.43 GtC/y)- according to the scenario A1 AIM proposed by the IPCC for the decade 2000-2010, ocean absorption around 30% of the human emission. The A1 storyline and scenario family describes a future world of very rapid economic growth, global population that peaks in mid-century and declines thereafter, and the rapid introduction of new and more efficient technologies. The comprehensive description of all scenarios may be found at http://www.ipcc.ch/pdf/special-reports/spm/sres-en.pdf. The initial value for the vegetal parameter is calculated in a way to fit the variation rate of the CO_2 equal to the data. To turn it explicit, the computer code lines are copied below: 1- The natural emissions, including the k factor: Qn= 90*(1.4)/(3.0)=42 2-The human emission, considering the IPCC scenario A1 AIM in 1999 qh=9.43*(1.4/3.0) 3- The CO_2 consumption consu=Qn +0.7*qh -1.17 where the factor 1.17 is the growing of the CO_2 atmospheric concentration between 1999 and 2000. The initial CO_2 atmospheric concentration is CO2i=368.31

The initial photosynthesis rate A= (CO2i-1.5*42.0)/(CO2i+3.0*42.0)

year	Ca Model constant emission	Ca Model extrapolation	Ca Data	Linear
1990	352.48	353.46	354.88	353,24
1995	360.93	360.84	360.88	362,09
2000	370.10	369.26	369.48	370,94
2005	379.78	378.94	379.91	379,79
2010	389.83	390.13	389.68	388,64
2015	400.15	403.16	-	397,49
2020	410.67	418.43	-	406,34
2025	421.32	436.46	-	415,19

TABLE I: Ca predictions, according to the present model and historical data. The supposed model conditions are $qh = 7.53$GtC/y between 1990 and 2000, and $qh = 9.43$ between 2000 and 2025 (first column); on the second column, the conditions used to generate the figure 3 are considered on the extrapolation, on the third column, we show the observed data. We notice that there is an almost linear evolution. To compare with the linear approx and extrapolation, given by Ca= 1,77*(year-1990)+ 353,24, it's included the last column

P= consu/A

vegetal death rate b=0.683 and f =0.100 and $\chi^2 = 0.02814$

The dashed line in Fig. 3 corresponds to an exponential increase in the emissions, takes into account a variation in the absorptions due the increase of the b factor, and also the increase of the photosynthesis rate. So with the b factor being "'time dependent"', growing exponentially from the equilibrium with photosynthesis rate to the present value , constant f parameter and with simulated exponential emissions (not free, they should comes from historical data) we obtain a smooth fit to historical data. We must note, in the historical data, the period 1938-1946, when a constant value of Ca is observed. Another important point is that only with raising emissions it is not possible to fit the data curve completely. Only considering variations in absorptions, due the vegetable death, the curve may be reasonably fitted. This means, for example, that burning forest contributes twice, by one side CO2 concentration increases due to emissions (burning) and by the other side it also increases due to reduced absorption (deforesting). As an additional remark about the features of the model, we notice that the parameters to fit short range data may be different to those needed to fit a long historical period, this is intrinsic to the non-linearity of the set of coupled equations.

IV. DISCUSSION

In this work we presented a mathematical model, based on prey-predator equations, to estimate the time evolution of the atmospheric CO2 concentration. We consider the effect of a growing photosynthesis rate when CO2 atmospheric concentration increases. In order to simplify, we study only one kind of photosynthesis (C3) and the ambient temperature was kept constant. Using the photosynthesis rate function in a adapted Lotka-Volterra system of equations, where CO2 where prey and plants are predators we simulated several scenarios and show that the parameters can be choice to reproduce the historical data and the made some predictions. It is important to note that our model has only two free parameters f (related to the derivative dP/dt) and the most relevant b, vegetable death rate. We note that if $0 < b < 1$ and when $A(Ca, T) = b$ we have equilibrium. That is, the photosynthesis rate in the equilibrium is equal to vegetable death rate. This is the reason because it is easy to fit the initial an final values in the model. With f we fit the derivatives, and with b the end concentration (stabilization). We also observe that the model consider a different way to compute only the absorptions of CO2, keeping the emissions as an external input. The model in fact, is an initial value problem, where the parameters (b,f) are used to fit the initial values of the functions (P,Ca) and the respective derivatives. The IPCC prediction for 2010,made at 1990, under the scenario bussines as usual was $Ca = 390$ ppm. This is also the result obtained on the present work. Here is a crucial point to be discussed in a future work: the difference between the adapted prey-predator model and the IPCC model, also known as Bern model(Siegenthaler and Joos, 1992). The detailed comparison will be presented in a further work. As another future extension of this study, we intend to consider other effects, like temperature variation, different photosynthesis models and climate forcing. Applications of the model on the different scenarios are a very issue problem and will be present soon. In conclusion, this very simple model, with only two free parameters and equations (or eventually, exponential functions to emission and vegetal death rate) works very well on reproduce the historical data.

V. RESUMO E PALAVRAS-CHAVE

Neste trabalho fazemos uma adaptação do Modelo presa-predados de Lotka-Volterra para o sistema em que plantas são predadoras do CO_2 atmosférico. A quantidade de CO_2 absorvido pelas plantas é proporcional à taxa de fotossíntese. Com este modelo e informações adicionais sobre a emiss

ao de CO_2 na atmosfera e absorção pelos oceanos, conseguimos ajustar uma curva com as concentrações entre 1800 e 2000 e ainda fazer predições sobre as concentrações em um futuro próximo, desde que os cenários supostos se matenham. Palavras-Chave: Concentração do

CO_2 atmosférico, emissão de CO_2, modelo presa-predador, taxa de fotossíntese.

VI. ACKNOWLEDGMENTS

The authors thank Prof. R. A. Clemente for criticism and suggestions. F.M.M. Luz thanks the State University of Ponta Grossa for financial support.

VII. REFERENCES

CDIAC- Carbon doixide information analysis -www.cdiac.ornl.gov

Keeling C.D., Bacastow R.B ., Carter A. F., Carter S. C., Whorf T.F. , Heimann M.., Mook W.G ., and Roeloffzen H (1989) - DC (1989) Atmospheric carbon dioxid record from Mauna-Loa. Geophysical Monograph, 55, AGU, Washington, 165

Kirschbaum, M.U.F (1994). The sensitivity of C3 photosynthesis to increasing CO2 concentration: a theoretical analysis of its dependence on temperature and background CO2 concentration Plant, Cell and Environment.,17-7:747

Laisk A , Edwards G.E ,. (2000) A mathematical model of C4 photosynthesis: The mechanism of concentrating CO2 in NADP-malic enzyme type species . Photosynthesis Research 66 N3 , 199 and references therein

Meszaros E. , Global and Regional changes in Atmospheric Composition (2000) Lewis publisher, Boca Raton, Pg 1,2 15 Neftel, A., Moor, E., Oeschger, H. and Stauffer, B. (1985) Vostok Ice core provides 160000 year record of atmospheric CO2 Nature 315, 45-47

Sabine C.L et al, , (2004) , The Oceanic sink for anthropogenic CO2 Science 305, n 5682 , 367- (2004)

Siegenthaler U., Joos F., Use of a simple model for studying oceanic tracer distribuition and global carbon cycle, Tellus,44B, 186-207,1992.

Tans P. P. , Inez F.Y. , Takahashi T , (1990) - Observational Contrains on the Global Atmospheric Co 2 Budget Science 247 , . no. 4949, pp. 1431 - 1438 DOI: 10.1126/science.247.4949.1431

Volterra A. , V.,Mark Kot (editor) (1931) Variations and fluctuations of the number of individuals in animal species living toghether. In Animal ecology, MacGraw-Hill , Elements of Mathematical Ecology; Cambridge University Press

Wahlen M,.,.Fischer H., Smith J., Mastroianni D. ., Deck B. ,., 2000, Vostok Ice Core CO2 Data, 1105-2856m., IGBP PAGES/World Data Center-A for Paleoclimatology data Contribution Series 2000-003. NOAA/NGDC Paleoclimatology Program, Boulder CO, USA